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Application of Point Processes in Modelling Credit Derivatives

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Morgan Stanley Hungary Analytics Ltd.

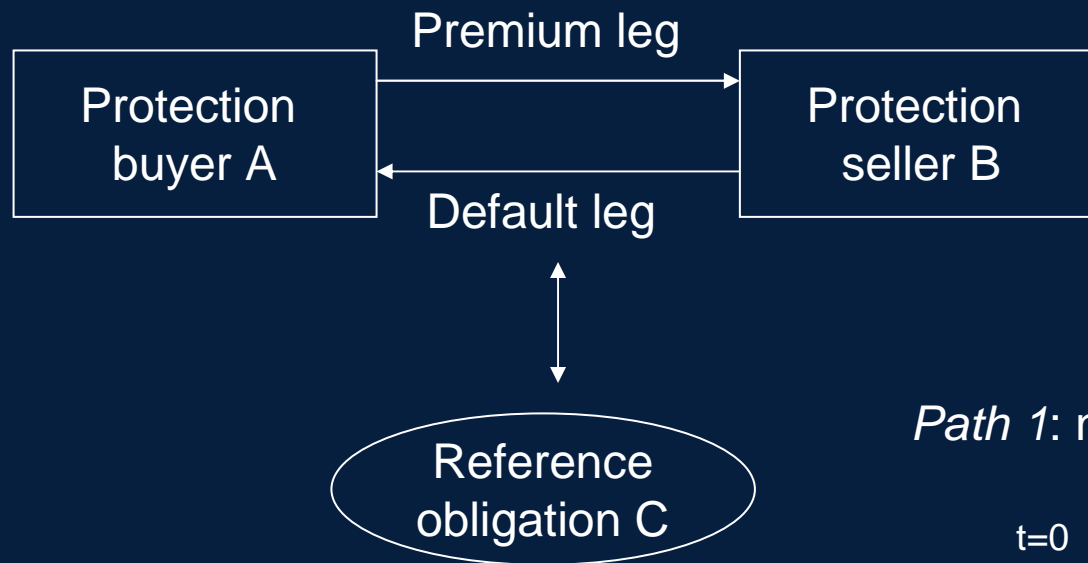
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We will discuss

- Credit Default Swaps (CDS)
- Multi-name Credit Derivatives
- Simple Poisson Process
- Compound Poisson Process
- Mixed Poisson Process
- Poisson Process with Stochastic Intensity (Cox Process)
- Self-exciting Poisson Process (Hawkes Process)

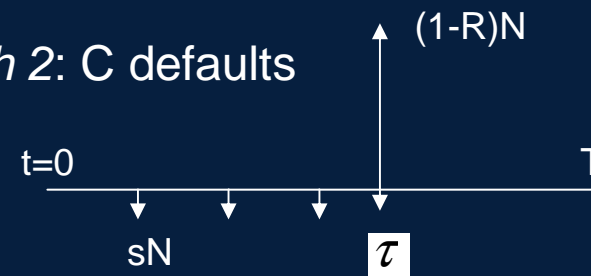
CDS - Kinetics



Path 1: no default of C

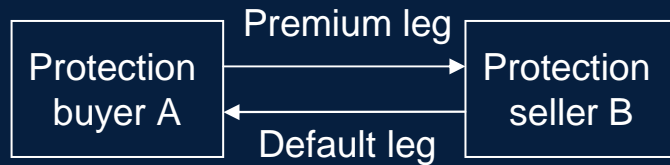


Path 2: C defaults



N – notional [\$]
 s – spread [bps/year]
 R – recovery [1]
 T – maturity [year]

CDS – Constant hazard rate (toy) model



- Default intensity $h = \text{const}$
- Continuous premium payment
- Fixed recovery
- Const interest rate

Default probability

$$P(t < \tau < t + \Delta t) = \underbrace{e^{-ht}}_{\text{survives up to } t} \cdot \underbrace{h\Delta t}_{\text{defaults in interval}}$$

defaults in interval
survives up to t

Default leg

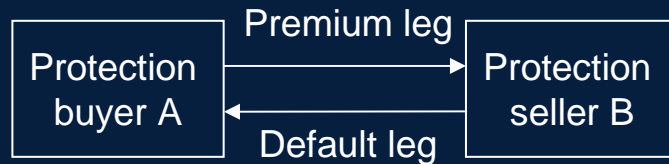
$$\text{DefPV} = N \int_0^T e^{-rt} \cdot \underbrace{(1-R)e^{-ht} h dt}_{\text{expected default payment at } t}$$

Premium leg

$$\text{PremPV} = N \int_0^T e^{-rt} \cdot \underbrace{se^{-ht} dt}_{\text{expected premium payment at } t}$$

$$\text{PremPV}_1 = N \int_0^T e^{-rt} \cdot e^{-ht} dt$$

CDS – Fair spread



The fair spread s_{fair} makes the contract worth 0

$$\text{DefaultPV} = s_{\text{fair}} \cdot \text{PremPV}_1$$



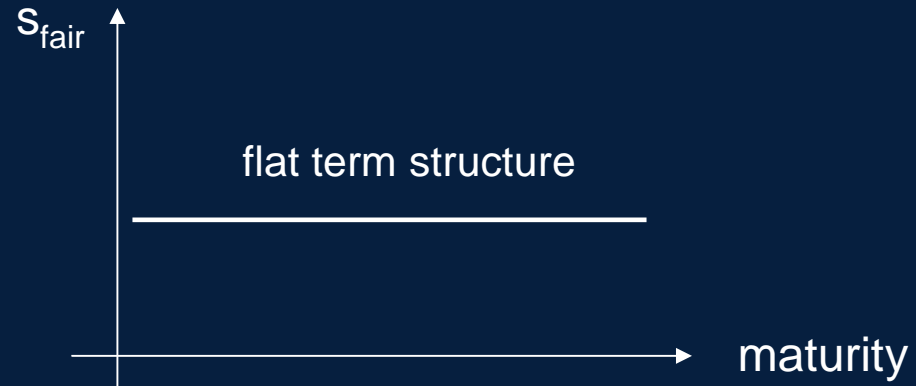
$$s_{\text{fair}} = \frac{\text{DefaultPV}}{\text{PremiumPV}_1} = (1 - R)h$$

CDS - Term structure

Simple model:

$$S_{\text{fair}} = (1 - R)h$$

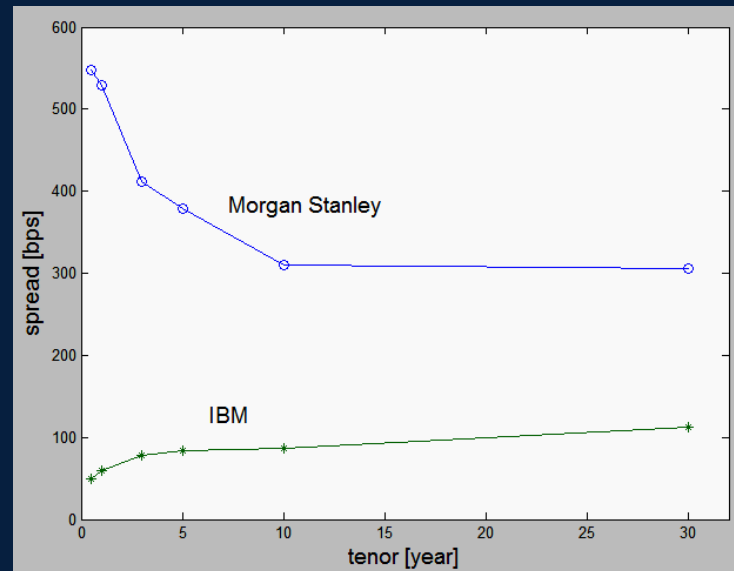
Does not depend on maturity T



Market term structure of CDS spreads on 26/2/2009

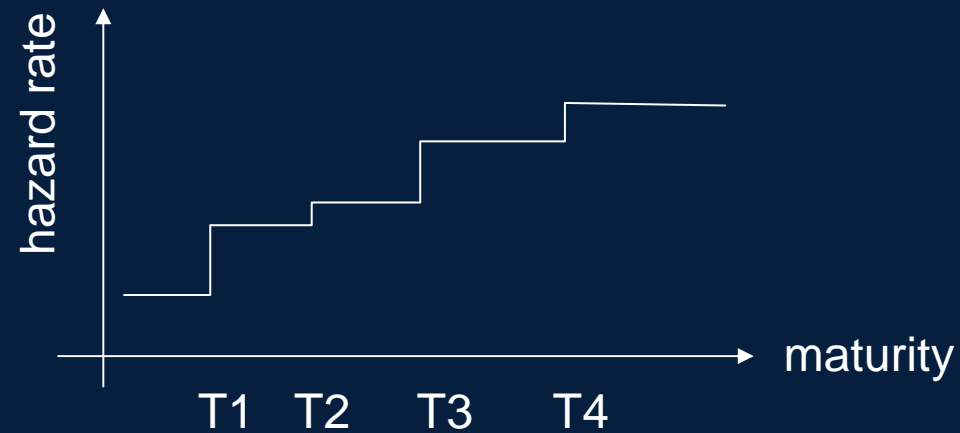


Default intensity is time dependent



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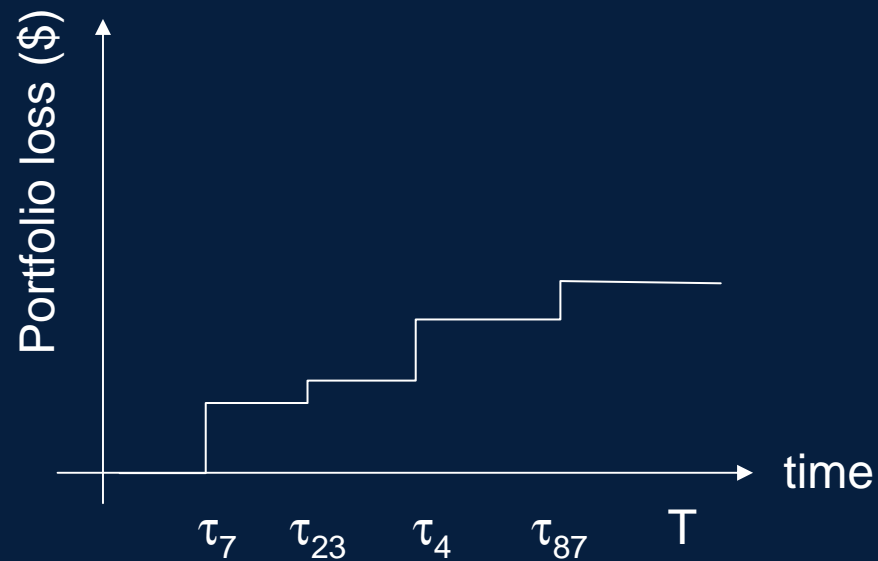
CDS - Calibration



Piece-wise constant hazard rate model can be calibrated to market term structure by bootstrapping

$$\{s_1, s_2, \dots, s_T\} \rightarrow h_1 \rightarrow h_2 \rightarrow \dots \rightarrow h_T$$

Multi-name credit

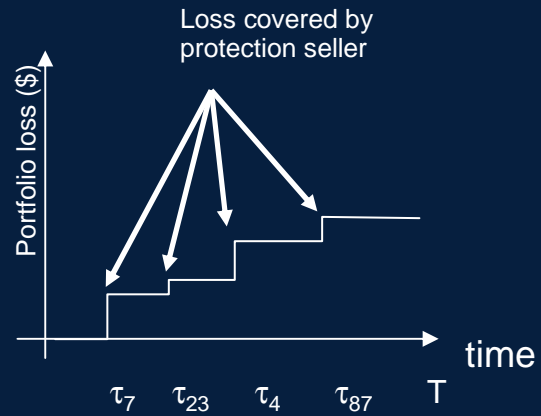


Sample path of portfolio loss

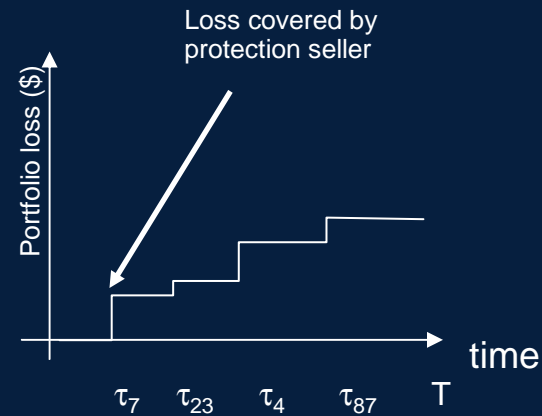
Portfolio of names

CDX-NA-IG index:
125 North American
investment grade
CDSs

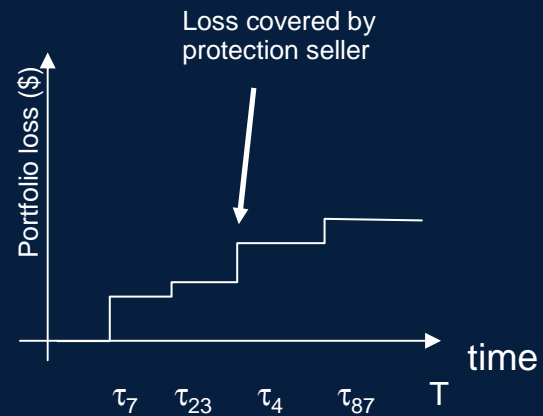
Multi-name credit - Products



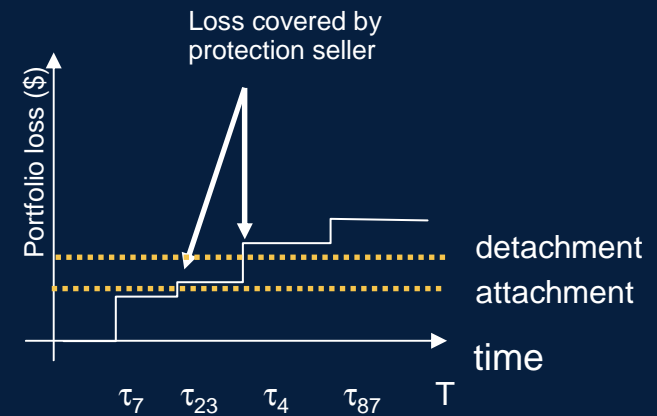
Index swap



FTD (1st to default basket)

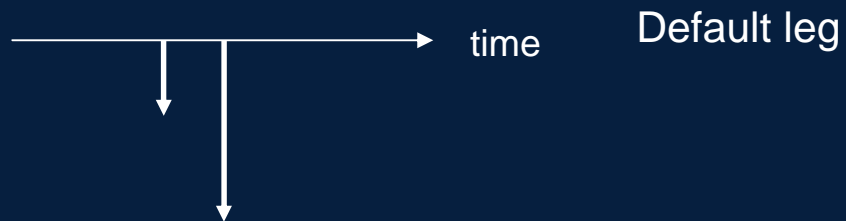
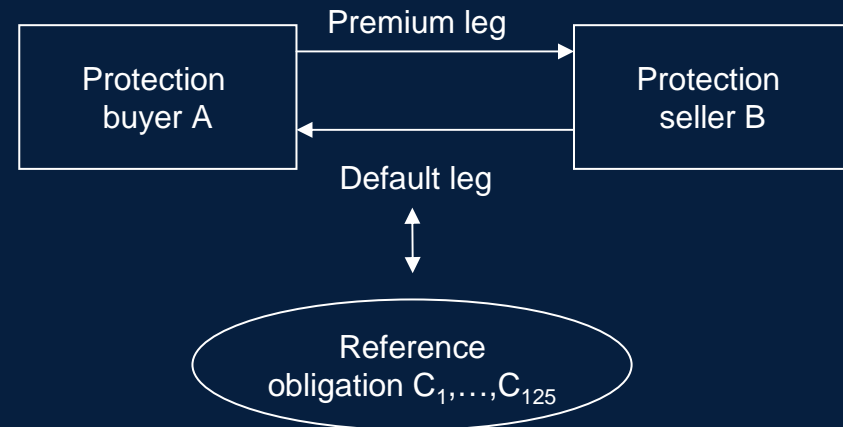
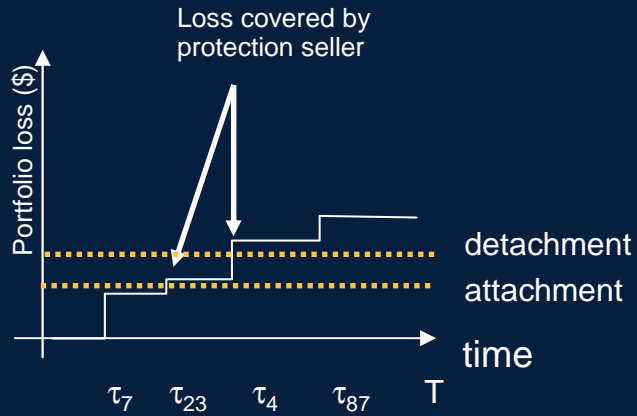


n-th to default basket



CDO (tranche swap)

Multi-name credit - Kinetics



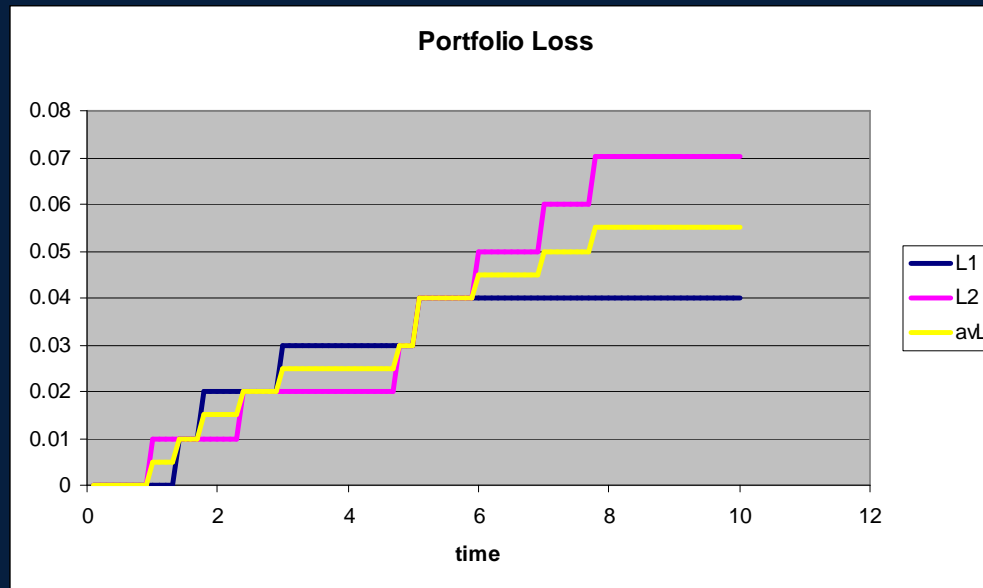
Portfolio Loss – Poisson Model

$$dL_t = J_0 dN_t^{(\lambda)}$$

J_0 Constant jump, corresponds to a single default

$dN_t^{(\lambda)}$ Increment of Poisson process with intensity

λ



Time-dependent loss distribution: Loss surface

Given in closed form

Problem:

No correlation,
weak tail for loss
distribution

Portfolio Loss – Compound Poisson Model

$$dL_t = J dN_t^{(\lambda)}$$

J

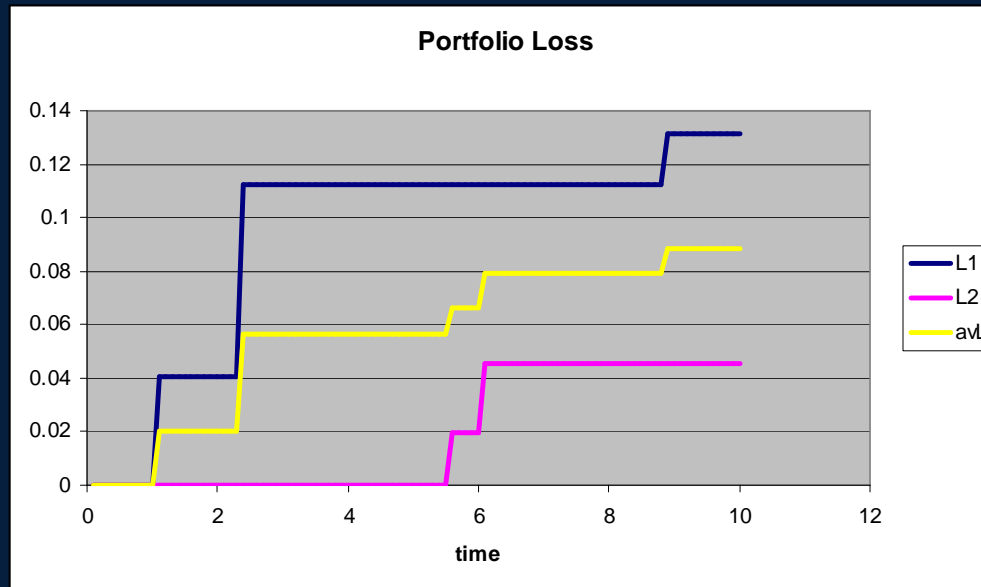
Random jump, corresponds to a cluster of defaults

Loss surface

Can be computed numerically

Panjer recursion

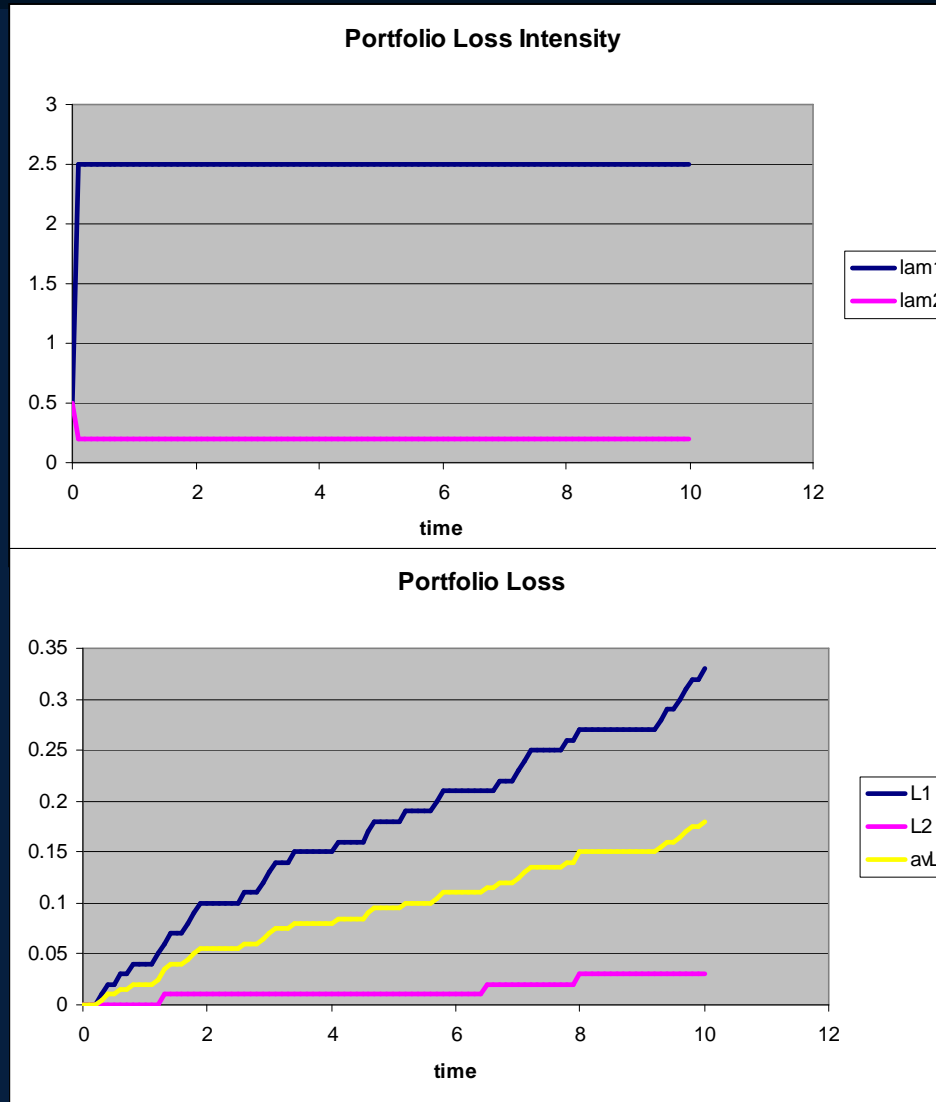
Closed form for characteristic function + FFT



Problem:

Simultaneous defaults unrealistic in some applications

Portfolio Loss – Mixed Poisson Model



$$dL_t = J_0 dN_t^{(\lambda)}$$

λ Random intensity

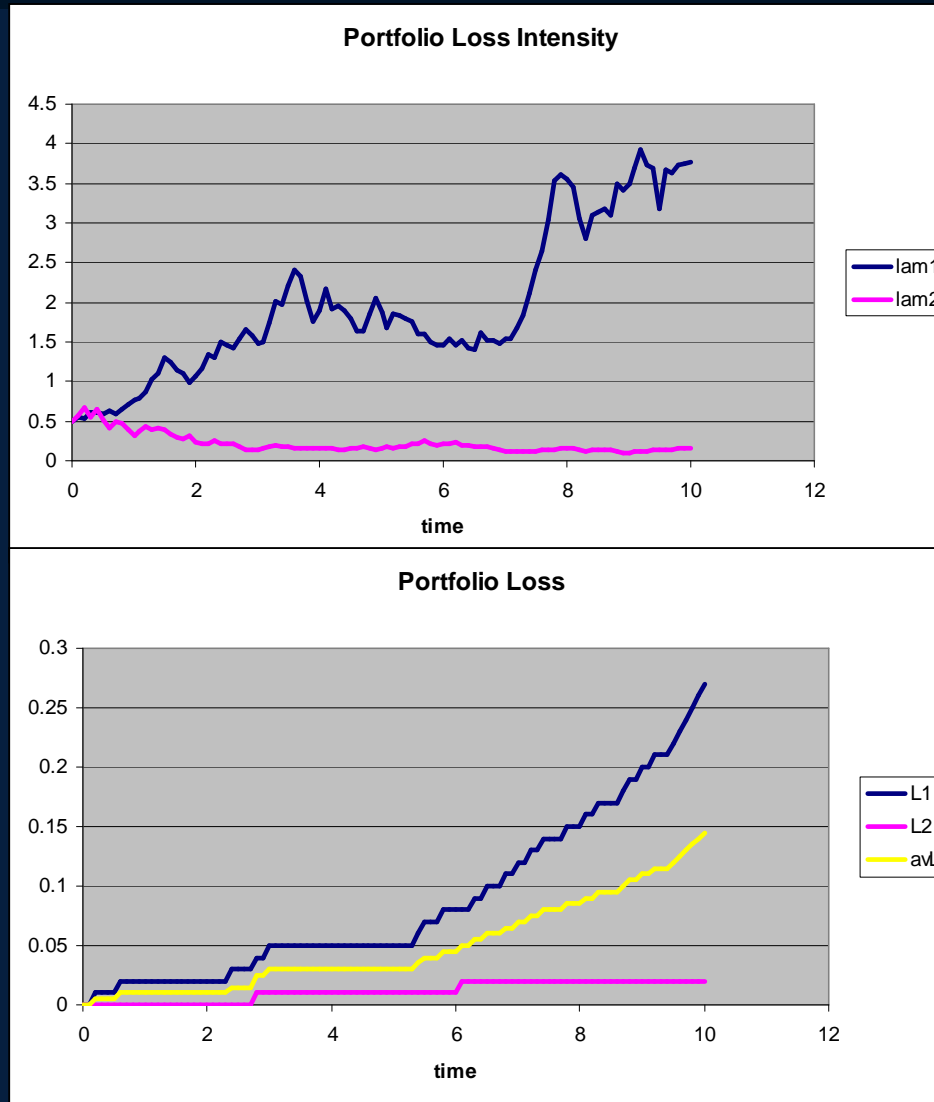
Loss surface

Given in closed form +
numerical integration over
intensity distribution

Problem:

Value of intensity is
determined at start -
dynamically not realistic

Portfolio Loss – Doubly stochastic (Cox) Model



$$d\lambda_t = \sigma \sqrt{\lambda_t} dW_t$$
$$dL_t = J dN_t^{(\lambda_t)}$$

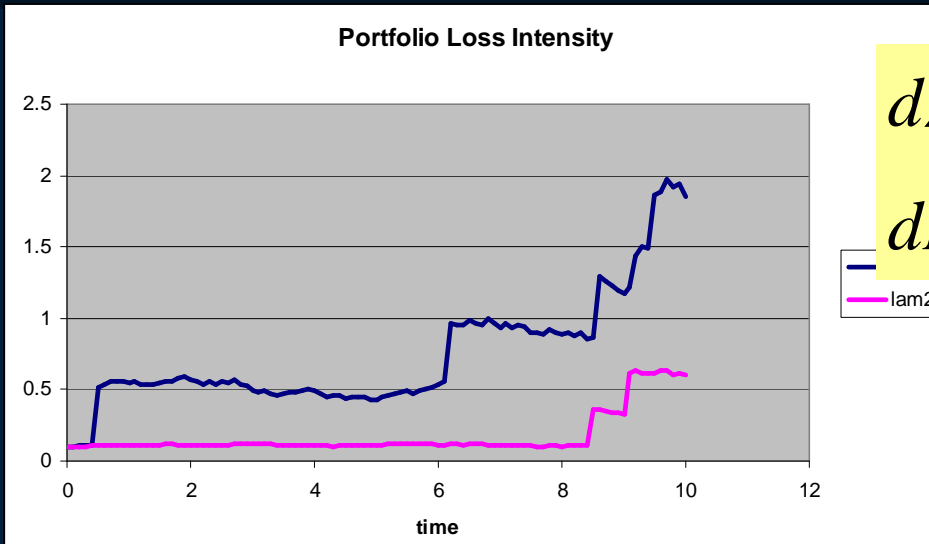
Loss surface

Complicated closed form
for characteristic function +
FFT

Problem:

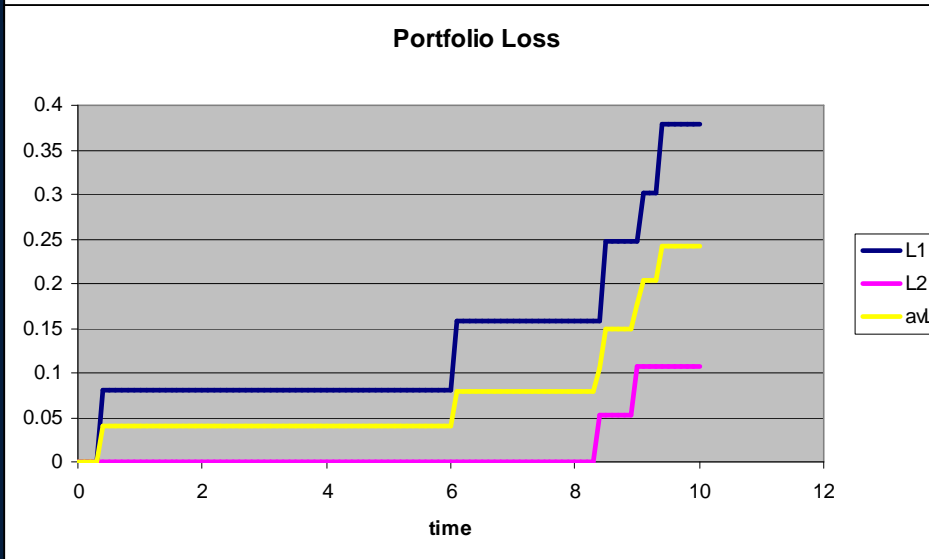
No explicit interaction
between names

Portfolio Loss – Self-exciting (Hawkes) Model



$$d\lambda_t = \sigma \sqrt{\lambda_t} dW_t + D dN_t^{(\lambda_t)}$$

$$dL_t = J dN_t^{(\lambda_t)}$$



Loss surface

Solving ODE
for characteristic function +
FFT

Problem:

Hard to find instruments
to calibrate

We are currently hiring Mathematical Modelers

- Ph.D. or near-Ph.D. in a quantitative area
- Finance background not required
- Must love analytic thinking

Let (X_t) be cadlag, defined by

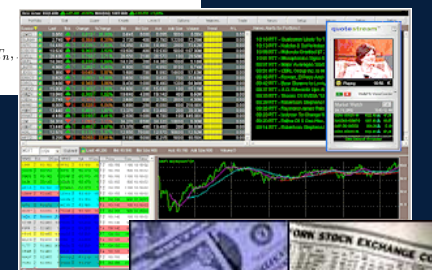
$$dX_t = \int_E \delta(t, y) p(dt, dy).$$

Let $F(t, x)$ be a (C^1, C^2) function. Then

$$dF(t, X_t) = [F(t, X_{t-} + \delta(t, Y_t)) - F(t, X_{t-})] dN_t.$$

Integrating from 0 to t we get:

$$F(t, X_t) = \sum_{n=1}^{N_t} [F(T_n, X_{T_n}) - F(T_n, X_{T_n-})]$$



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