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# Application of Point Processes in Modelling Credit Derivatives

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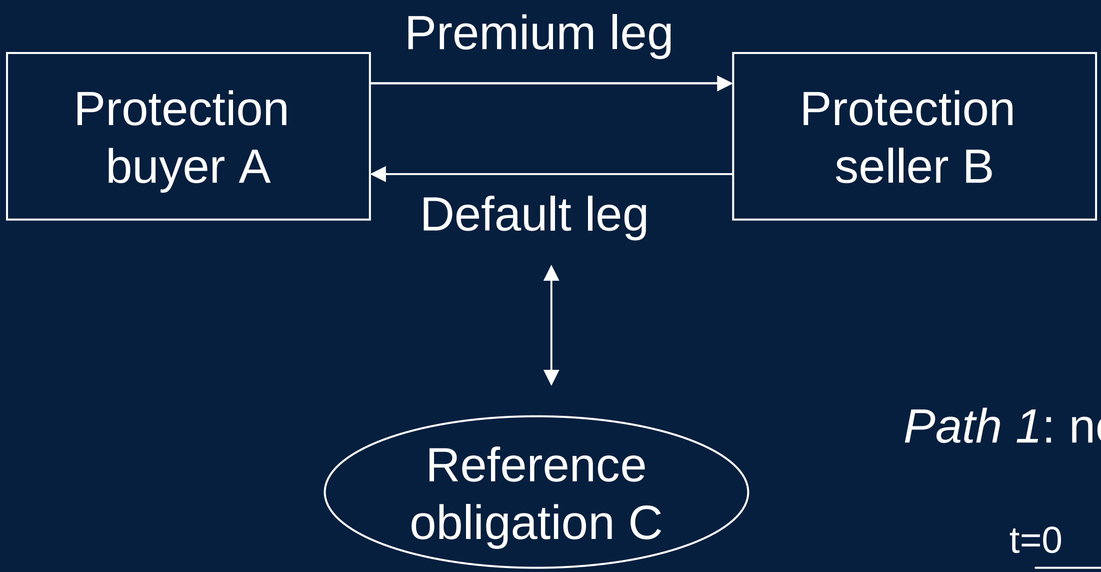
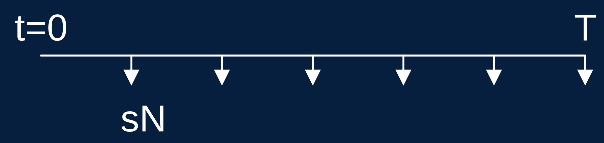
# We will discuss

- Credit Default Swaps (CDS)
- Multi-name Credit Derivatives
- Simple Poisson Process
- Compound Poisson Process
- Mixed Poisson Process
- Poisson Process with Stochastic Intensity (Cox Process)
- Self-exciting Poisson Process (Hawkes Process)

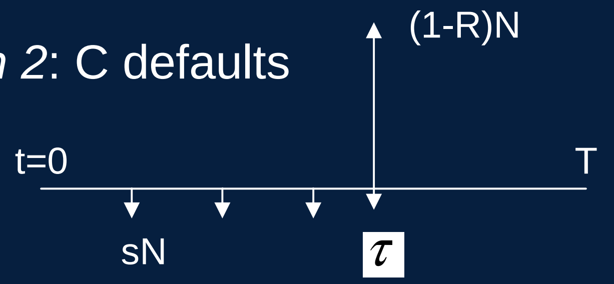
# CDS - Kinetics




*Path 1*: no default of C

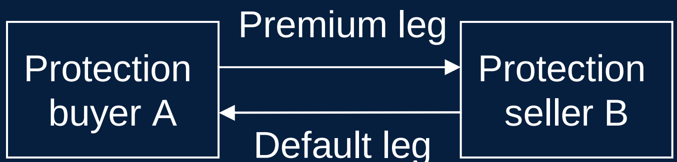



*Path 2*: C defaults




N – notional [$]
s – spread [bps/year]
R – recovery [1]
T – maturity [year]

# CDS – Constant hazard rate (toy) model




- Default intensity h=const
- Continuous premium payment
- Fixed recovery
- Const interest rate

Default probability

$$P(t < \tau < t + \Delta t) = e^{-ht} \cdot h\Delta t$$

$h\Delta t$ → defaults in interval

$e^{-ht}$ → survives up to t

Default leg

$$\text{DefPV} = N \int_0^T e^{-rt} \cdot (1-R)e^{-ht} h\,dt$$

$(1-R)e^{-ht}h\,dt$ → expected default payment at t

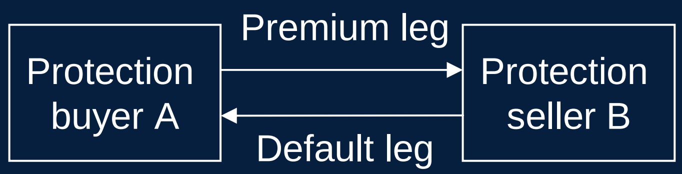
Premium leg

$$\text{PremPV} = N \int_0^T e^{-rt} \cdot s e^{-ht}\,dt$$

$se^{-ht}\,dt$ → expected premium payment at t

$$\text{PremPV}_1 = N \int_0^T e^{-rt} \cdot e^{-ht}\,dt$$

# CDS – Fair spread




The <u>fair spread</u> $s_{fair}$ makes the contract worth 0

$$\text{DefaultPV} = s_{fair} \cdot \text{PremPV}_1$$

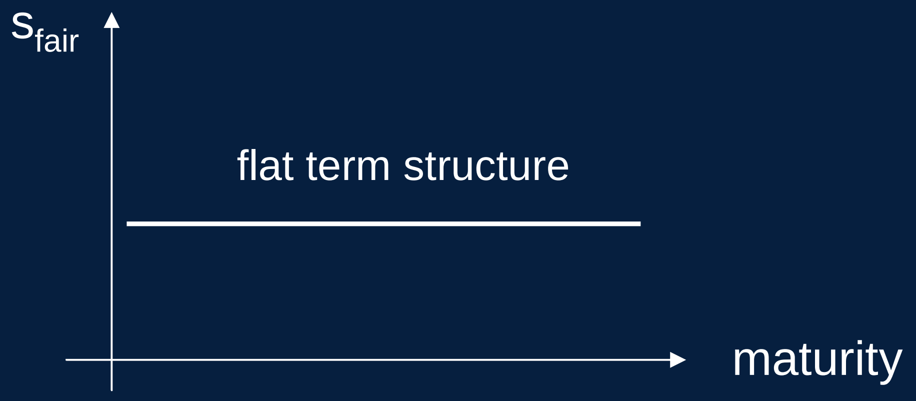
$$s_{fair} = \frac{\text{DefaultPV}}{\text{PremiumPV}_1} = (1 - R)h$$

# CDS - Term structure

Simple model:

$$s_{\text{fair}} = (1 - R)h$$

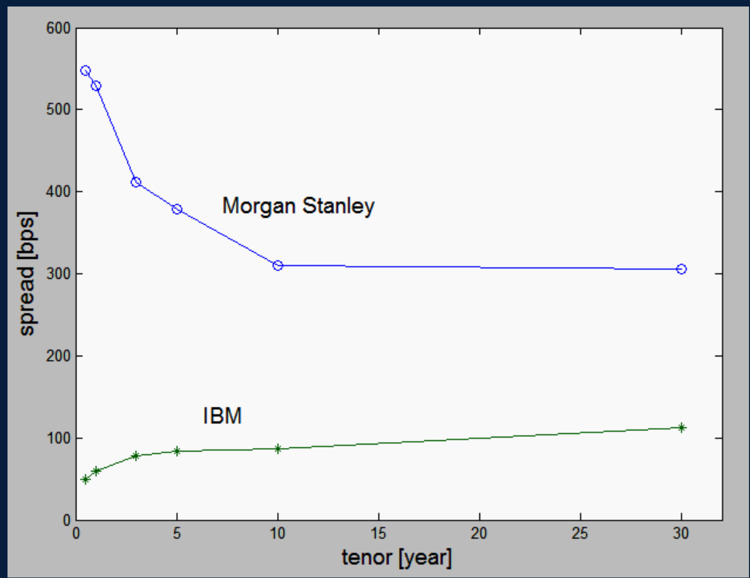
Does not depend on maturity T



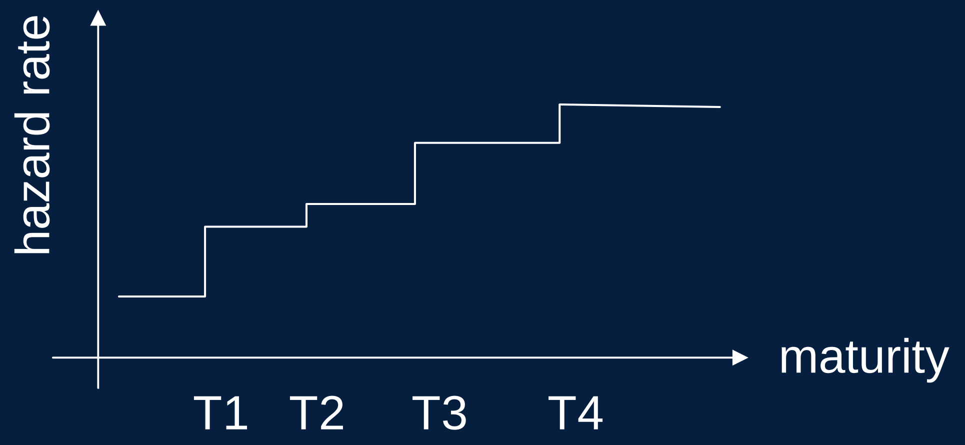

Market term structure of CDS spreads on 26/2/2009

⇩

Default intensity is time dependent



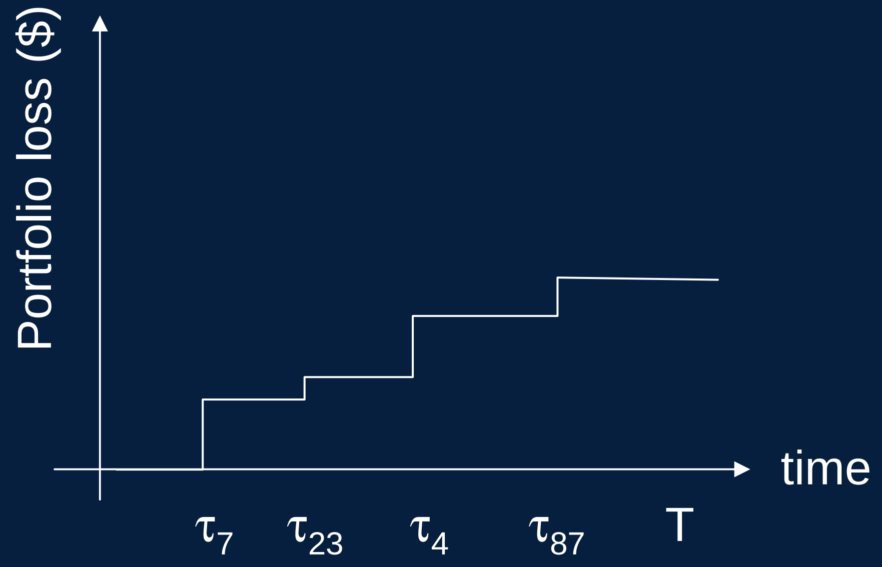
# CDS - Calibration




Piece-wise constant hazard rate model can be calibrated to market term structure by bootstrapping

$$\{s_1, s_2, \ldots, s_T\} \to h_1 \to h_2 \to \ldots \to h_T$$

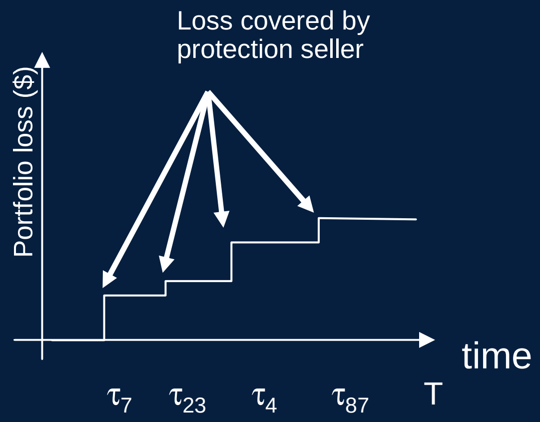
# Multi-name credit



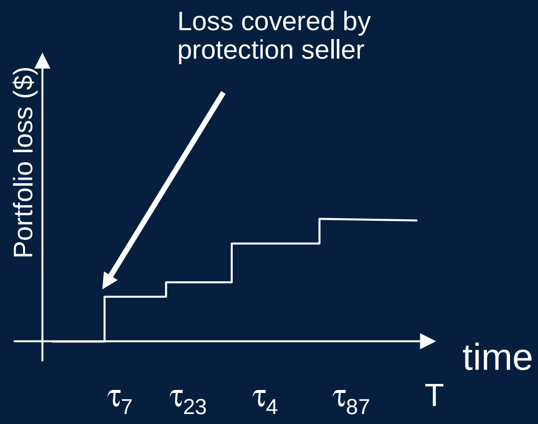

Sample path of portfolio loss

Portfolio of names

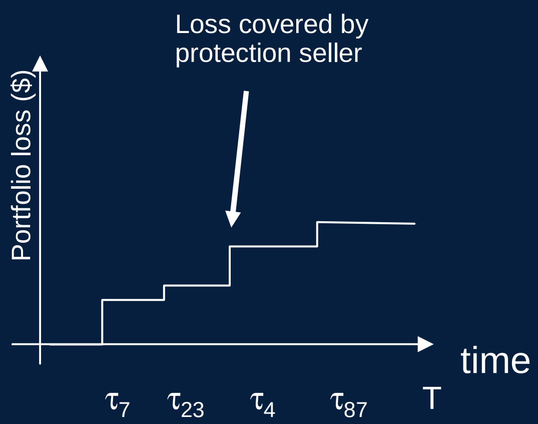
# Multi-name credit - Products



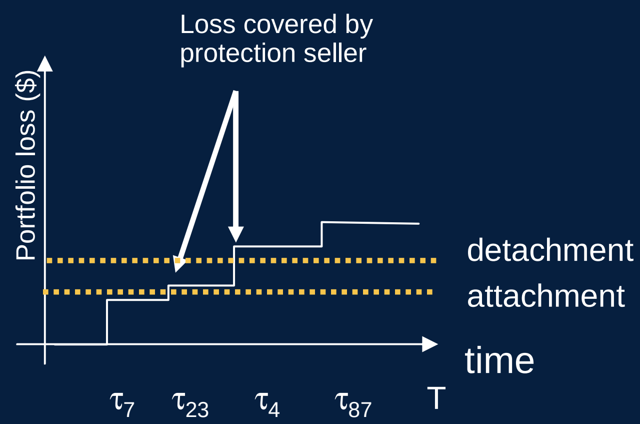

Index swap




FTD (1st to default basket)

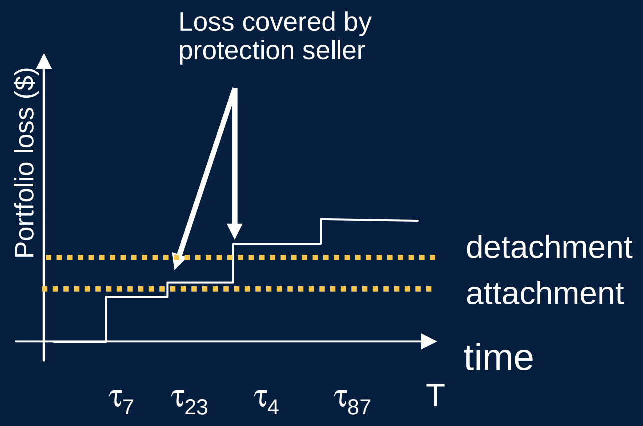


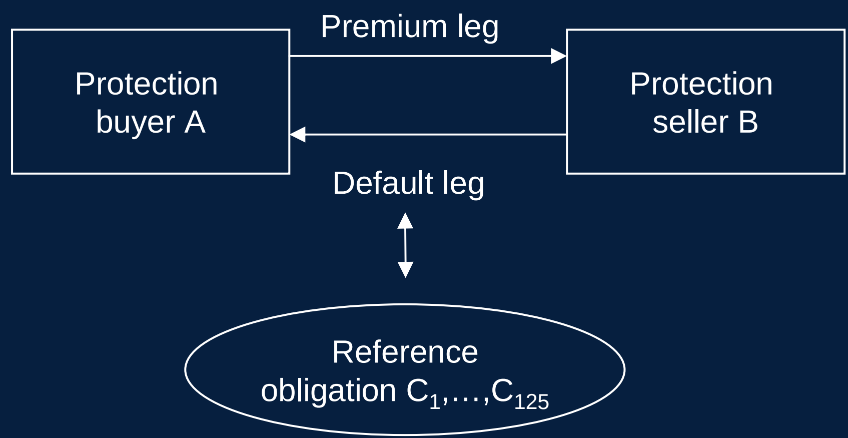

n-th to default basket




CDO (tranche swap)

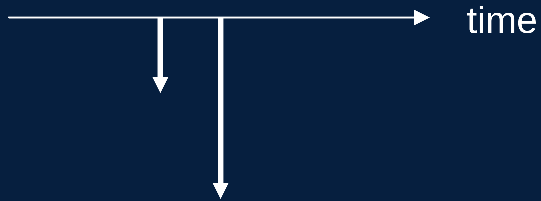
# Multi-name credit - Kinetics





Premium leg



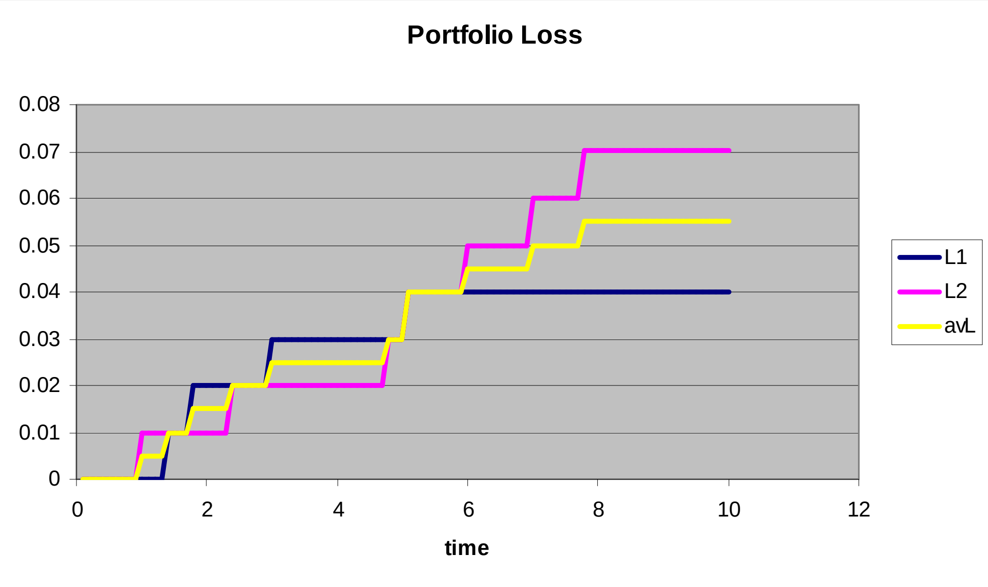

Default leg

# Portfolio Loss – Poisson Model

$$dL_t = J_0 dN_t^{(\lambda)}$$

$J_0$ Constant jump, corresponds to a single default

$dN_t^{(\lambda)}$ Increment of Poisson process with intensity $\lambda$




Time-dependent loss distribution: Loss surface

Given in closed form

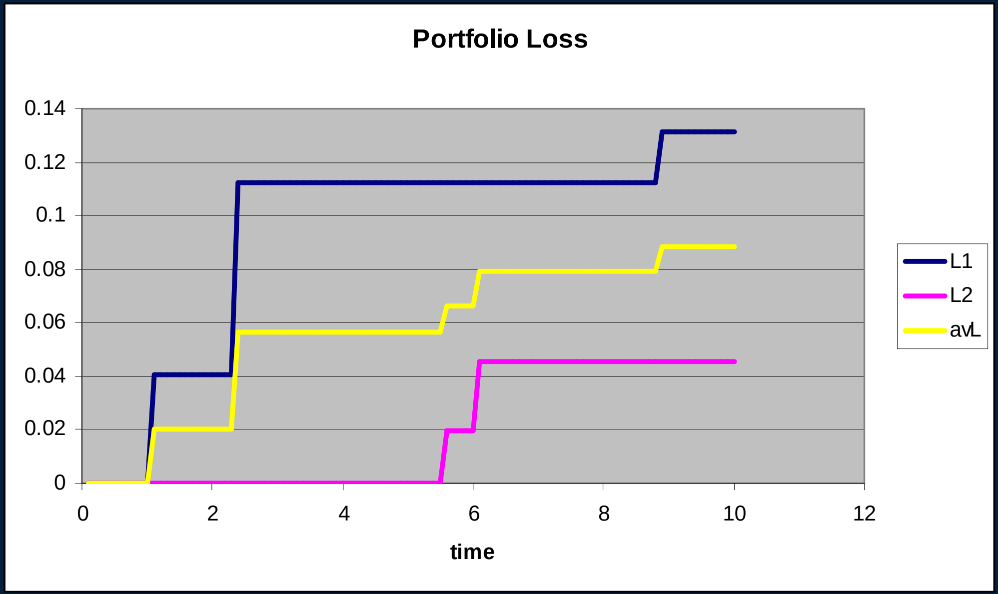
Problem:

No correlation, weak tail for loss distribution

# Portfolio Loss – Compound Poisson Model

$$dL_t = JdN_t^{(\lambda)}$$

$J$ Random jump, corresponds to a cluster of defaults




Loss surface

Can be computed numerically

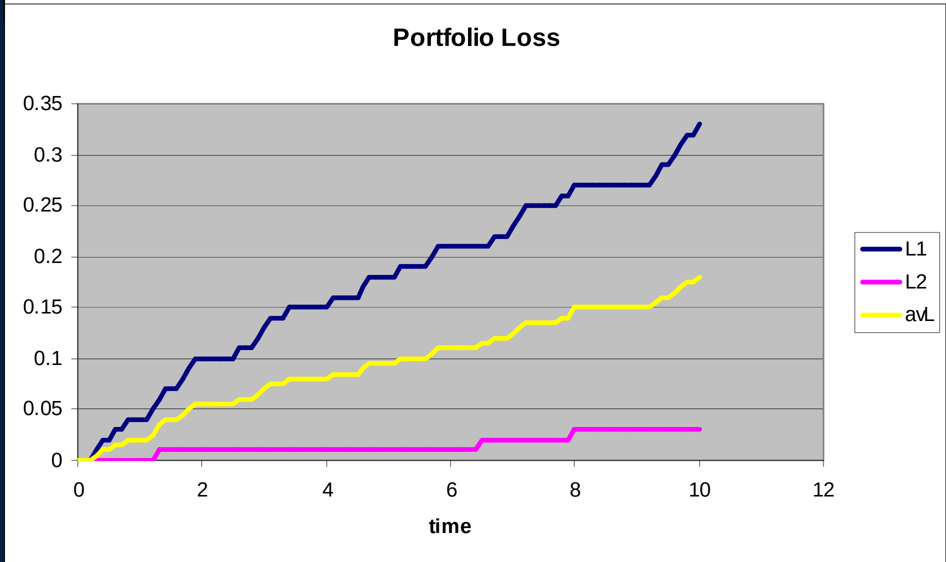
Panjer recursion

Closed form for characteristic function + FFT

Problem:

Simultaneous defaults unrealistic in some applications

# Portfolio Loss – Mixed Poisson Model


$$dL_t = J_0 dN_t^{(\lambda)}$$

$\lambda$ Random intensity

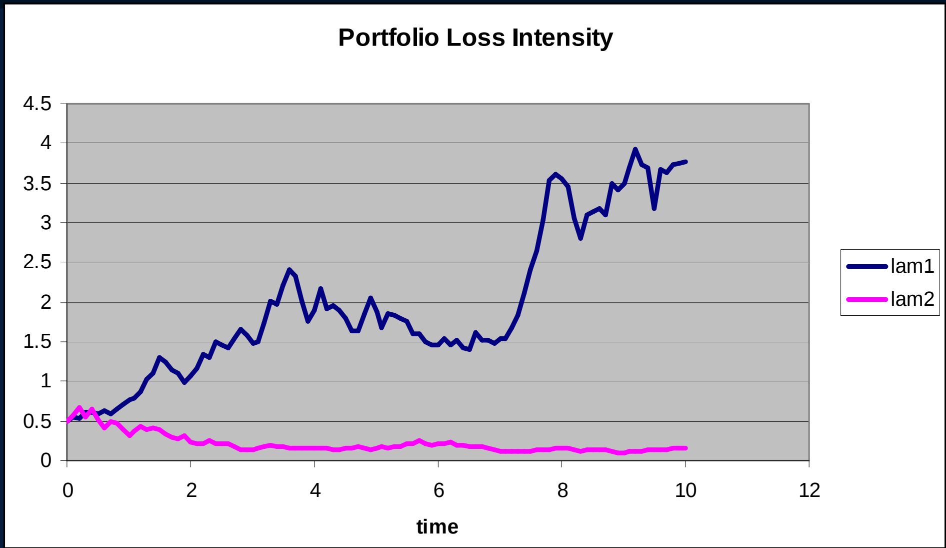
Loss surface

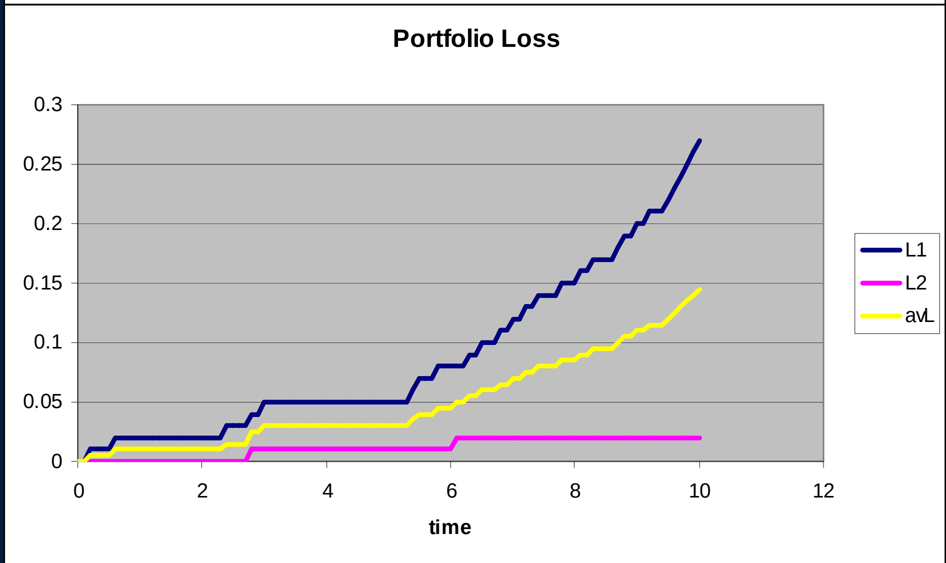
Given in closed form + numerical integration over intensity distribution

Problem:

Value of intensity is determined at start - dynamically not realistic

# Portfolio Loss – Doubly stochastic (Cox) Model




$$d\lambda_t = \sigma\sqrt{\lambda_t}\,dW_t$$

$$dL_t = J\,dN_t^{(\lambda_t)}$$

Loss surface

Complicated closed form for characteristic function + FFT

Problem:

No explicit interaction between names

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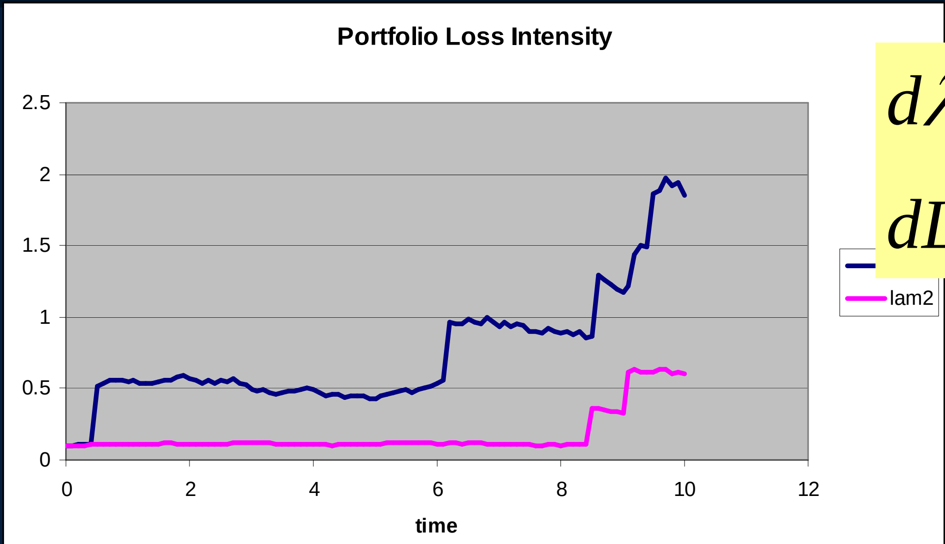
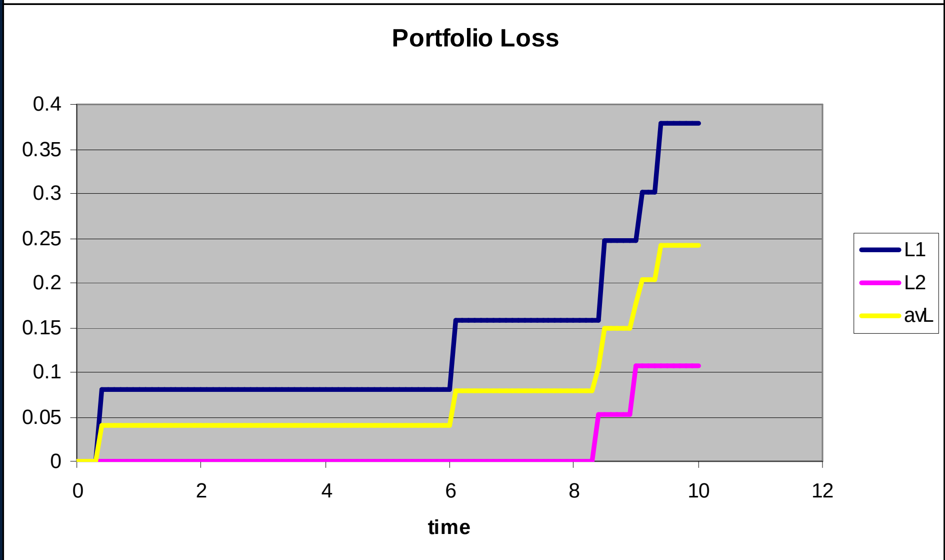
# Portfolio Loss – Self-exciting (Hawkes) Model




$$d\lambda_t = \sigma\sqrt{\lambda_t}\,dW_t + D\,dN_t^{(\lambda_t)}$$

$$dL_t = J\,dN_t^{(\lambda_t)}$$




Loss surface

Solving ODE
for characteristic function +
FFT

Problem:

Hard to find instruments
to calibrate

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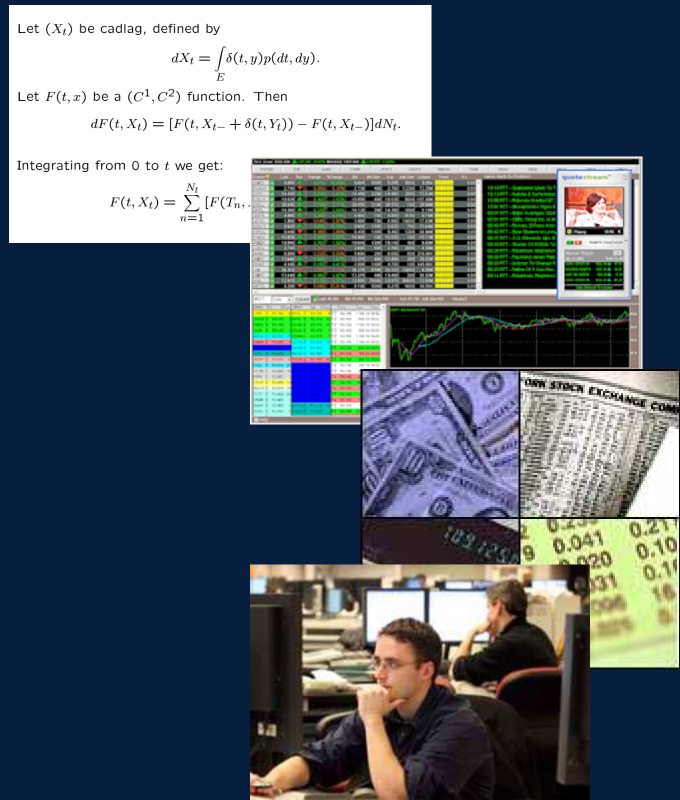
# We are currently hiring Mathematical Modelers

- Ph.D. or near-Ph.D. in a quantitative area
- Finance background not required
- Must love analytic thinking

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