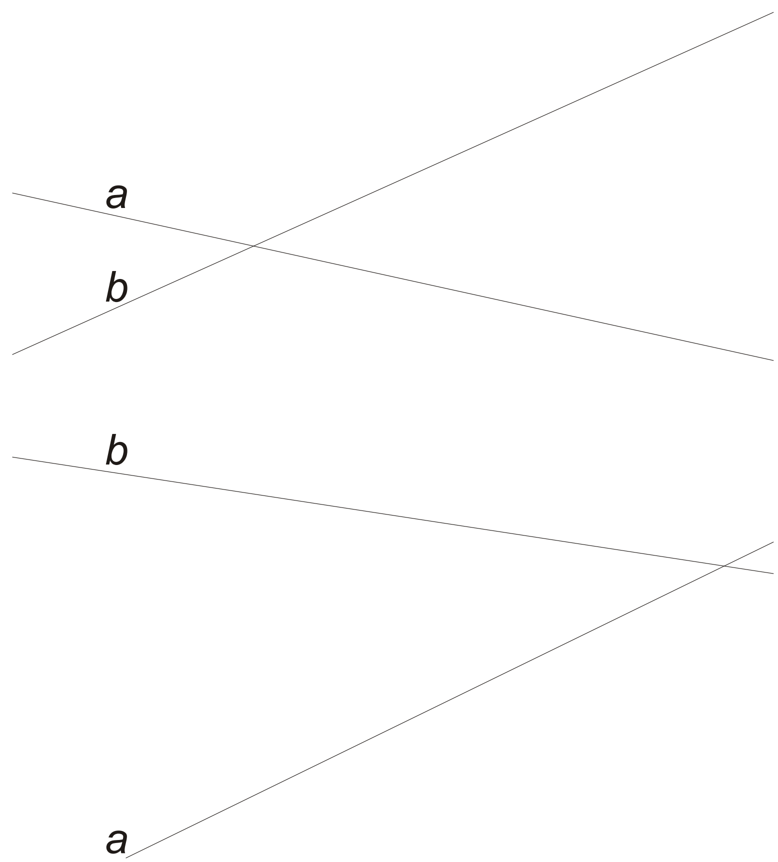
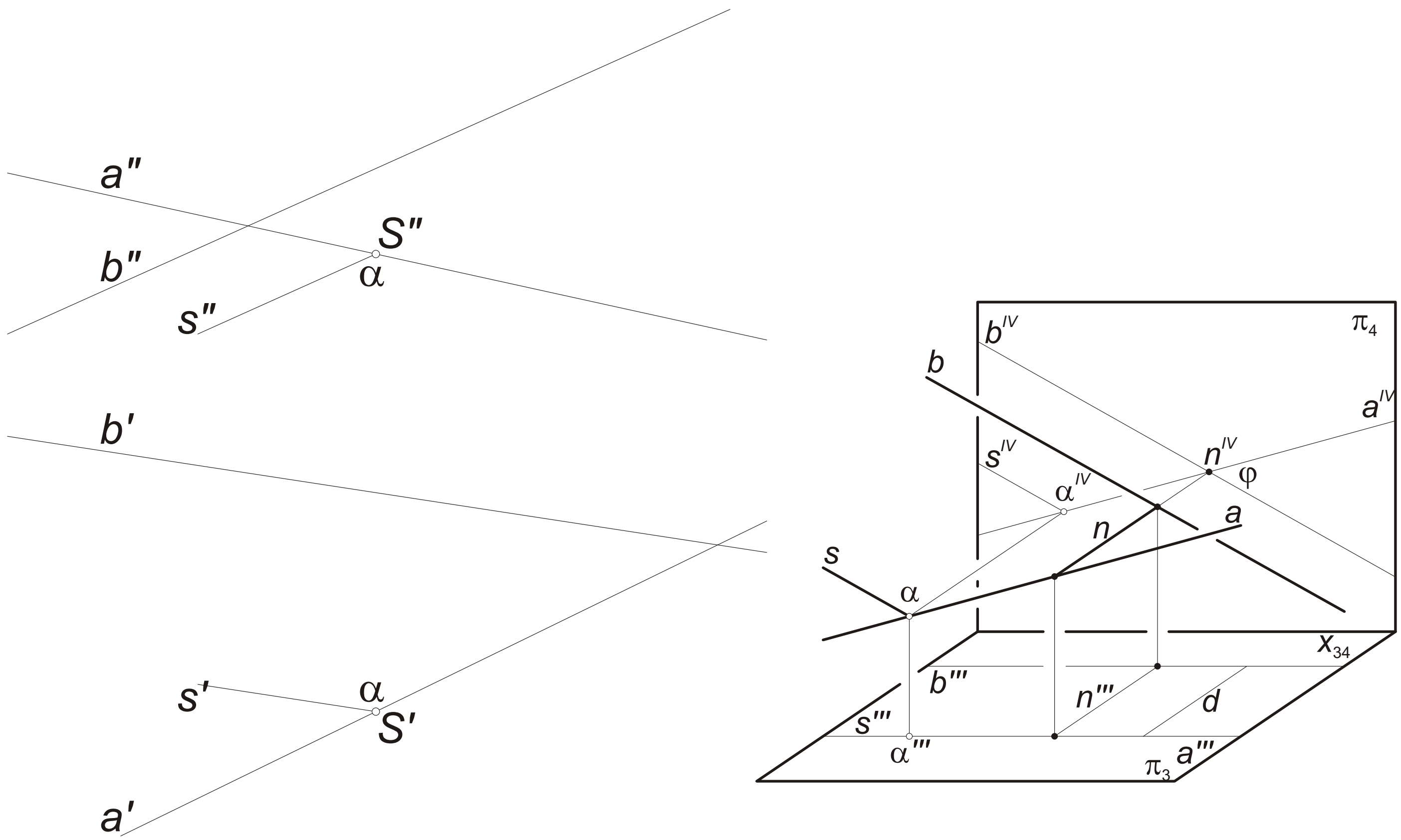
Transformation of a plane of projection

The distance, the angle, and the normal transversal of two skew lines

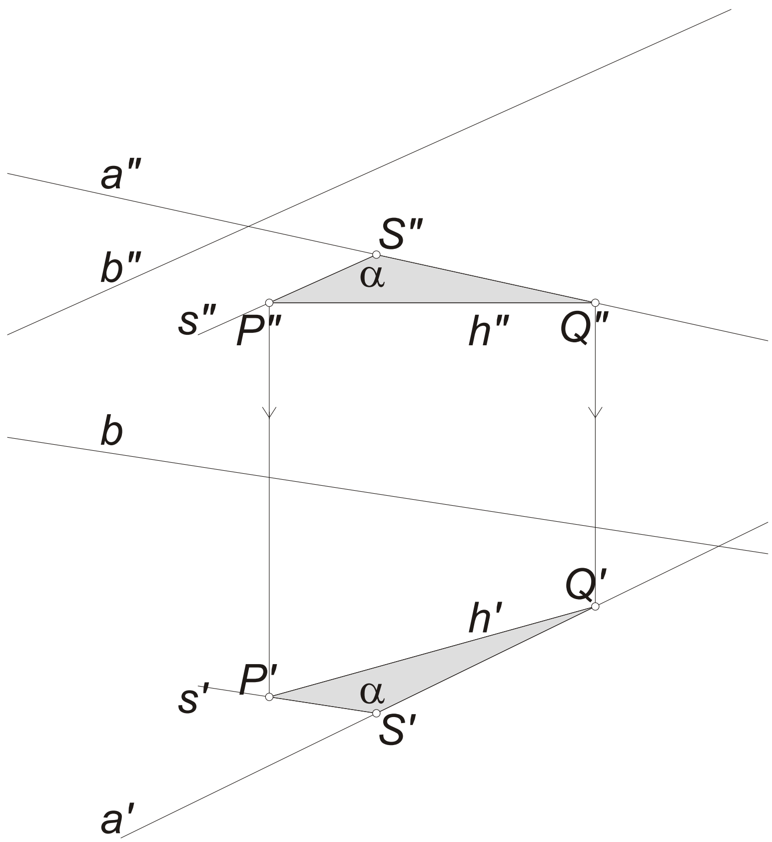
Exercise. *Construct the distance d, the angle ϕ and the normal transversal n of the given skew lines a and b.*



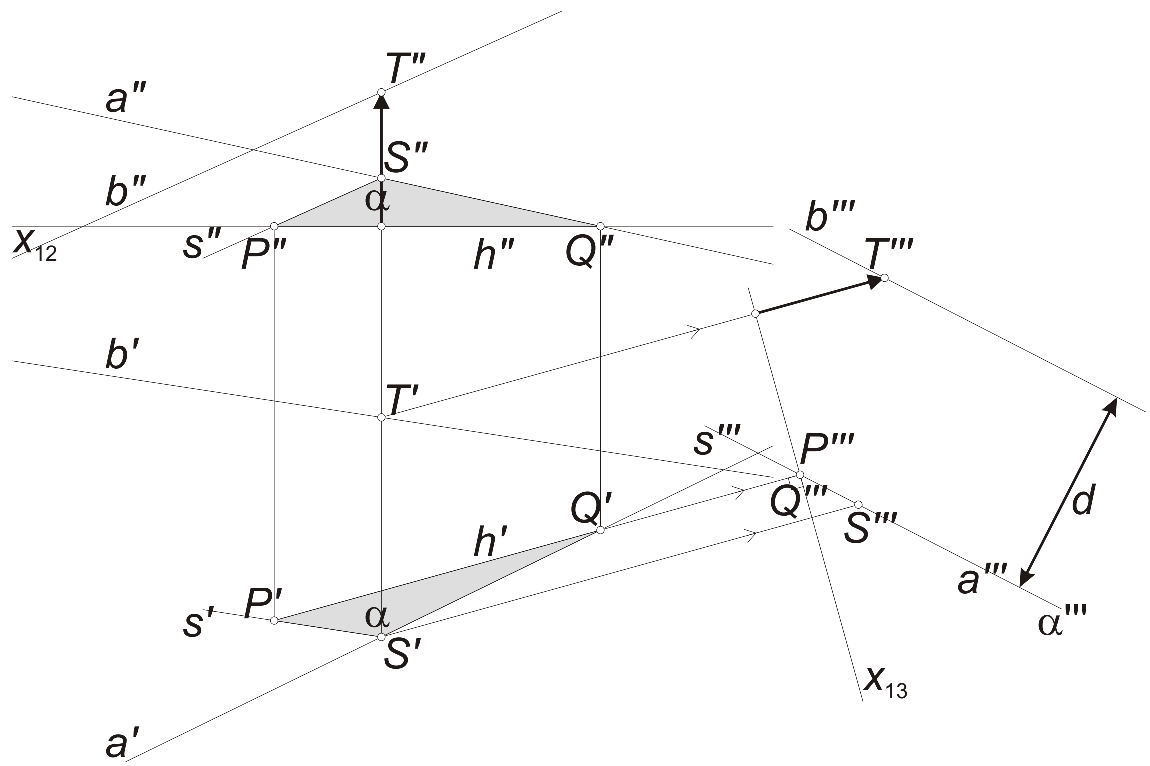
On one of the lines, say on a, we choose a point S, and a line s through it, parallel to b. Then the plane $\alpha = [a, S]$ is perpendicular to the normal transversal, because $n \perp a$ and $n \perp b || s$, and therefore n is perpendicular to any two intersecting lines in α. Thus, transforming α into a III^{rd} projecting plane, n will be a III^{rd} parallel line, and finally, transforming α into a IV^{th} parallel plane, n^{IV} will be a single point, the intersection point of a^{IV} and b^{IV}, since it will be a IV^{th} projecting line.



In the plane α we choose a horizontal (parallel) line h. We draw the vertical (second) projection h'' of h horizontally, and its horizontal (first) projection is determined by the construction of the intersection points P and Q, with the lines a and s, respectively. Hence $h = PQ$ is a horizontal line of α.



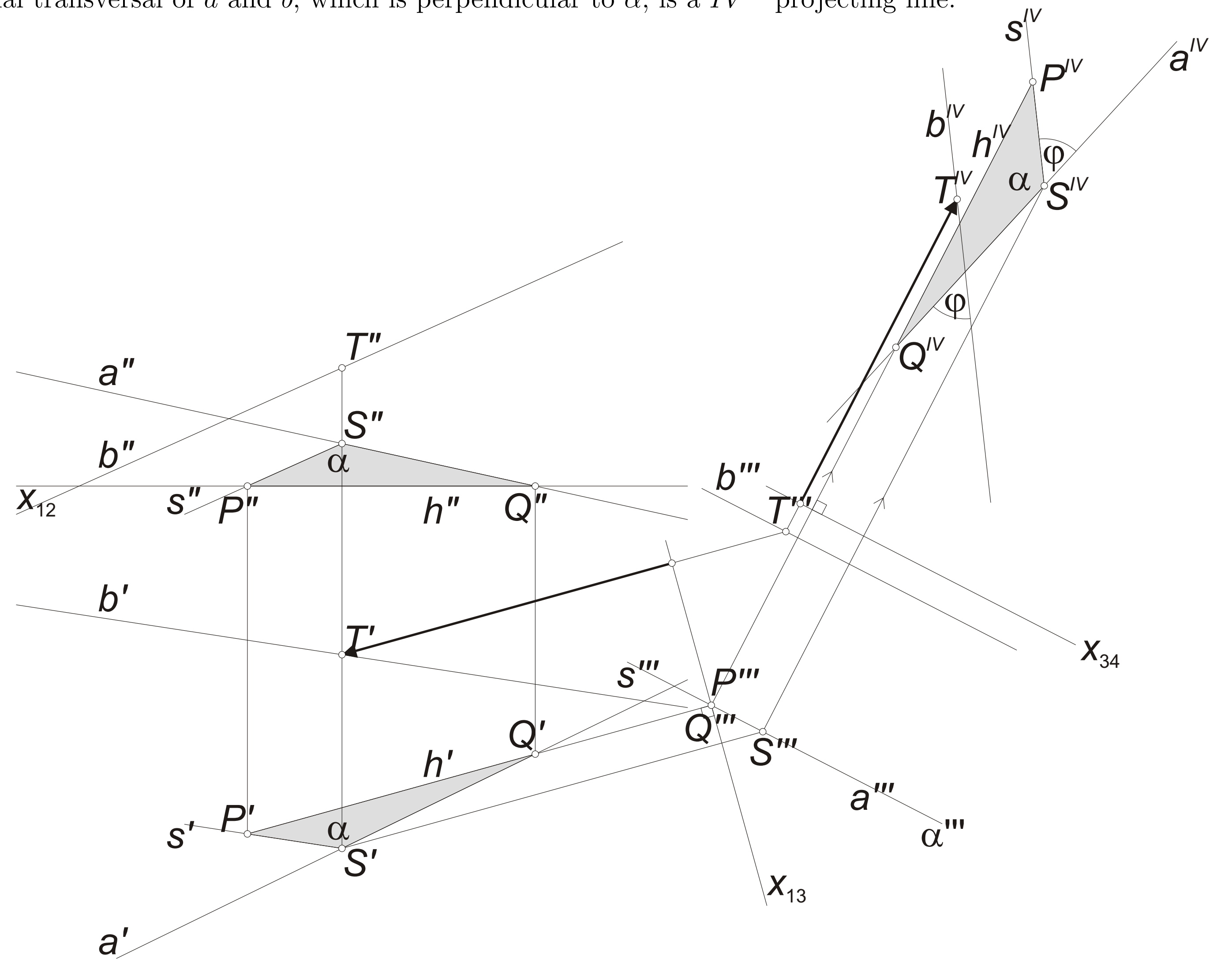
For the transformation we choose a point T of b, for example in such a way that its line of recall coincides with that of S. We choose the axis x_{12} as well, in our picture it contains the points P'' and Q''. Finally, we choose the new axis x_{13} as a line perpendicular to h'.
To construct S''' and T''', we measure the (signed) distances of S'' and T'' from x_{12}, on the new lines of recall of S and T, from x_{13}. Since P'' and Q'' lie on x_{12}, P''' and Q''' will be on x_{13}. Then α is a III^{rd} projecting plane: $\alpha''' = a''' = s'''$. Since $b||s$, b''' is the line parallel to s''', and passing through T'''. Hence $a'''||b'''$.
The normal transversal is perpendicular, and thus, the distance of a and b is equal to the distance of a''' and b'''.



The new, IV^{th} plane of projection will be parallel to α: $x_{34}||\alpha'''$. The distance of the IV^{th} projection of a point from x_{34} is equal to the distance of its I^{st} projection from x_{13}, as it is shnown in the picture for the transformation of T. Since in the space $b||s$, we have that b^{IV} is parallel to s^{IV}, and passes through T^{IV}.

Hence, the plane α, as well as the lines a, b and s, are IV^{th} parallel lines. We can see then the angle of a and s in its real size, and so can we the required angle ϕ of a and b.

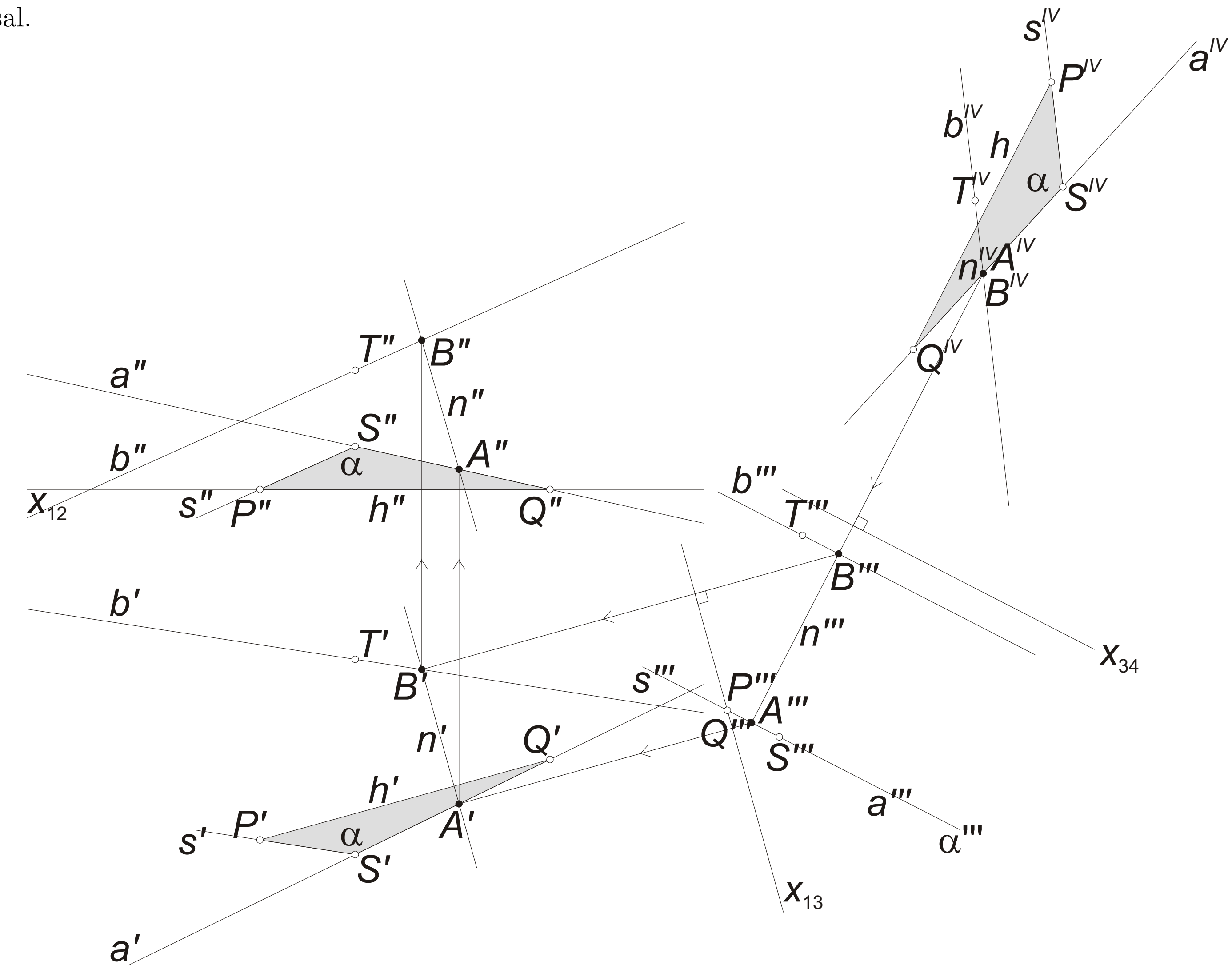
The normal transversal of a and b, which is perpendicular to α, is a IV^{th} projecting line.



The normal transversal now is a IV^{th} projecting line, and thus its IV^{th} projection is a single point. By exclusion, it must be the intersection point of a^{IV} and b^{IV}, which coincides with the IV^{th} projections of the two endpoints A and B of the normal transversal segment connecting a and b:

$$n^{IV} = A^{IV} = B^{IV} = a^{IV} \cap b^{IV}.$$

Drawing the lines of recall of A and B in the systems IV-III, III-I and I-II, we obtain the III^{rd} I^{st} and II^{nd} projections of the points. By connecting the two points we have the corresponding projections of the normal transversal.



Finally we determine the visibility of the two lines and the transversal segment. In the picture the right angle signs indicate that the corresponding two lines our perpendicular in space, and not necessarily in the projections: the normal transversal intersects both lines perpendicularly.