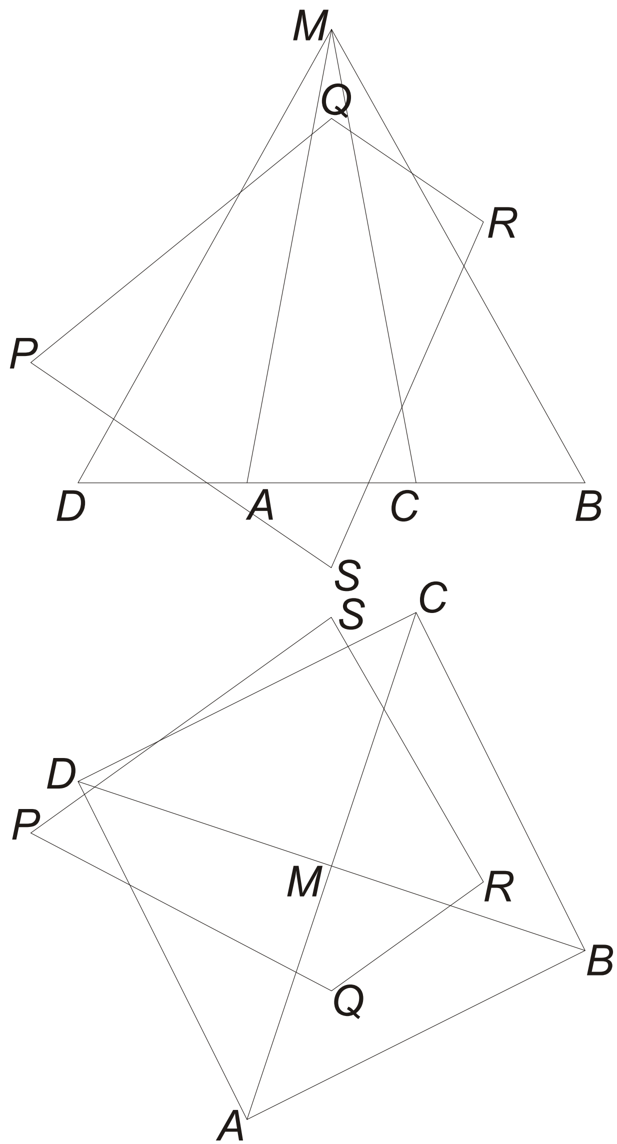
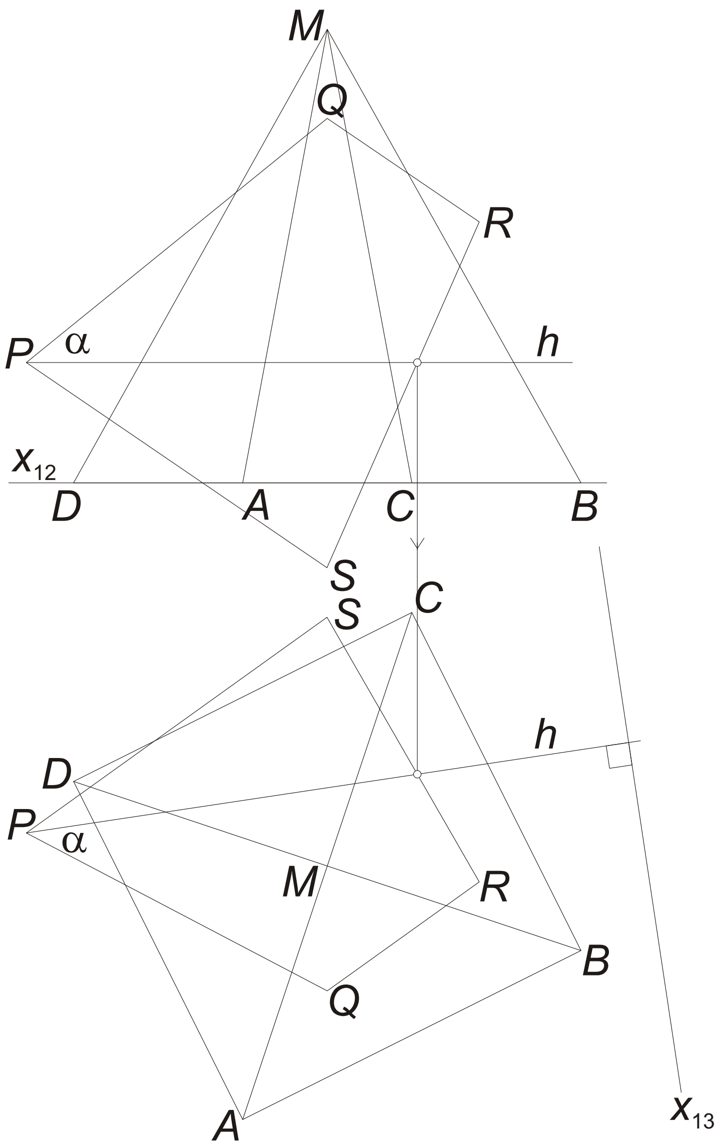
Planar section of a polyhedron

Intersection of a square-based pyramid with a trapezoid lying on an oblique plane

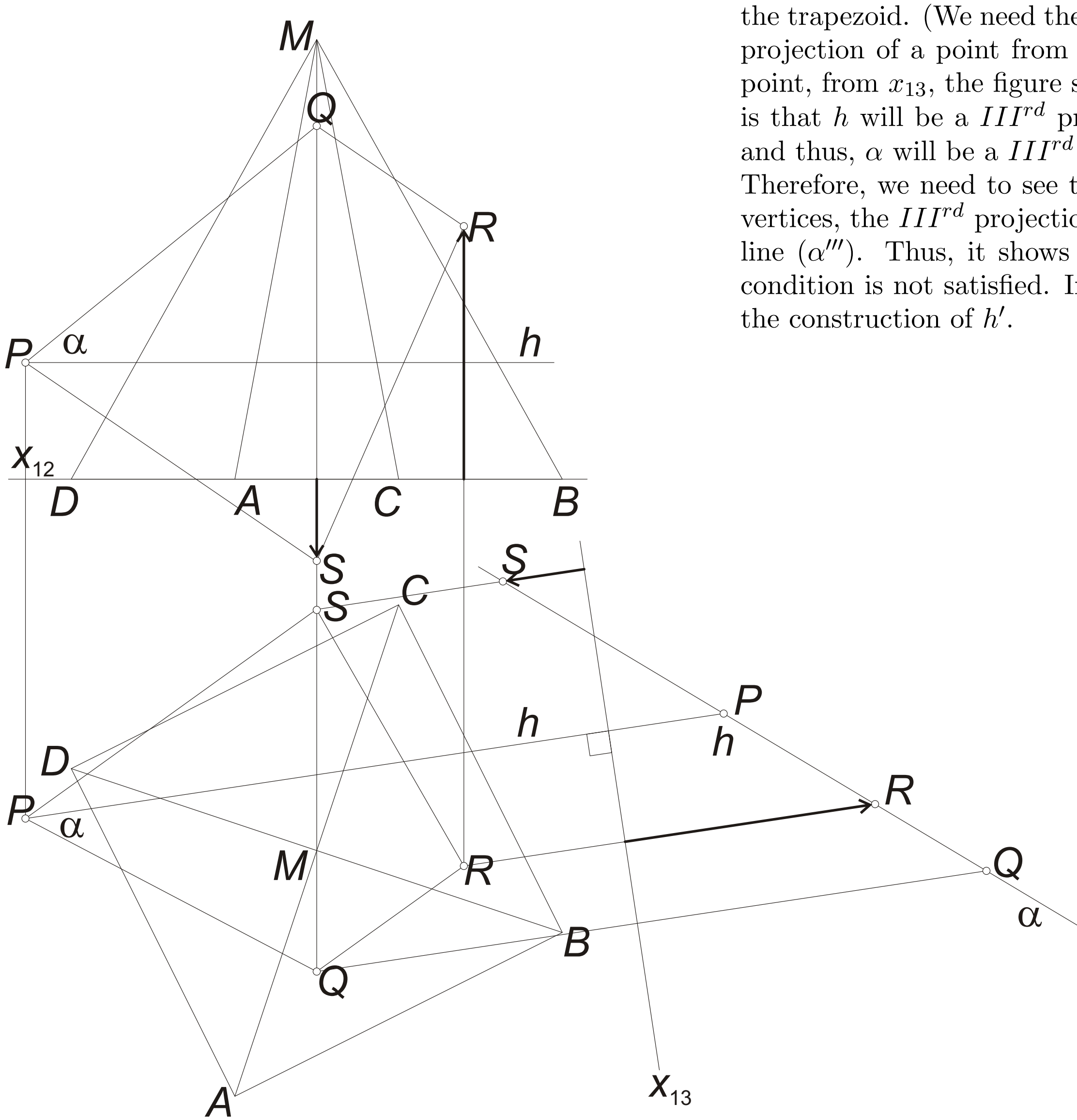


Exercise. *Given a regular square-based pyramid $ABCDM$ with its base lying on a horizontal (parallel) plane, and a trapezoid $PQRS$. Construct the intersection of the pyramid with the trapezoid. Show visibility assuming that the pyramid is a solid and that the trapezoid is* not *removed after the intersection.*

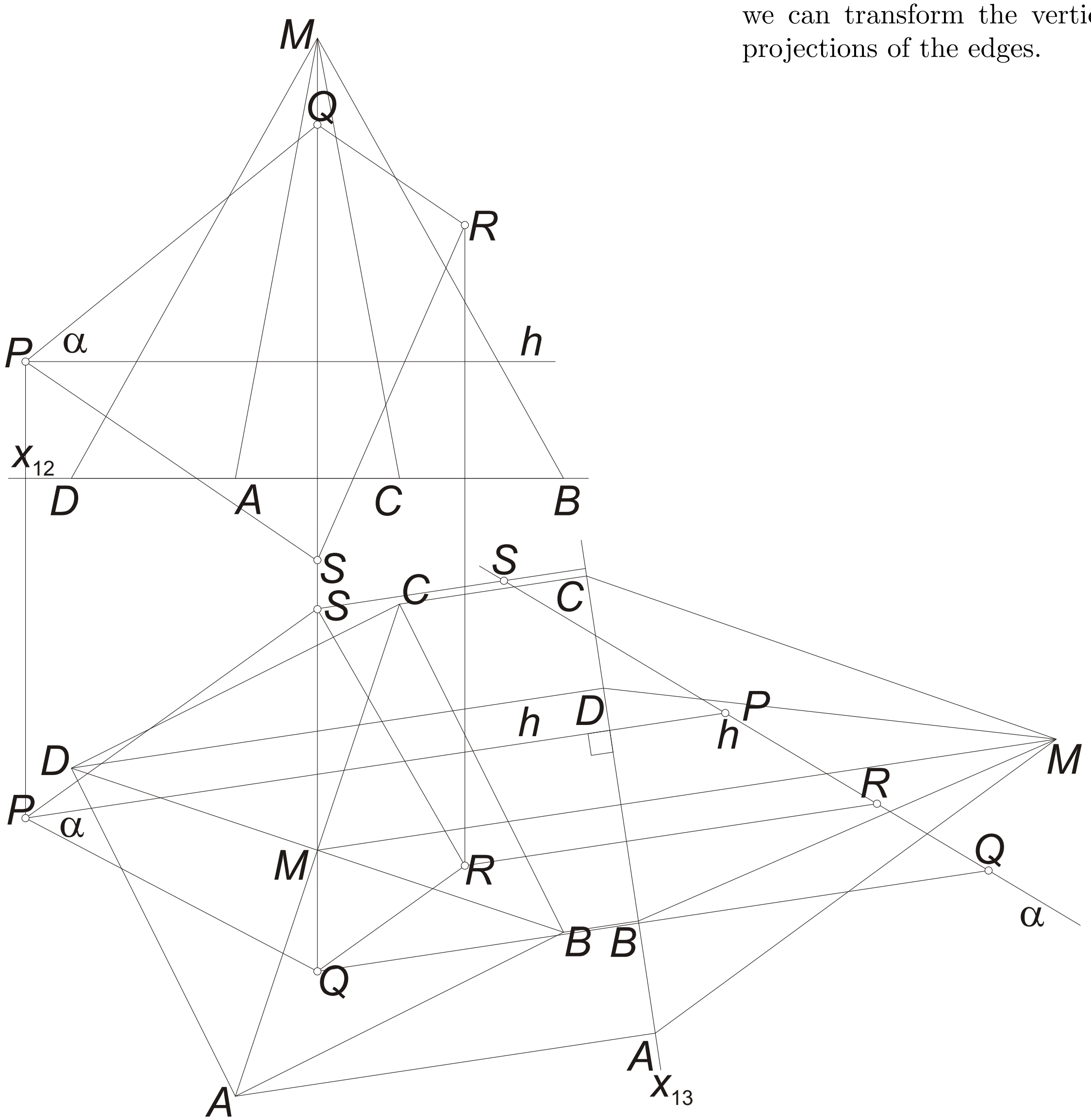


First we construct the intersection polygon of the pyramid with the *whole* plane containing the trapezoid. To obtain the vertices of this polygon, we need to transform α into a projecting plane. To do this, we construct a horizontal line h of the plane α, for instance the one passing through the vertex P of the trapezoid. (We draw h'' perpendicularly to the direction of the lines of recall, find its intersection point with the edge RS, then construct the horizontal projection of this point. The projection h' of h connects this point with P'.)
We choose the axis x_{13} of the $I-III$ system as a line perpendicular to h' ($x_{13} \perp h'$), and fix x_{12} in such a way that the base of the pyramid is contained in the horizontal plane of projection.

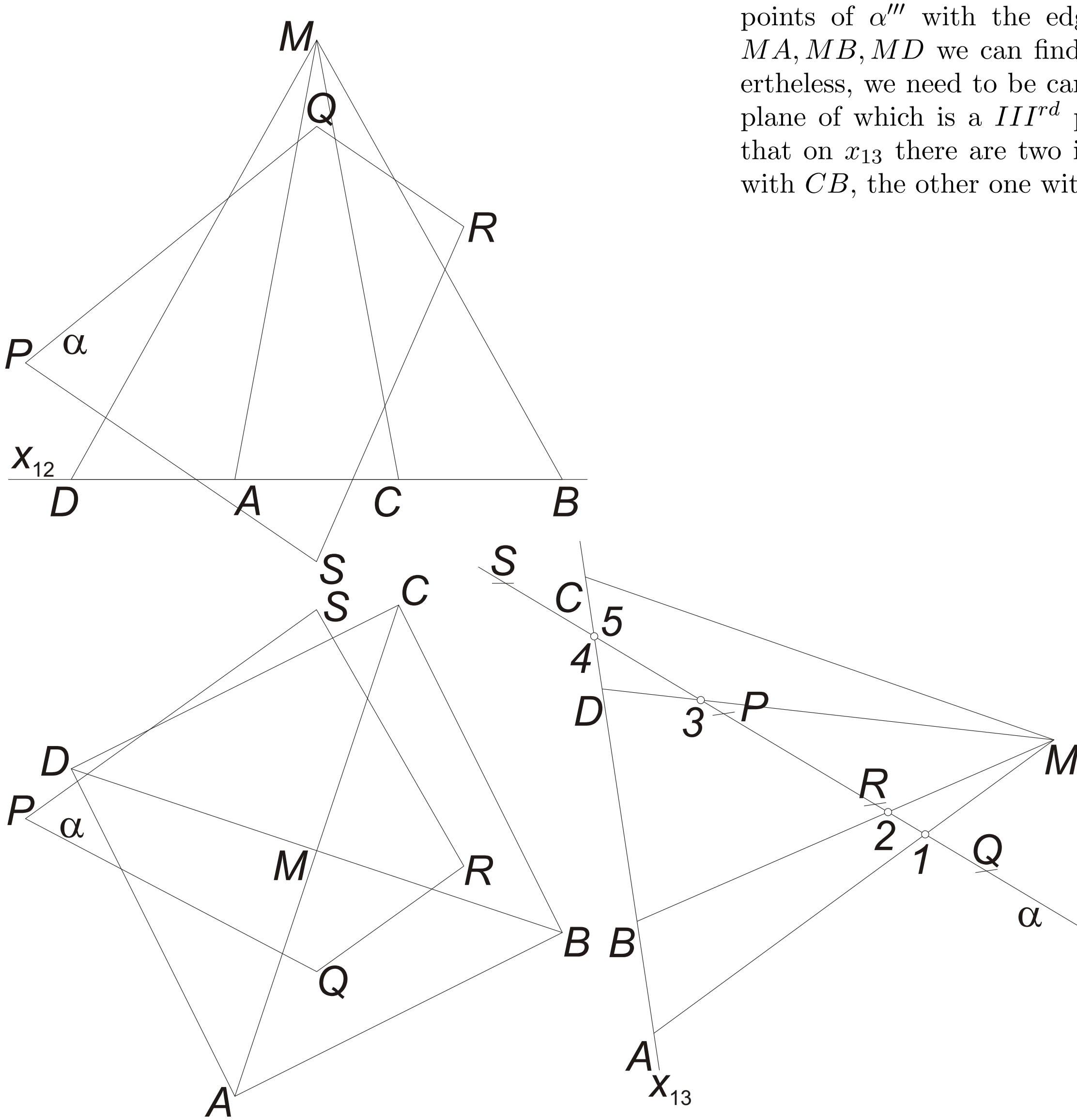
It is useful to carry out the transformation for the vertices of the trapezoid. (We need the measure the distance of the vertical projection of a point from x_{12} on the new line of recall of the point, from x_{13}, the figure shows this for R and S.) The reason is that h will be a III^{rd} projecting line (h''' is a single point), and thus, α will be a III^{rd} projecting plane (α''' is a line). Therefore, we need to see that, after the transformation of the vertices, the III^{rd} projections of the points will lie on the same line (α'''). Thus, it shows an error in the construction if this condition is not satisfied. In this case usually we need to check the construction of h'.



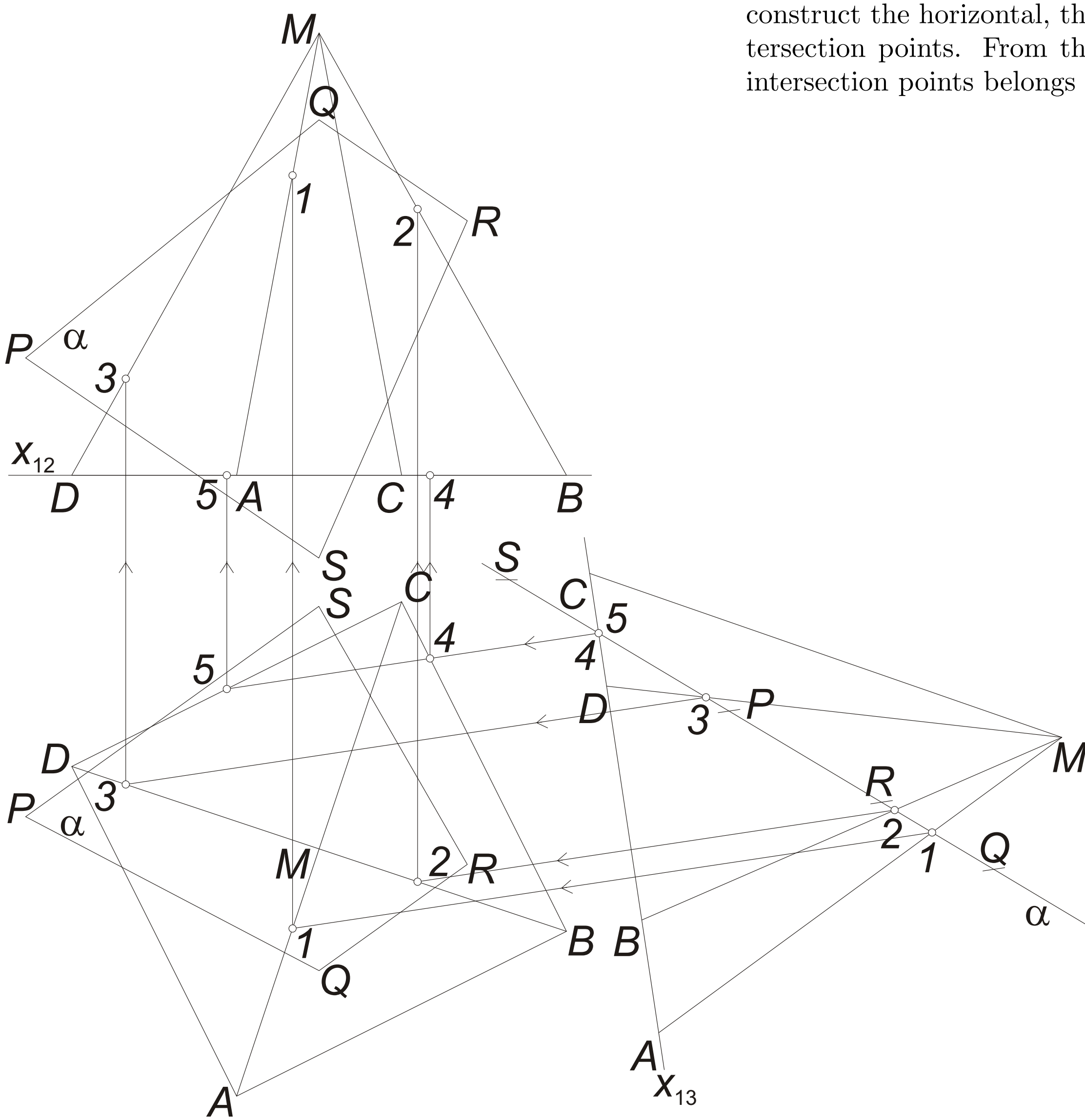
If the construction of the plane α of the trapezoid is correct, we can transform the vertices of the pyramid, and draw the projections of the edges.



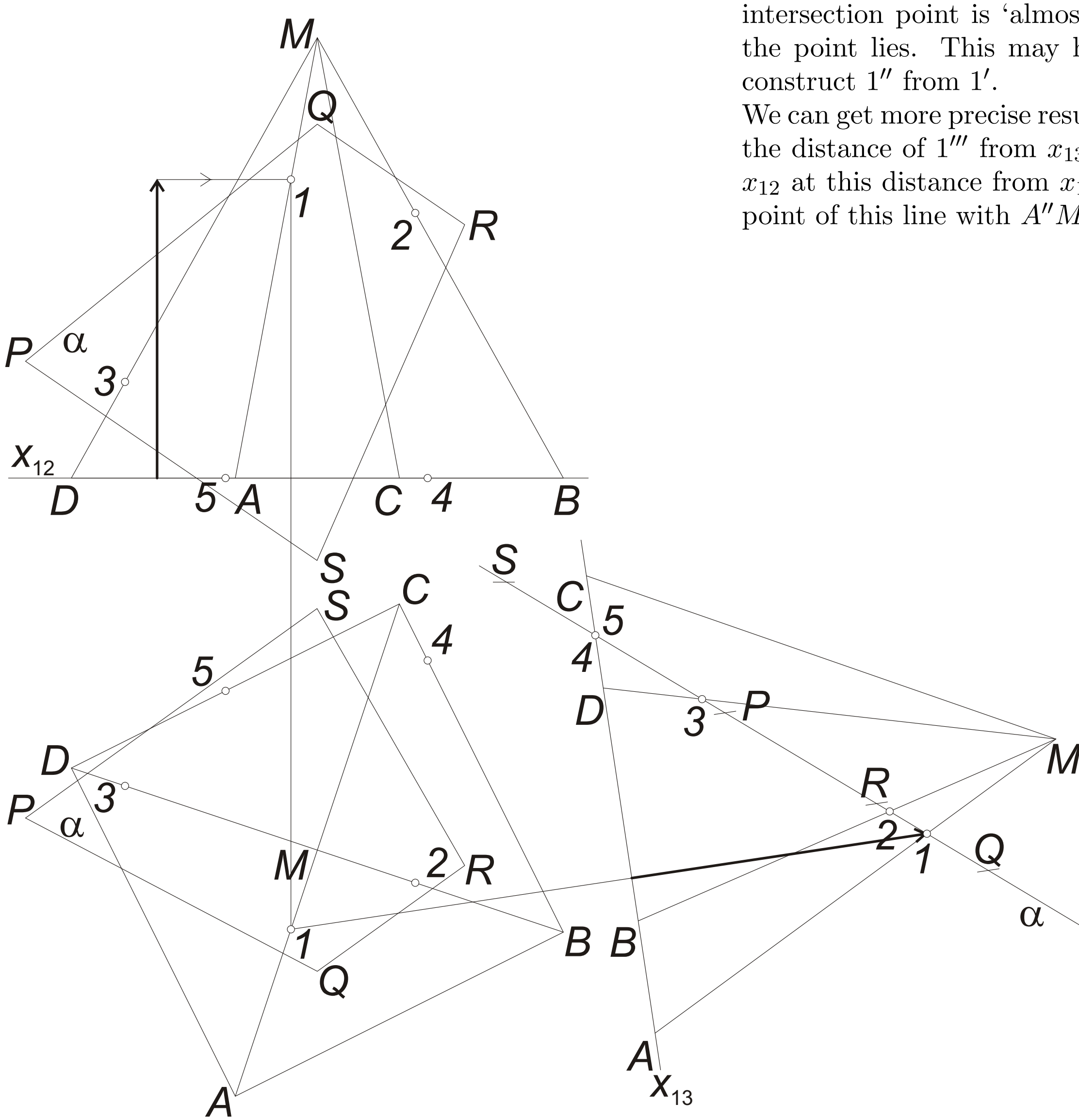
The vertices of the intersection polygon are the intersection points of α''' with the edges of the pyramid. On the lines MA, MB, MD we can find the vertices $1, 2, 3$ explicitly. Nevertheless, we need to be careful with the edges of the base, the plane of which is a III^{rd} projecting plane. We should notice that on x_{13} there are two intersection points: $4'''$ and $5'''$, one with CB, the other one with CD.

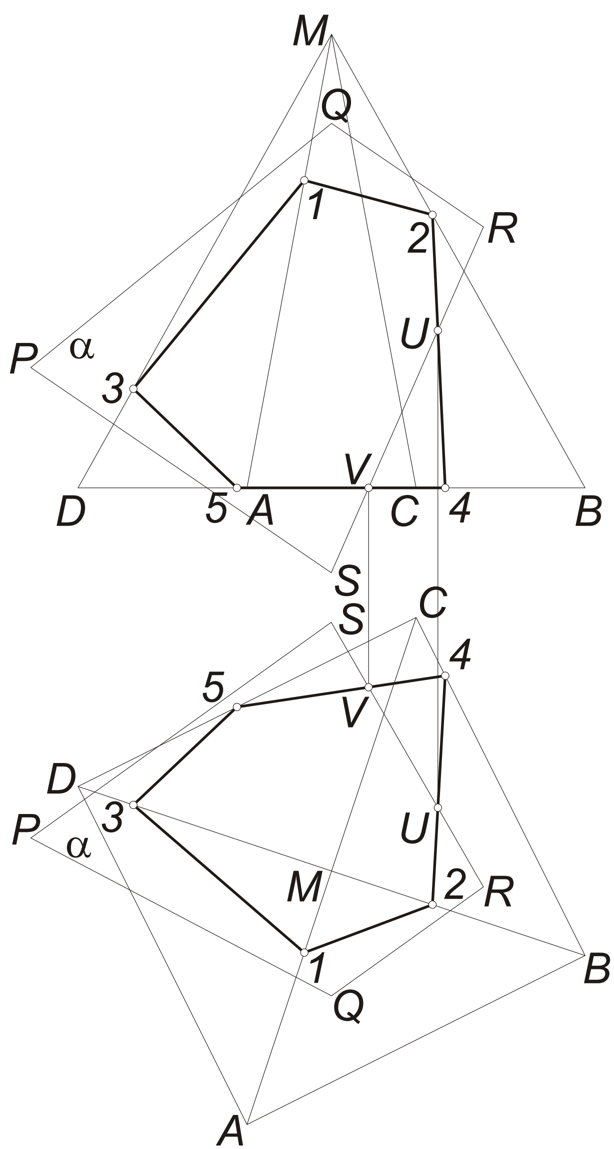


Using their lines of recall, from their III^{rd} projections, we can construct the horizontal, then the vertical projections of the intersection points. From the III^{rd} projections we know which intersection points belongs to which edge of the pyramid.

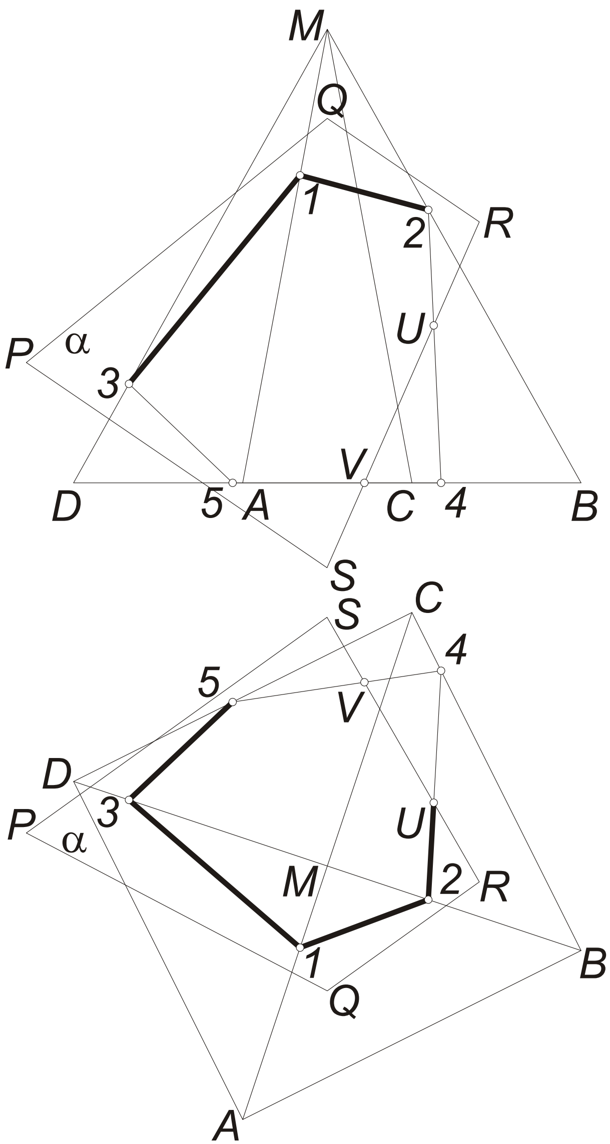


The construction can become unprecise if a line of recall of an intersection point is 'almost' parallel to the segment on which the point lies. This may happen, for example, if we want to construct $1''$ from $1'$.
We can get more precise result if we do the following: we measure the distance of $1'''$ from x_{13}, and construct the line, parallel to x_{12} at this distance from x_{12}. Then, $1''$ will be the intersection point of this line with $A''M''$.





In the third projection, we can see the segment EF in its real size, coinciding with the edge-length of the cube. On the othe hand, since e is a vertical line, in the vertical projection we can see the edge AA_x of the cube on the line e in its real size. Since the midpont of AA_x is E, in the vertical projection we need to measure half of the length of $E'''F'''$ from E'' in both directions. These points will be the vertical projections of A and A_x. The horizontal projections of these points can be obtained as the intersections of their lines of recall with e'.

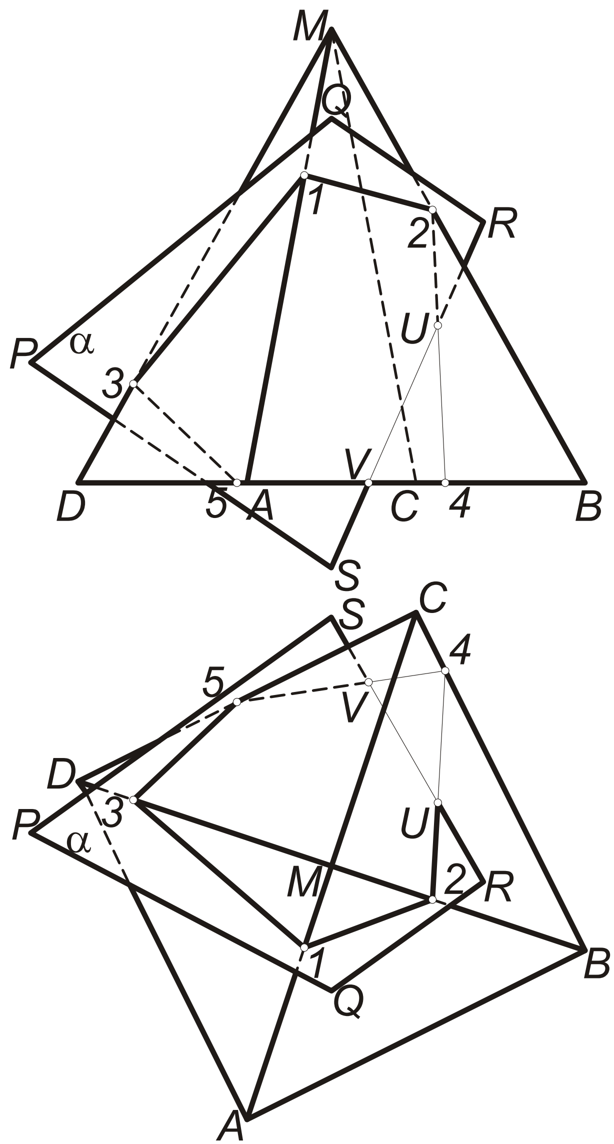


To construct the edges of the intersection polygon, we may apply the following general rules:

(1) Two vertices of the intersection polygon are connected if, and only if, the polyhedron has a face containing both points.
(2) If the polyhedron is convex, then the intersection polygon is convex as well.

Applying the first rule, we have, for example, that 1 and 2 can be connected, because both lie on the face ABM of the pyramid. Similarly, 4 and 5 can be connected because both lie on $ABCD$. Furthermore, the pairs 2 and 4, 5 and 3, 3 and 1 can be connected, since they are contained in the faces BCM, CDM, DAM, respectively. For instance, the vertices 3 and 4 cannot be connected,as there is no face of the pyramid containing both. Since the pyramid is convex, we may apply the second rule as well. According to this, the vertices $1, 2, 3, 4, 5$ should be connected such that they form a convex pentagon in α, and thus, in both the horizontal and the vertical projections. Using this prrinciple we obtain the same result: the whole plane α intersects the pyramid in the pentagon 12453.

After constructing the intersection with the whole plane α, we can find the intersection with the trapezoid $PQRS$. This intersection is the part of the pentagon contained in $PQRS$ in any of the two projections. In the figure, U and V denotes the points where the edges of the trapezoid intersect the surface of the pyramid. Since the horizontal and vertical projections of these points have been obtained independently, we should check if the two projections are on the same line of recall.



To examine the visibility of the edges of the intersection polygon, we may use the following rule: *Such an edge is visible if, and only if, the face of the polyhedron containing it is visible.*
In top view, all faces of the pyramid but its base are visible, thus, the visible edges in this view are 53, 31, 12 and $2U$. The base, and the edge $5V$ contained in it, is not visible.
In front view, the visible faces of the pyramid are DAM and ABM. Consequently, the visible edges are 31 and 12.