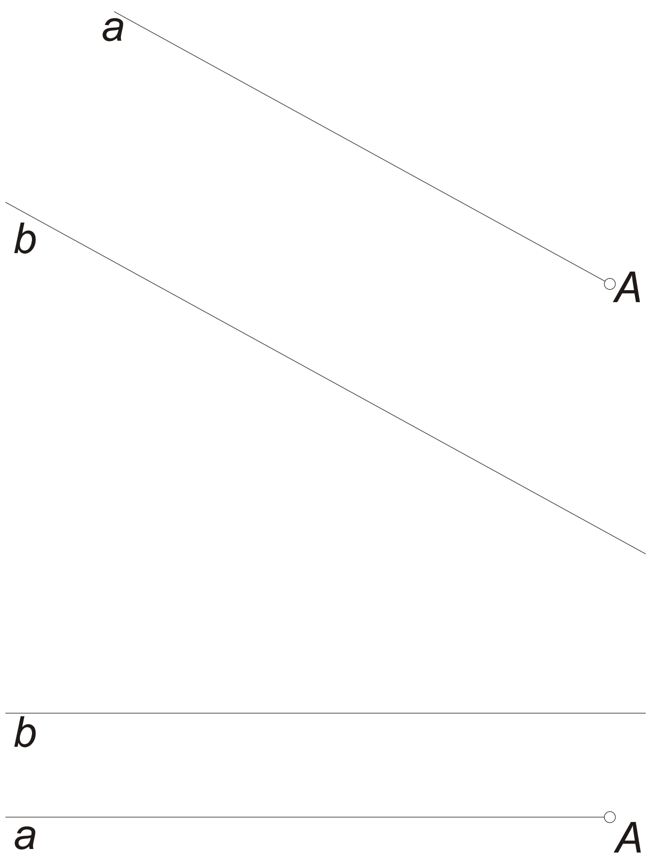
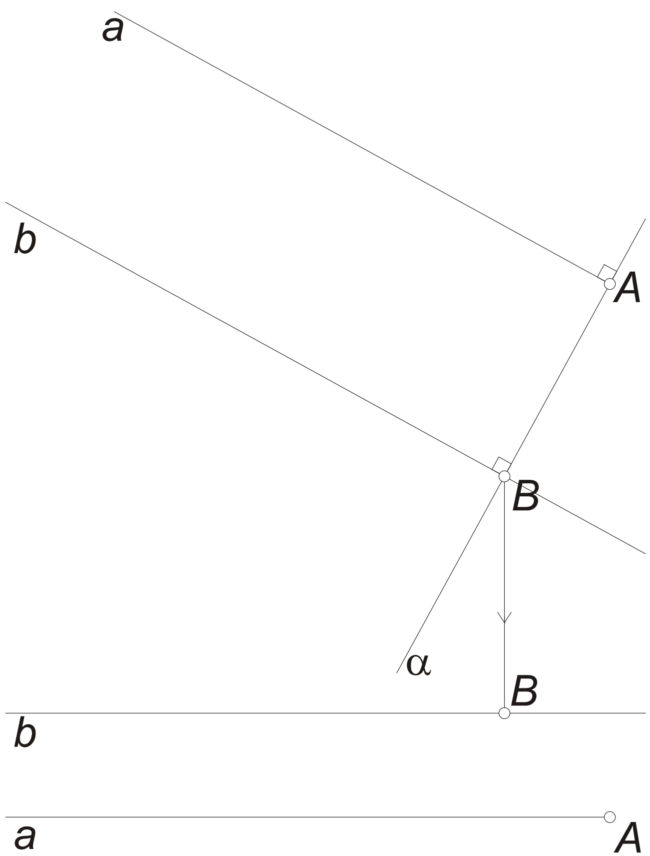
Metric construction

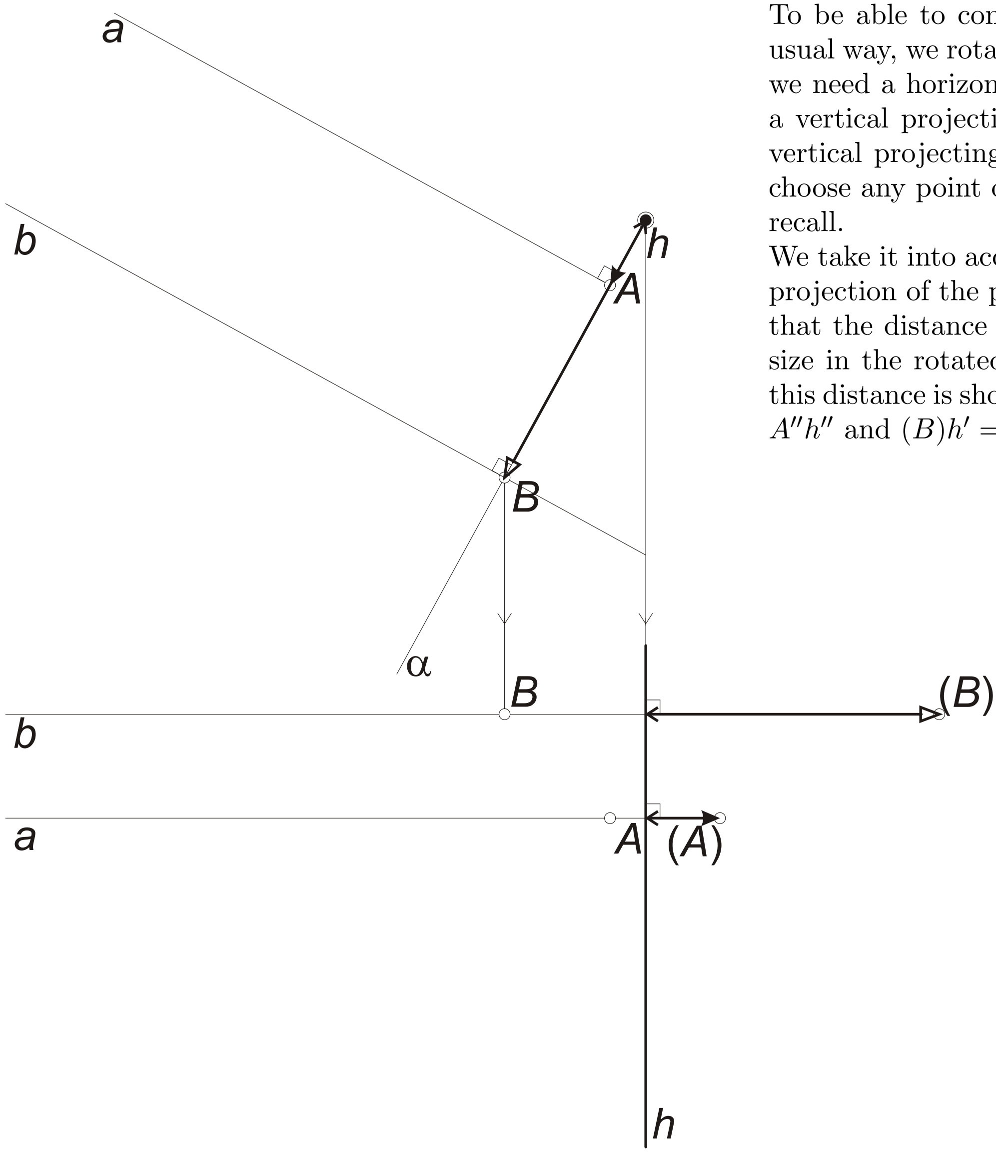
Construction of a pentagon-based regular prism with parallel lines as its lateral edges



Exercise. *Given the vertical half line a emanating from A, and the line b, parallel to a. Construct the pentagon-based regular prism* $ABCDEA_xB_xC_xD_xE_x$ *if its edge* AA_x *lies on* a*, and* BB_x *is contained in* b*. The height of the prism is equal to the length of the diagonals of the base. From amongst the geometrically feasible solutions, choose the one on the lower level in space. Show visibility assuming that the prism is a solid.*

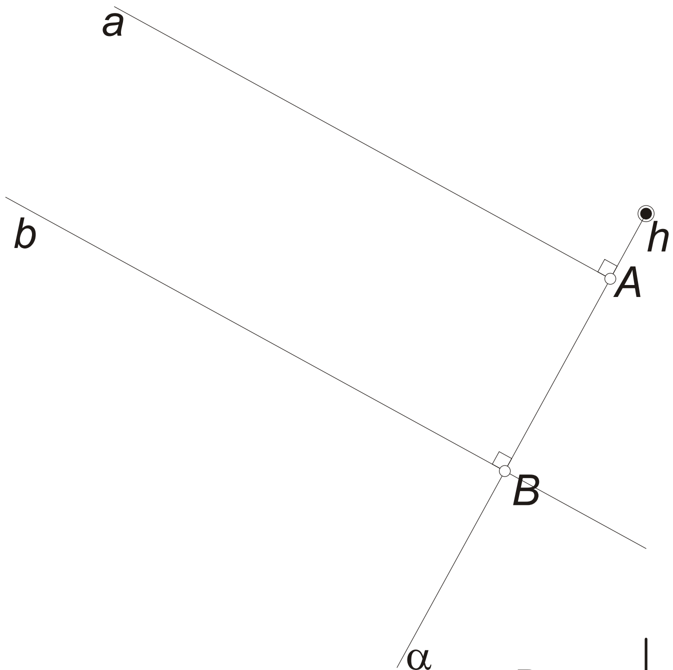


Since the prism is regular, its base $ABCDE$ is perpendicular to the direction of the lateral sides, and thus to a Therefore, as a is a vertical half line, the plane α of the base is a vertical projecting plane, and $\alpha'' \perp a''$.
We draw α'', using this property and the fact that α contains A, which intersects b'' at the vertical projection B'' of the vertex B. We obtain B' by drawing the line of recall of B.

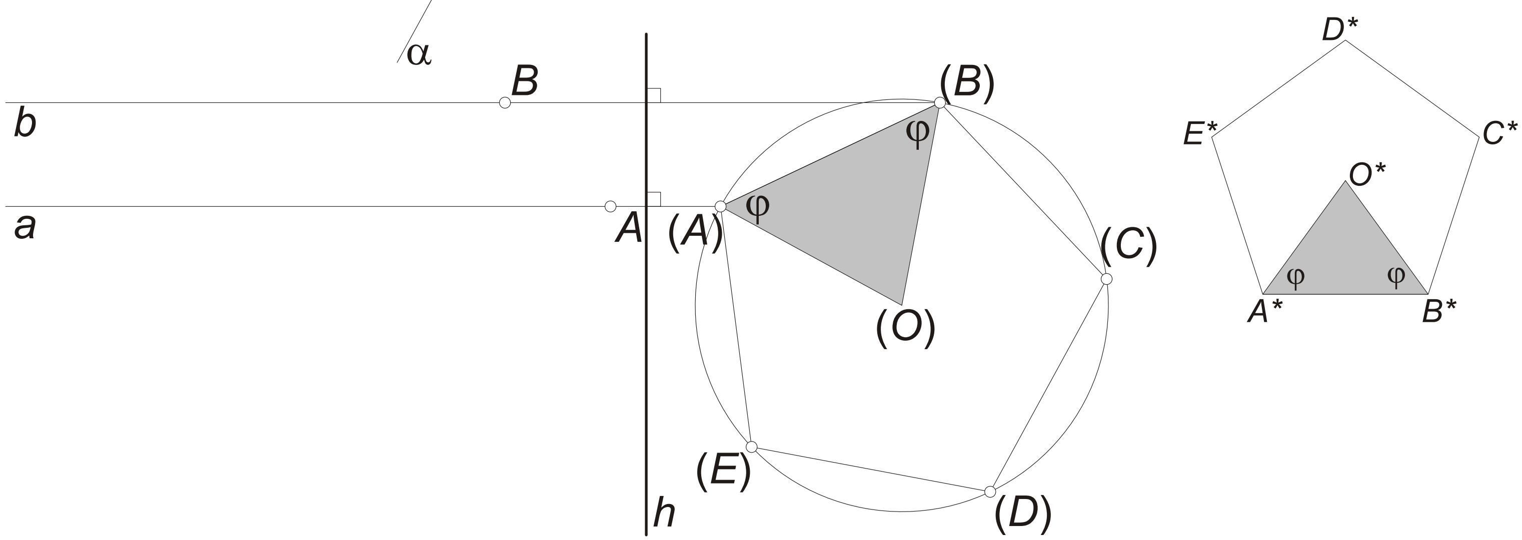


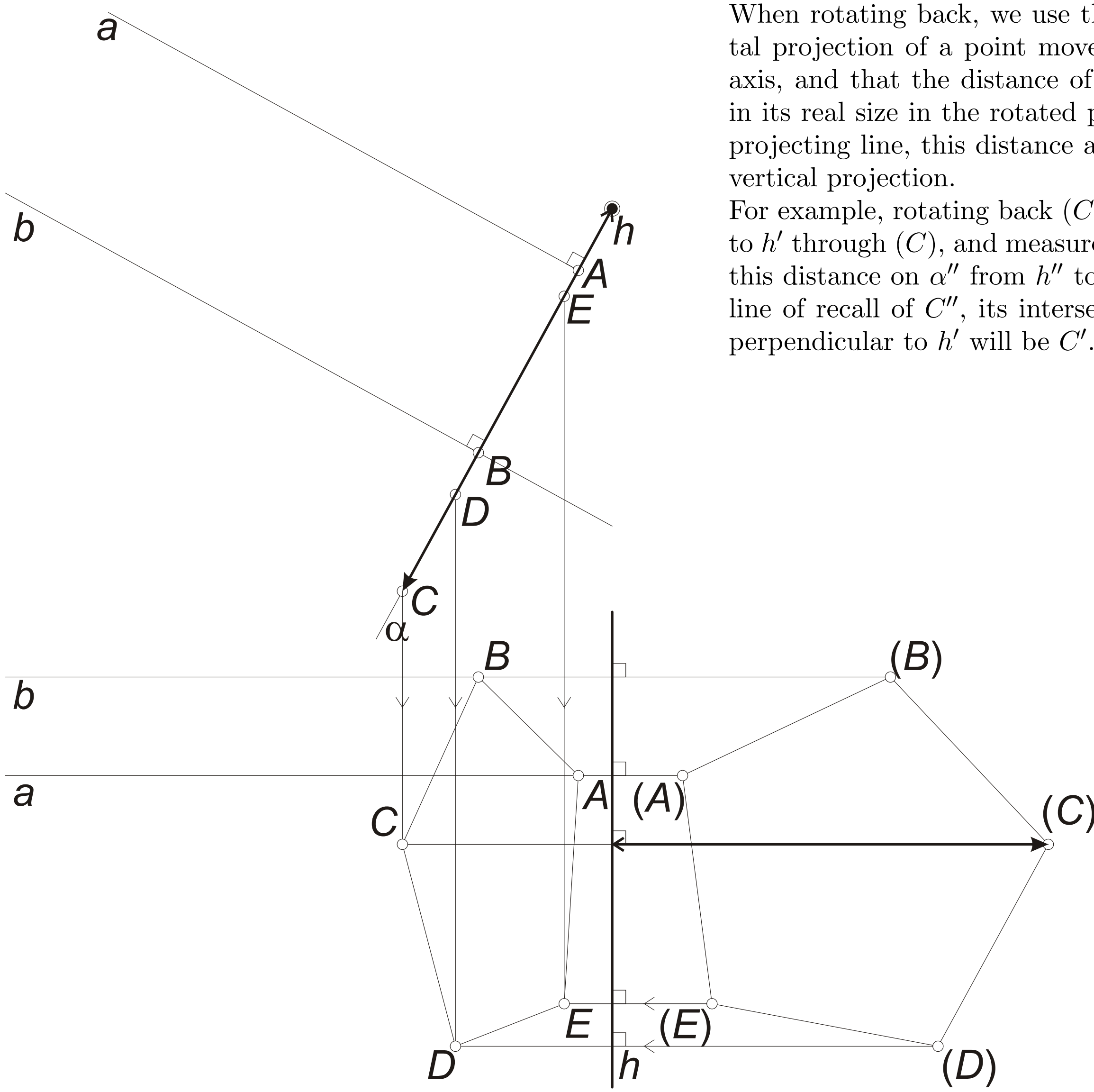
To be able to construct the regular pentagon $ABCDE$ in the usual way, we rotate its plane α into a horizontal position. First, we need a horizontal line h in α as the axis of the rotation. In a vertical projecting plane the horizontal lines are exactly the vertical projecting lines of the plane, and thus, as h'', we may choose any point of α'', and then h' is the corresponding line of recall.

We take it into account that, during the rotation, the horizontal projection of the point moves on a line perpendicular to h', and that the distance of the point from the axis appears in its real size in the rotated plane. Since h is a vertical projecting line, this distance is shown directly in the vertical projection: $(A)h' = A''h''$ and $(B)h' = B''h''$.



Using the similarity of regular pentagons, first, we construct an arbitrary regular pentagon $A^*B^*C^*D^*E^*$, with center O^*. From this, we can determine the angle ϕ on the base A^*B^* of the isosceles triangle $A^*B^*O^*$.
Because of the similarity, we need to construct this angle ϕ at the endpoints of segment $(A)(B)$ in the rotated plane. The intersection of the legs of these angles is the center (O) of the pentagon. Now we can draw the circle circumscribed about the pentagon, and, by drawing chords of length $(A)(B)$ on it, obtain the remaining vertices.
Notice that we could have chosen O on the other side of $(A)(B)$ as well. Nevertheless, we can consider that this choice would lead to the solution on a higher level in the space.

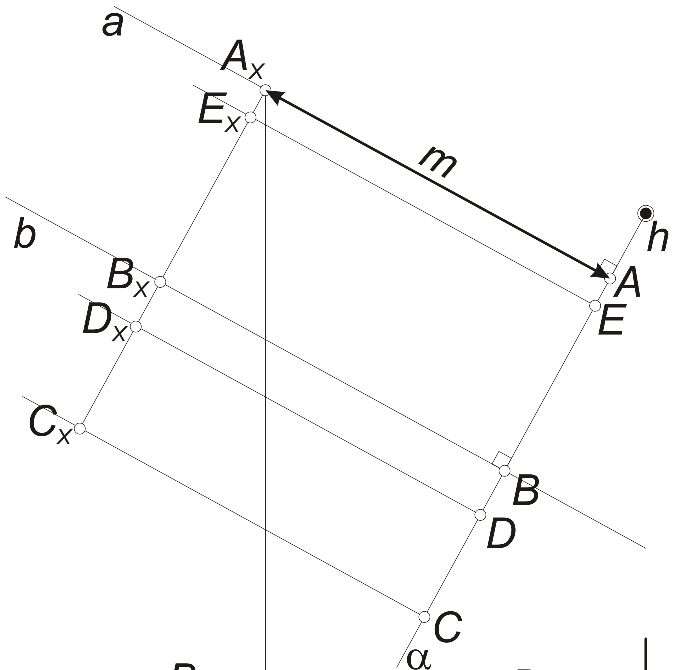


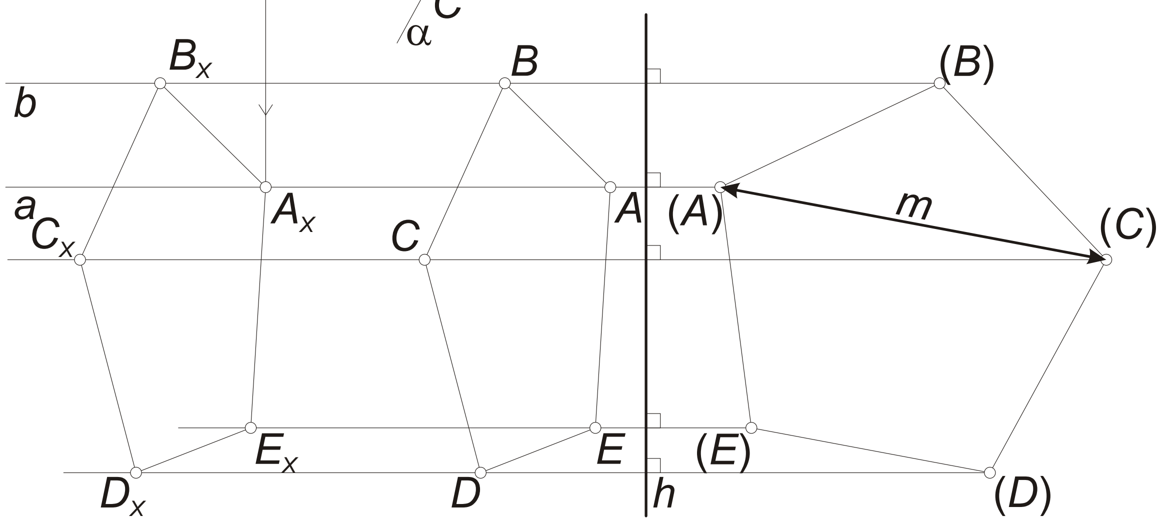


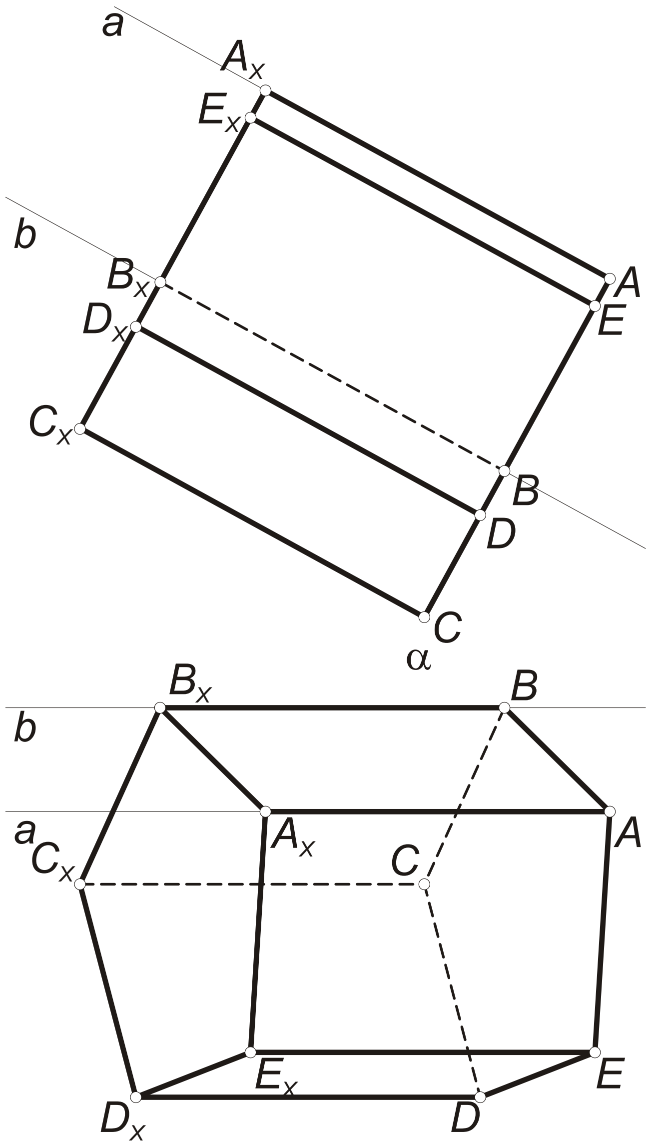
When rotating back, we use the facts, again, that the horizontal projection of a point moves on a line perpendicular to the axis, and that the distance of the point from the axis appears in its real size in the rotated plane. Since the axis is a vertical projecting line, this distance appears in its real size also in the vertical projection.

For example, rotating back (C), we draw the line perpendicular to h' through (C), and measure the $(C)h'$ distance. We measure this distance on α'' from h'' to obtain C''. Finally, drawing the line of recall of C'', its intersection with the line through (C), perpendicular to h' will be C'.

Through the points C'', D'', E'', we draw the lines containing the edges CC_x, DD_x and EE_x, parallel to a''. According to the text of the exercise, the height m of the prism is equal to the length of the diagonals of the pentagon. The diagonals are shown in their real size in the rotated pentagon $(A)(B)(C)(D)(E)$. On the other hand, since the lateral edges are contained in vertical lines, they are shown in their real size in the vertical projection. Thus, we can draw the height in this projection directly: $(A)(C) = m = A''A''_x$. We can obtain the vertical projections of the remaining vertices similarly.
To construct A'_x, we can use the line of recall of A_x. Since the projections of two parallel segments of the same length are parallel and equal, we can measure, in the horizontal projection, the length of $A'A'_x$ on the lines through the remaining points of the base. (The projections $B''_x, C''_x, D''_x, E''_x$ can be obtained also from the lines of recall of the points.) We draw the remaining edges of the prism as well.







To show visibility in the horizontal projection, we need to consider that, among the vertices, A_x is on the highest level, and thus, in the top view, we can see the roof of the pyramid.

For the vertical projection, we must observe that the closest face is DEE_xD_x, and thus, this face is visibe in the front view.