

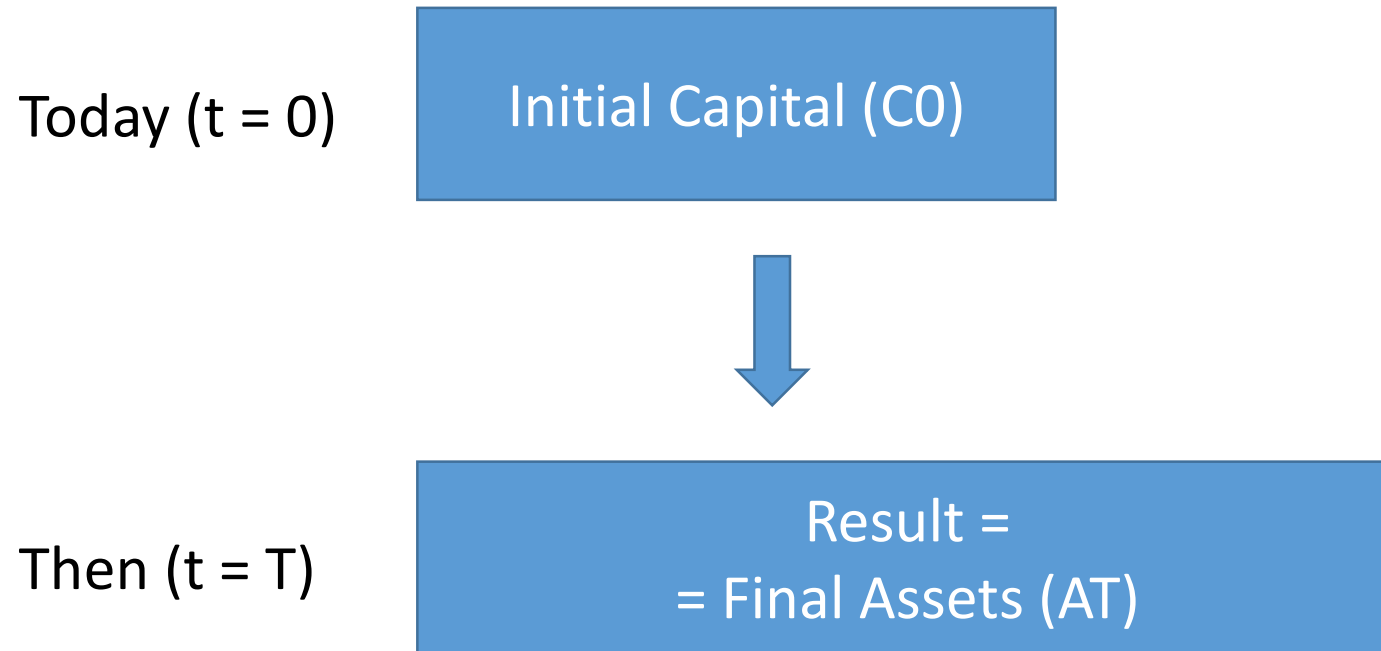
Modeling Optimal Financing Decisions with the Merton Model

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BME

2016

Financing a project



$$\text{Profit} = \text{AT} - \text{C0}$$

$$\text{Yield} = (\text{AT} - \text{C0}) / \text{C0}$$

Logarithmic Annual
Rate of Return on
Initial Investment

$$\text{ROI} = \ln(\text{AT} / \text{C0}) / \text{T}$$

$$\text{AT} = \text{C0} * \exp(\text{ROI} * \text{T})$$

Present Value with Required Yield (PV)

Today (t = 0)

Initial Capital (C0)
-> Initial Assets (A0)



Required Yield (r)

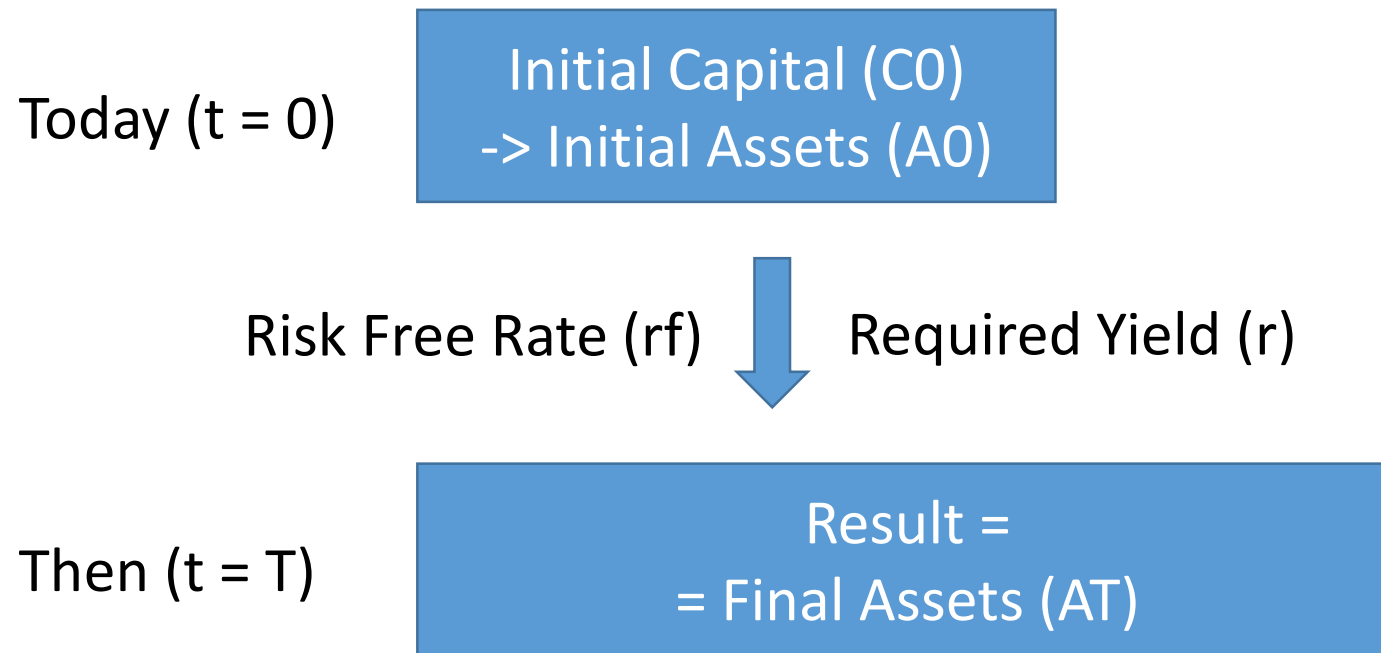
Then (t = T)

Result =
= Final Assets (AT)

$$\begin{aligned} PV(AT) &= A0 = \\ &= AT * \exp(-r * T) = \\ &= C0 * \exp((ROI - r) * T) \end{aligned}$$

Investment is worthy, if
 $A0 > C0$, i.e., if
 $ROI > r$

What if Result is random?



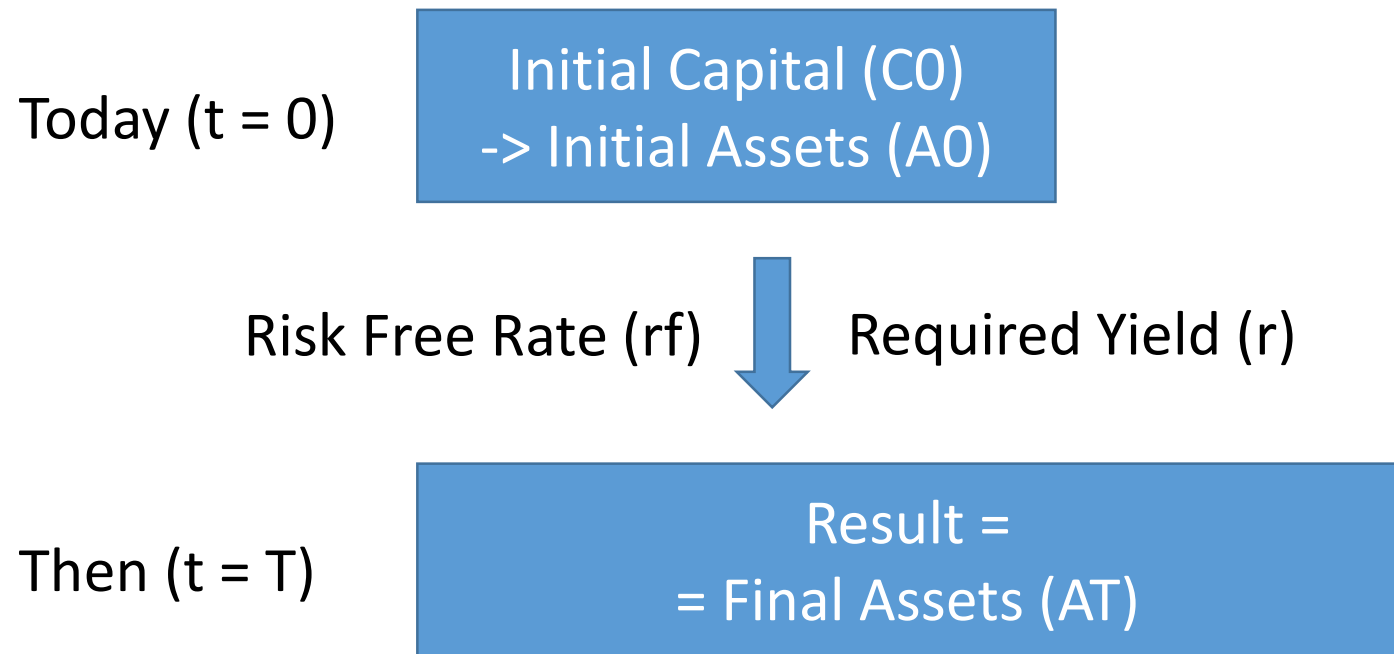
$$A_0 = \exp(-r * T) * E_P[AT]$$

$$A_0 = \exp(-r_f * T) * E_Q[AT]$$

We will use the risk-neutral measure (pricing measure) Q, to compute PV for all the different investments:

$$X_0 = \exp(-r_f * T) * E_Q[X_T]$$

Merton Model: Lognormal Result



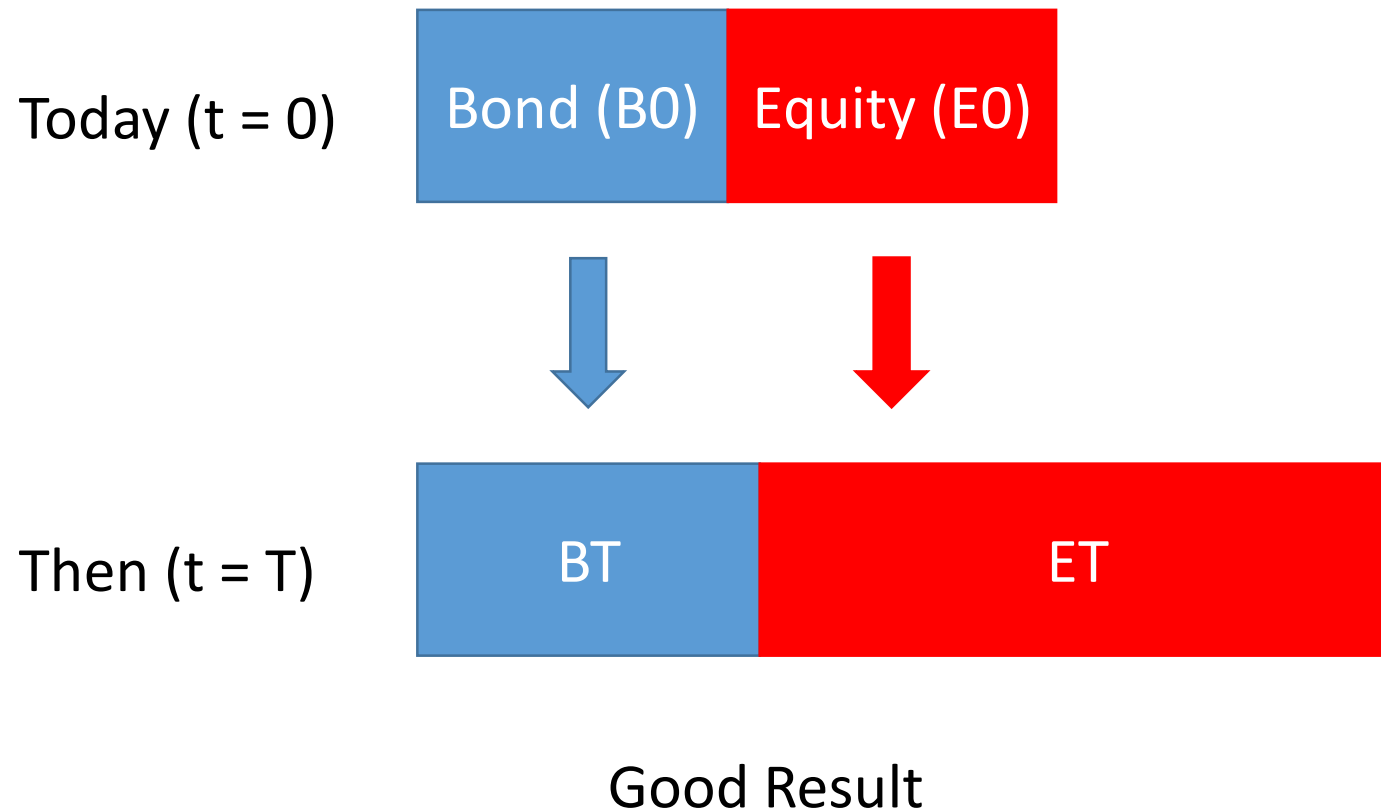
$$A_0 = \exp(-r * T) * E_P[AT]$$

$$A_0 = \exp(-r_f * T) * E_Q[AT]$$

$$AT = \exp(r_f * T) * \exp(\text{sig} * W_T - 0.5 * \text{sig}^2 * T)$$

$$X_0 = \exp(-r_f * T) * E_Q[X_T]$$

Bond versus Equity Investment



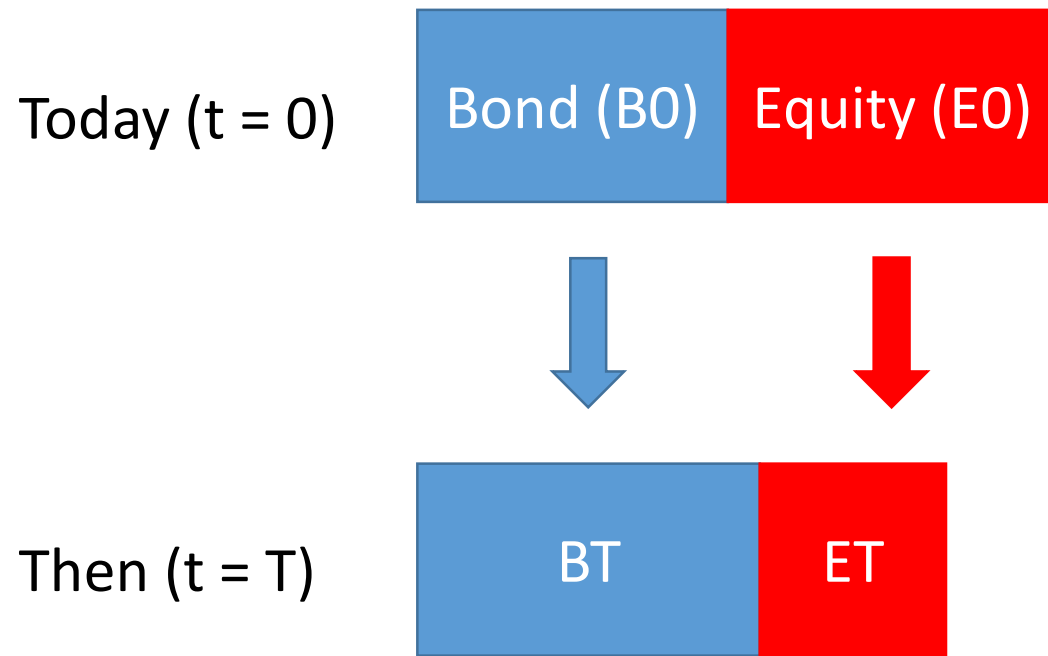
$$A_0 = \exp(-r * T) * E_P[AT]$$

$$AT = \exp(rf * T) * \exp(\text{sig} * W_T - 0.5 * \text{sig}^2 * T)$$

$$B_0 = \exp(-rf * T) * E_Q[BT]$$

$$E_0 = \exp(-rf * T) * E_Q[ET]$$

Bond versus Equity Investment



Bad Result

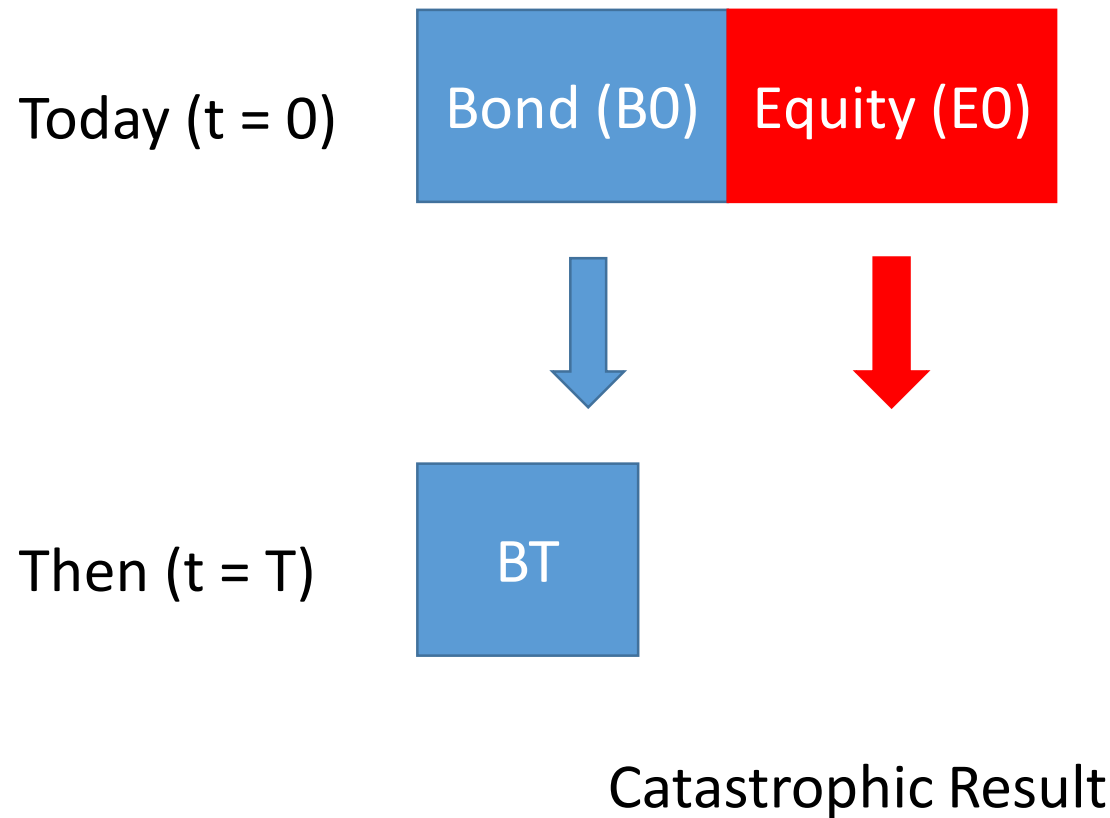
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Bond versus Equity Investment



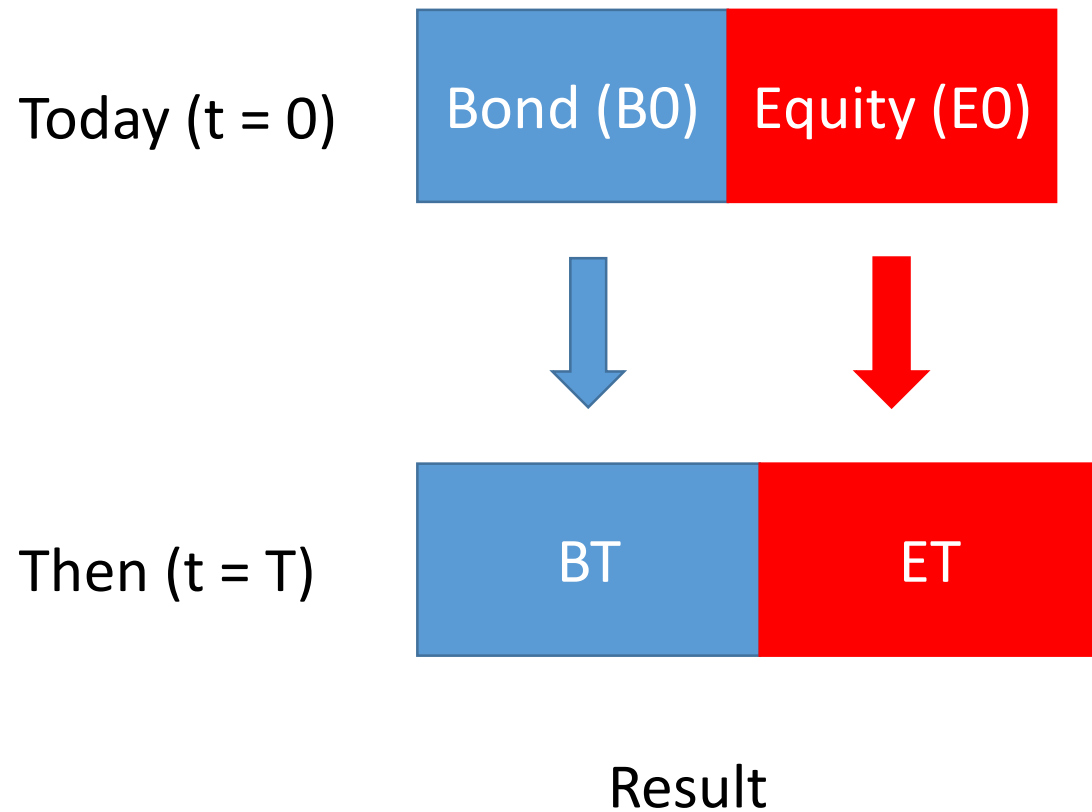
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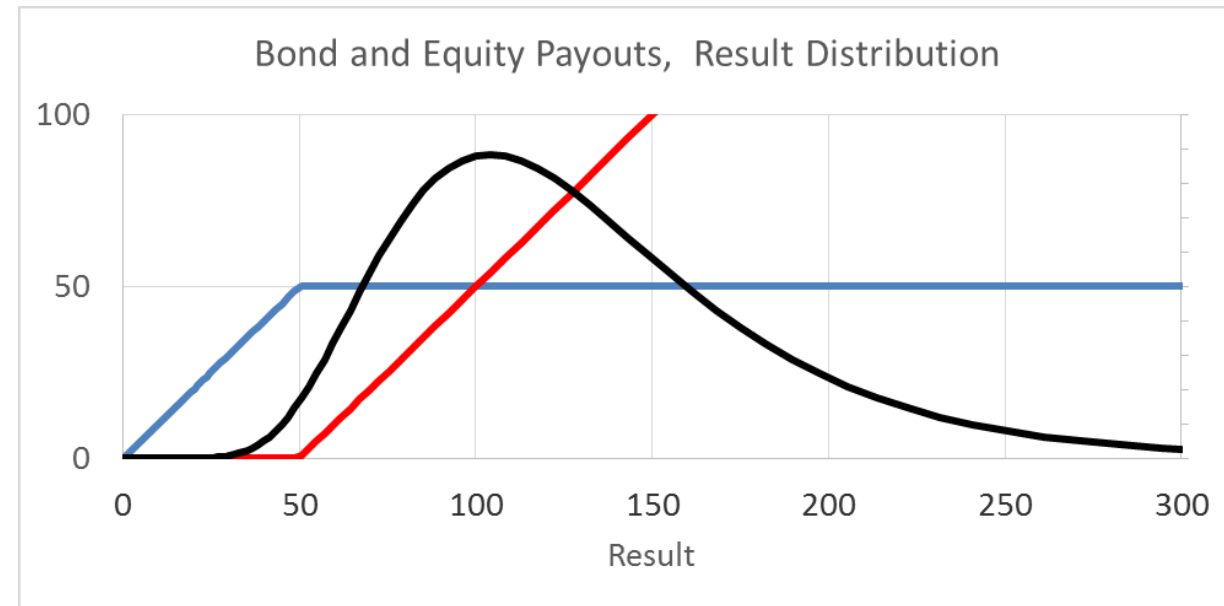
$$B_0 = \exp(-rf * T) * E_Q[BT]$$

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Bond versus Equity Investment Pricing



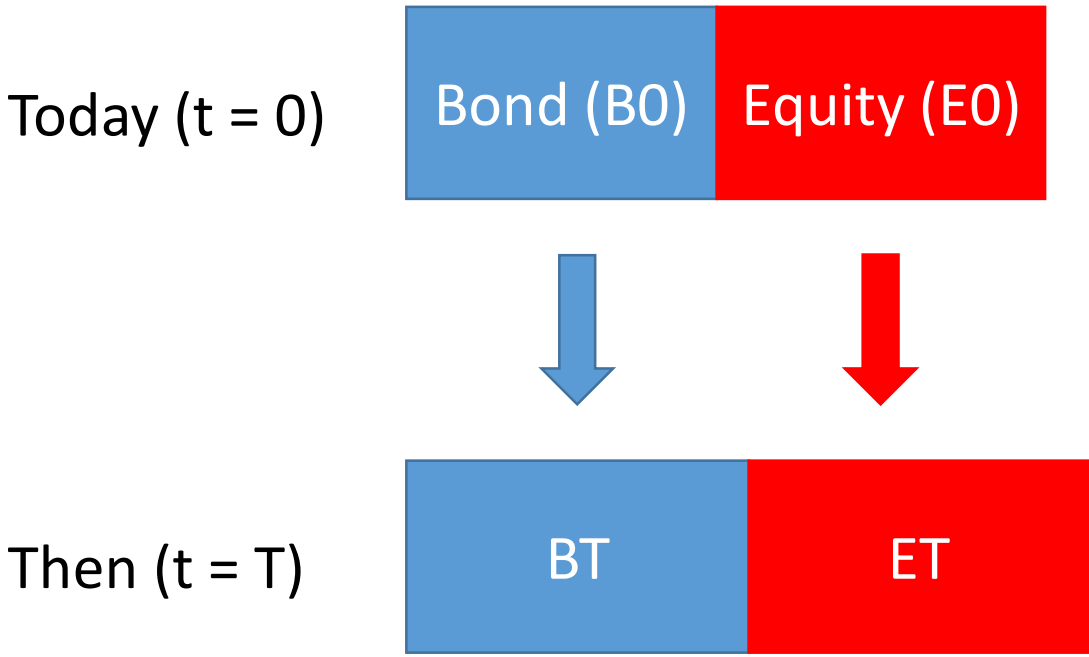
No cost of default



$$B0 = \exp(-rf * T) * E_Q[BT]$$

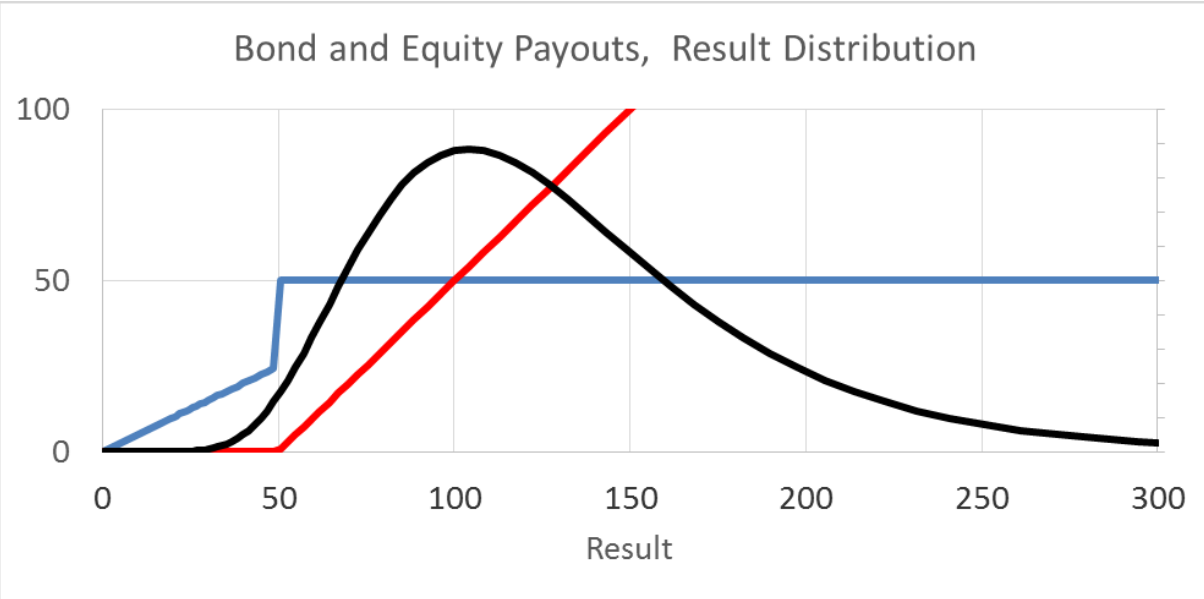
$$E0 = \exp(-rf * T) * E_Q[ET]$$

Bond versus Equity Investment Pricing



Result

With cost of default



$$B0 = \exp(-rf * T) * E_Q[BT]$$

$$E0 = \exp(-rf * T) * E_Q[ET]$$

Let us now experiment with excel ...