Algorithmic Mechanism Design in Matching Markets

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BME
Budapest
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Momentum research group: Mechanism Design, 2016-2021

Péter Biró, group leader

Mathematics MSc:	BME 2003
Economics MSc:	BCE 2007
Mathematics PhD:	BME 2008
Postdoc in Glasgow:	2007-2010
Guest at Stanford:	2014-2015

Ágnes Cseh, group member

Mathematics BSc:	BME 2010
Mathematics MSc:	Berlin 2012
Mathematics PhD:	Berlin 2015
Postdoc in Reykjavík:	2016

Computational Social Choice via matching schemes

We have an economic/social choice problem
with

- participants
- possible outcomes

+Some objective facts
+true preferences of the players over the possible outcomes.

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- participants: students and schools
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- decentralised (e.g., college admissions in the US)
- coordinated (e.g., college admissions in the UK)
- centralised (e.g., college admissions in Hungary and Spain)

Computational Social Choice via matching schemes

We have an economic/social choice problem : e.g., market with

- participants: buyers and sellers
- possible outcomes: matchings with prices

+Some objective facts (e.g., age of buyer)
+true preferences of the players over the possible outcomes.

We shall design such rules or mechanisms that lead to a 'good' solution given the objective facts and true preferences.

The mechanism may be

- decentralised (e.g. usual market)
- coordinated (e.g. eBay)
- centralised (e.g. Google's auction for TV ads in the US)

The main questions are:

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  answers by social scientists, economists, game theorists

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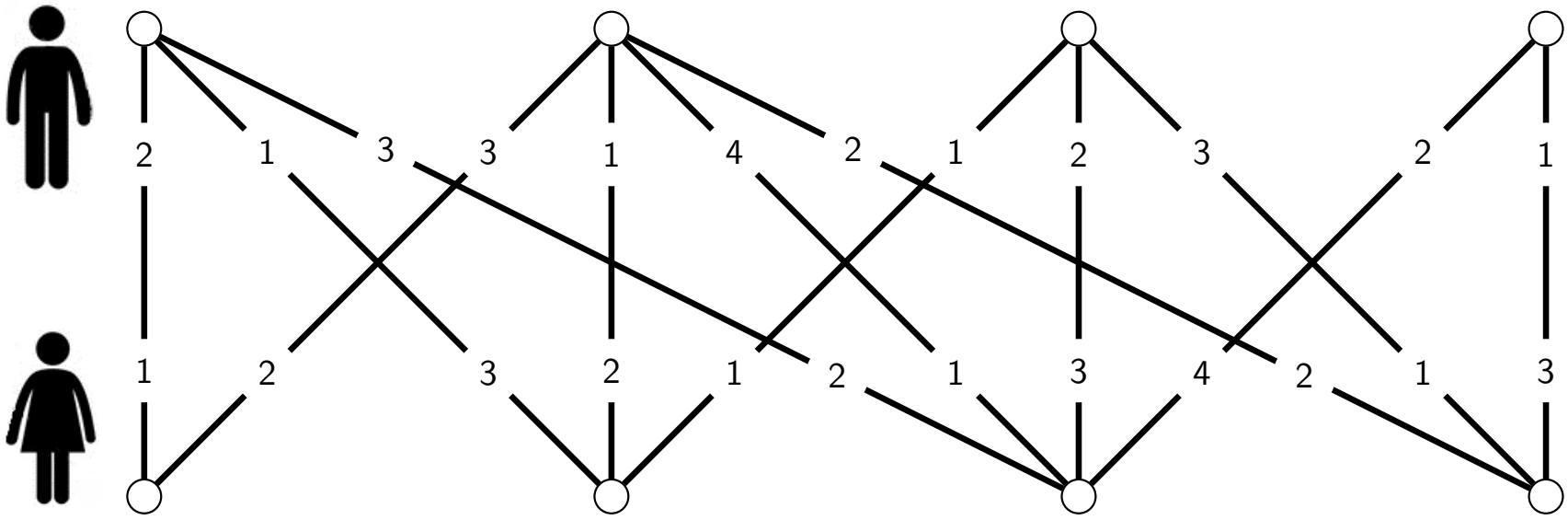
- **Is there any decentralised / coordinated / centralised mechanism that leads to a 'good' solution?**
  answers by game theorists, mechanism designers
- **Can we compute a 'good' solution efficiently via a centralised mechanism?**
  answers by computer scientists, mathematicians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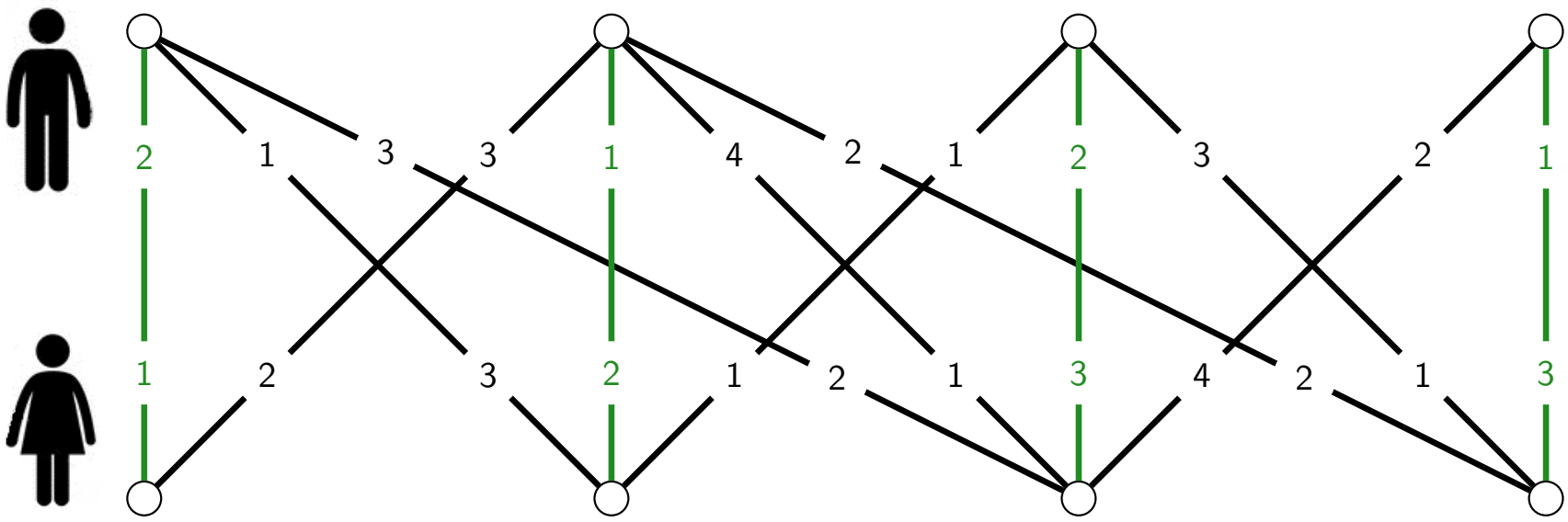
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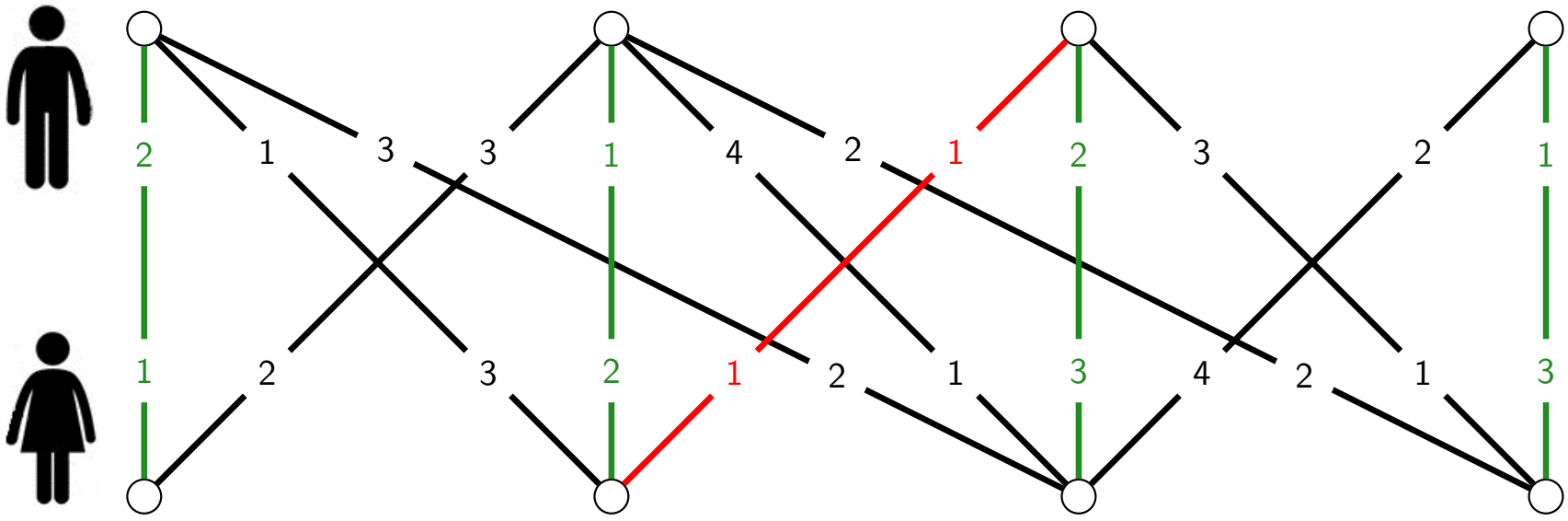

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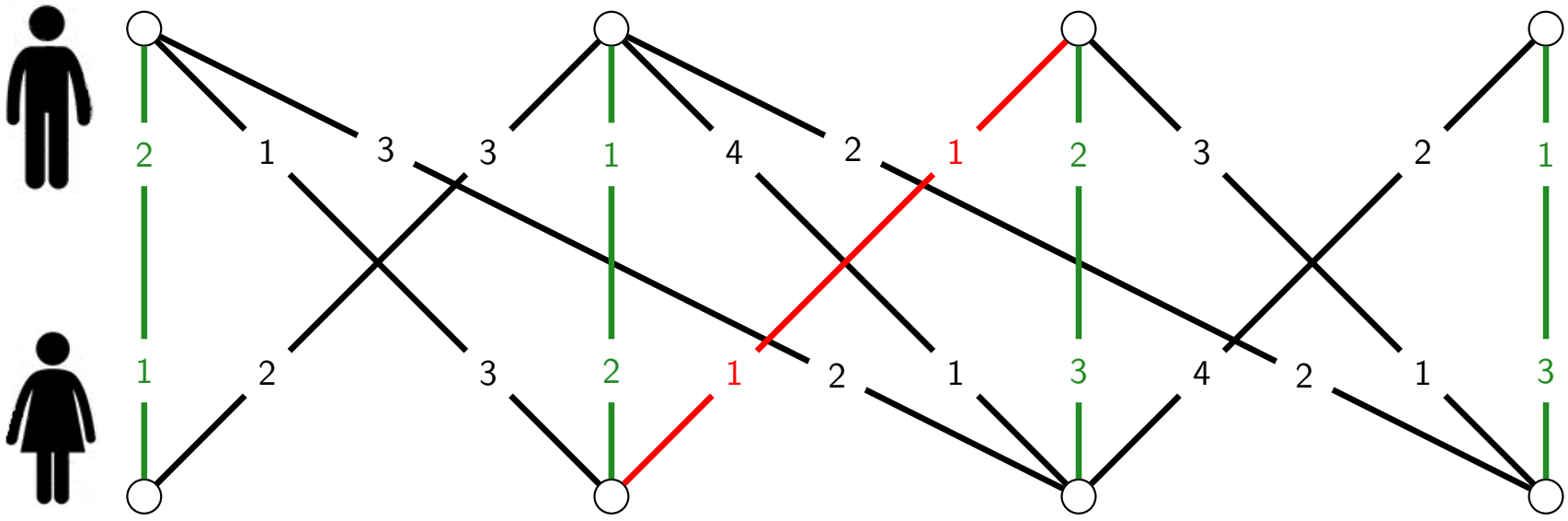

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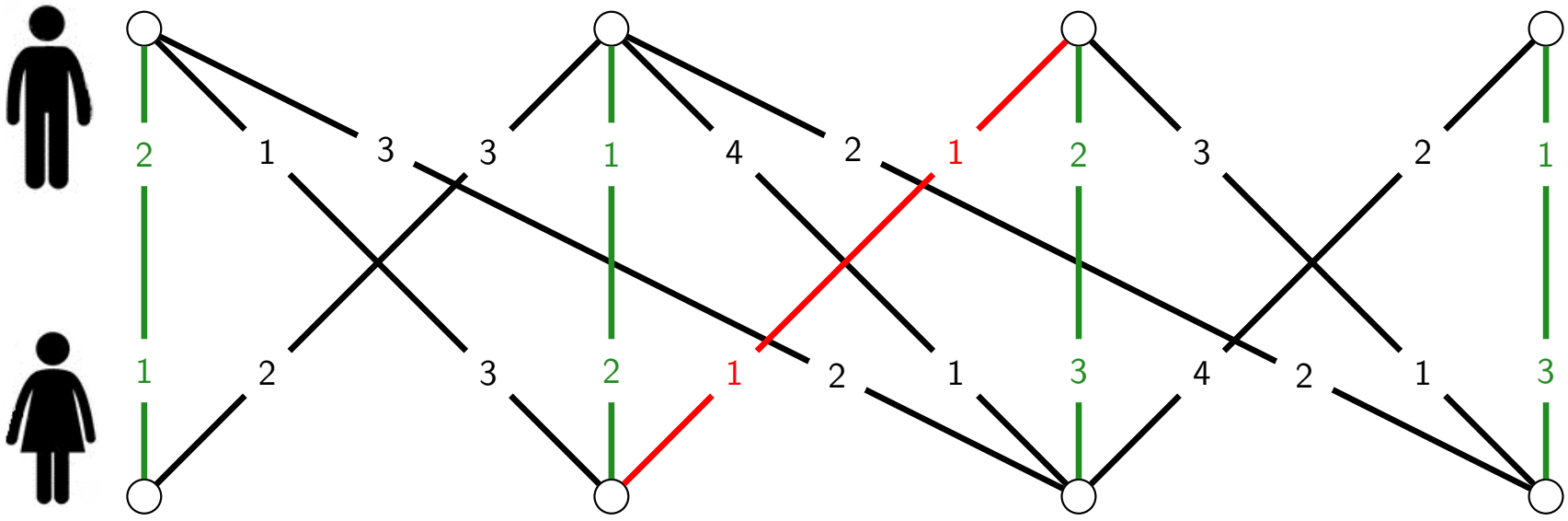

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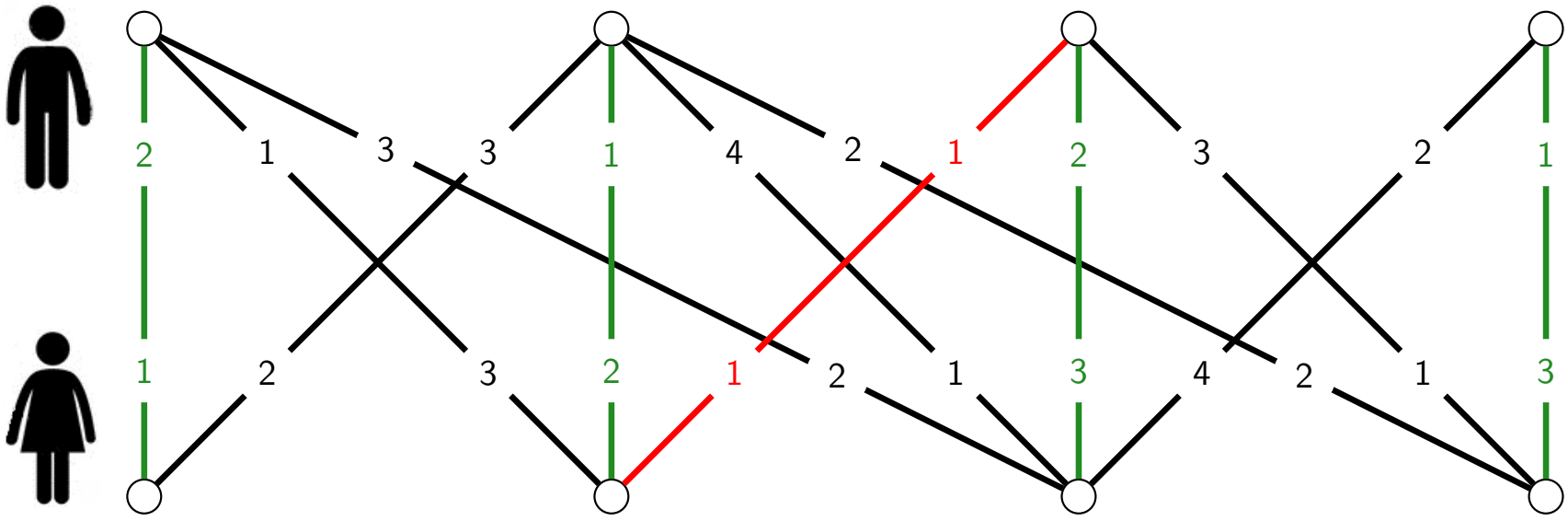

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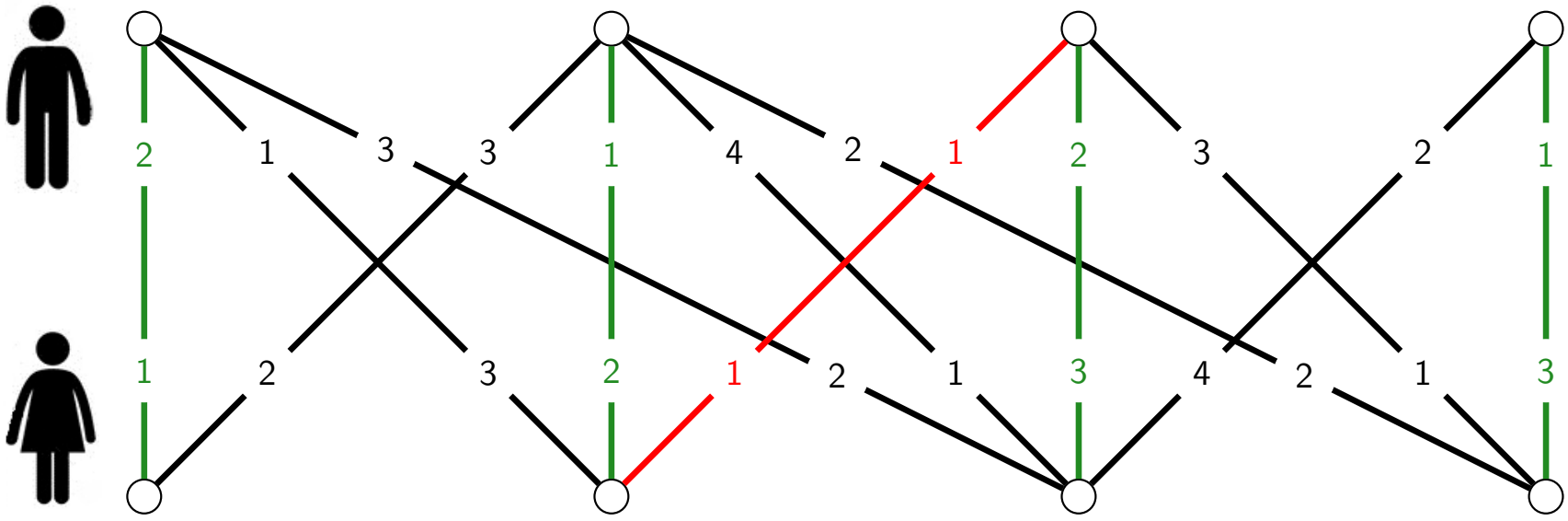

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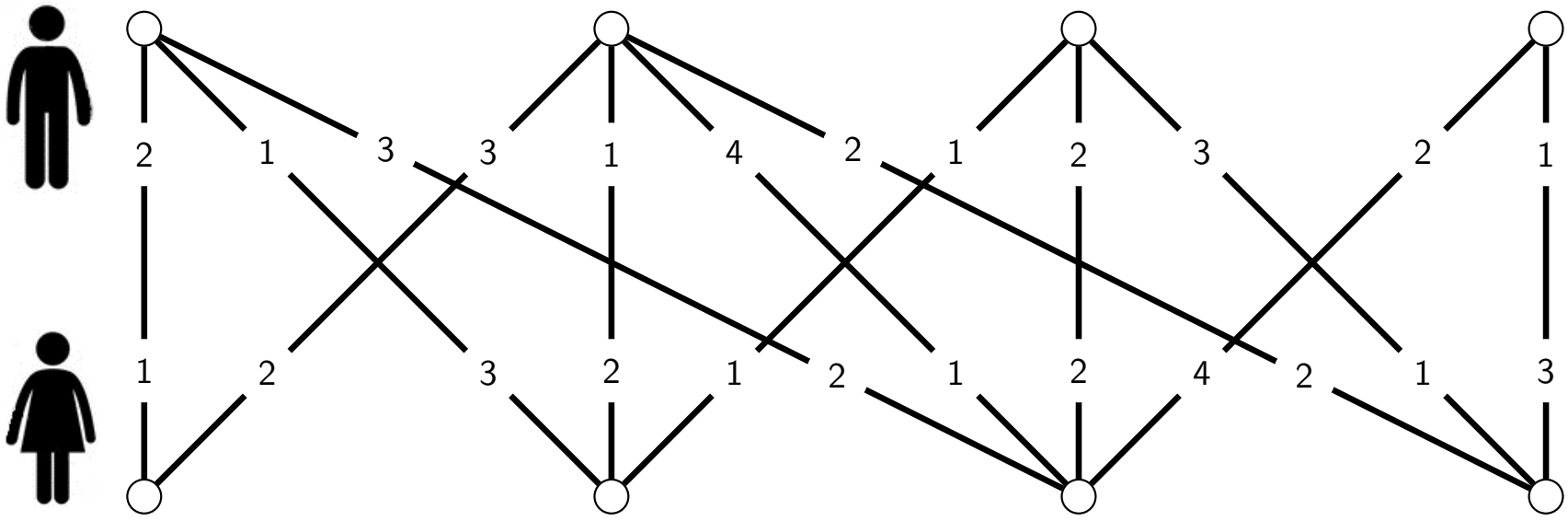
A stable matching always exists.



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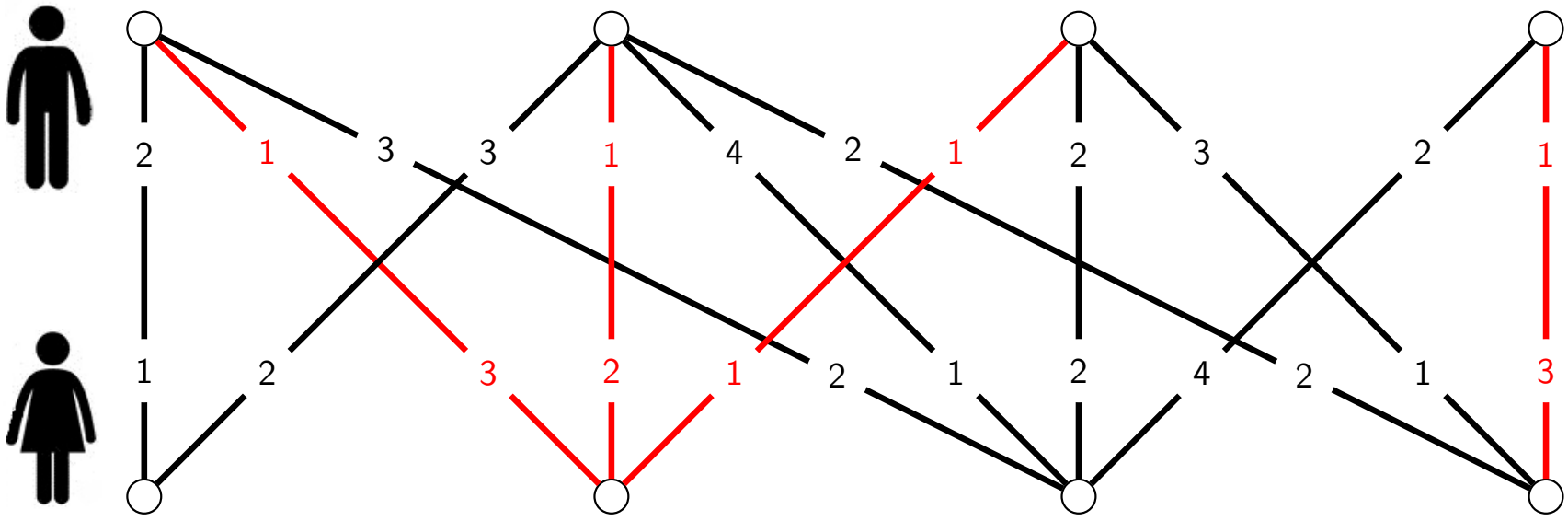

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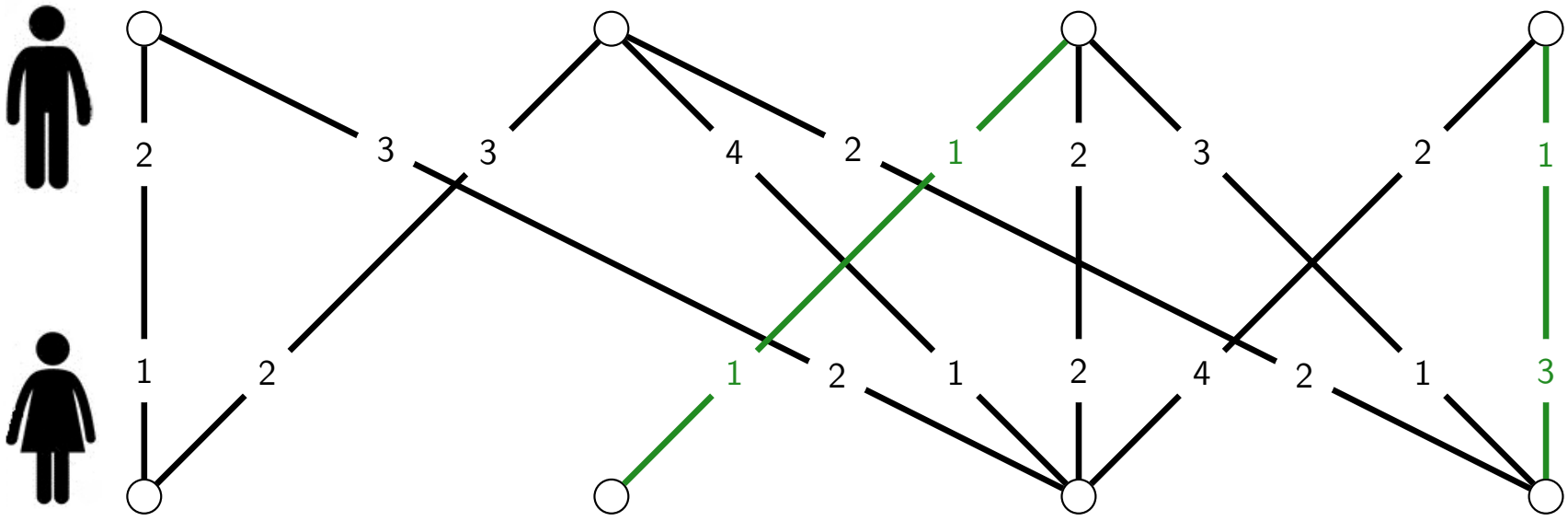

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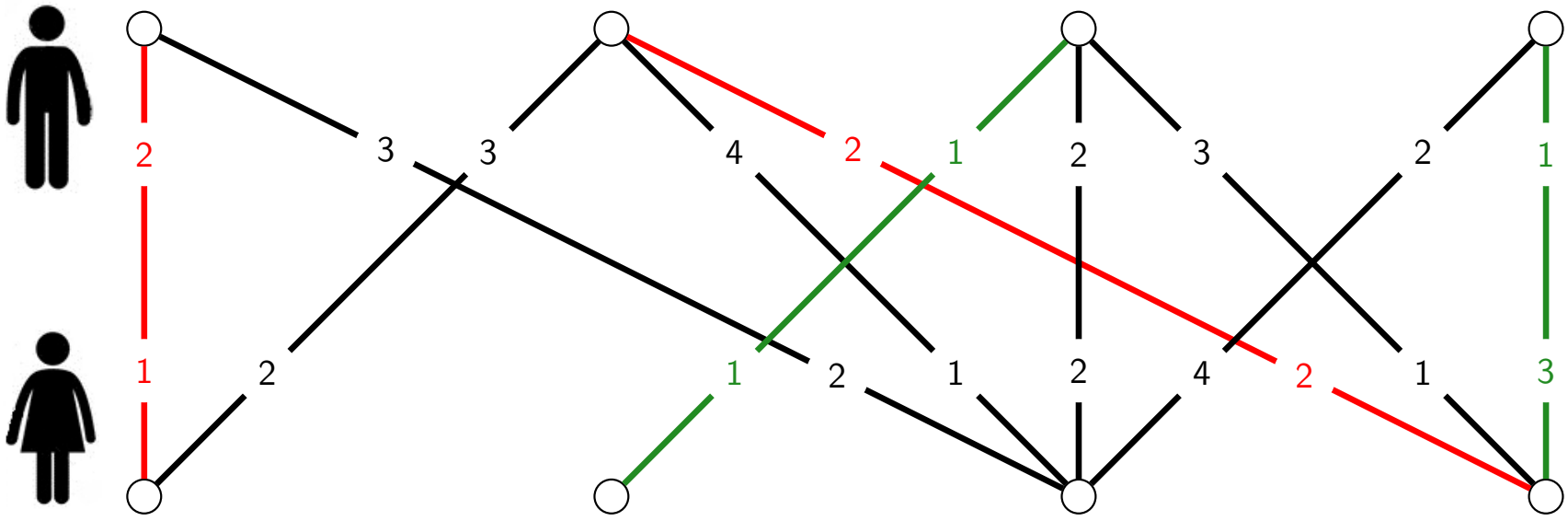

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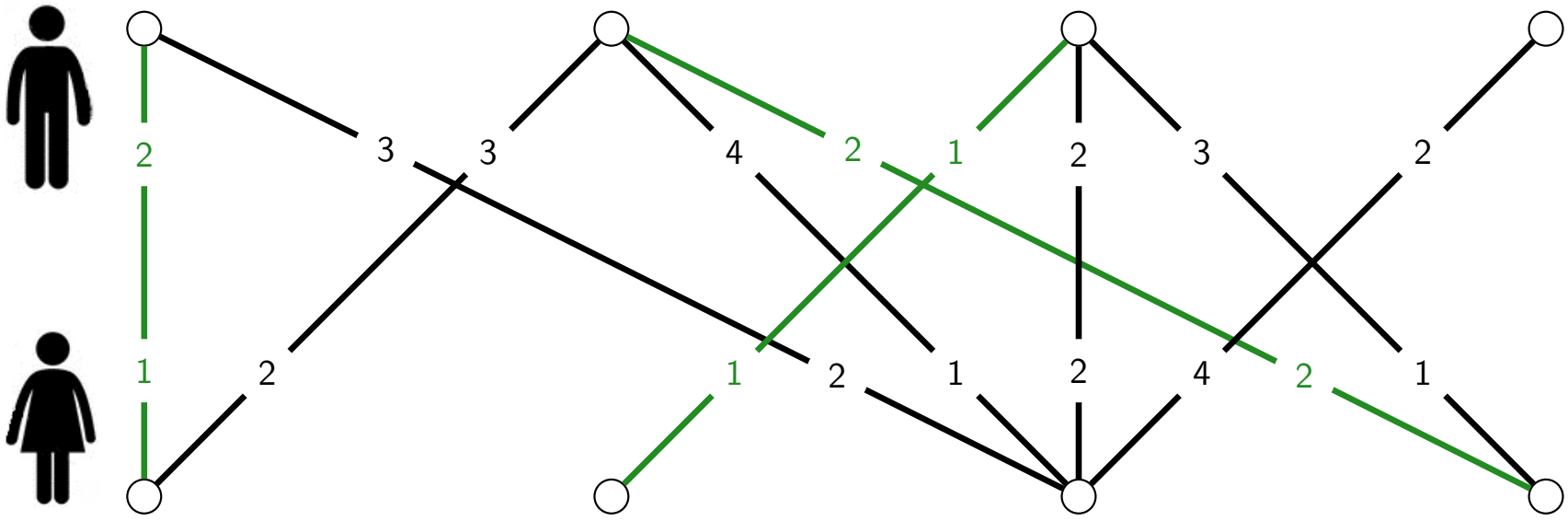

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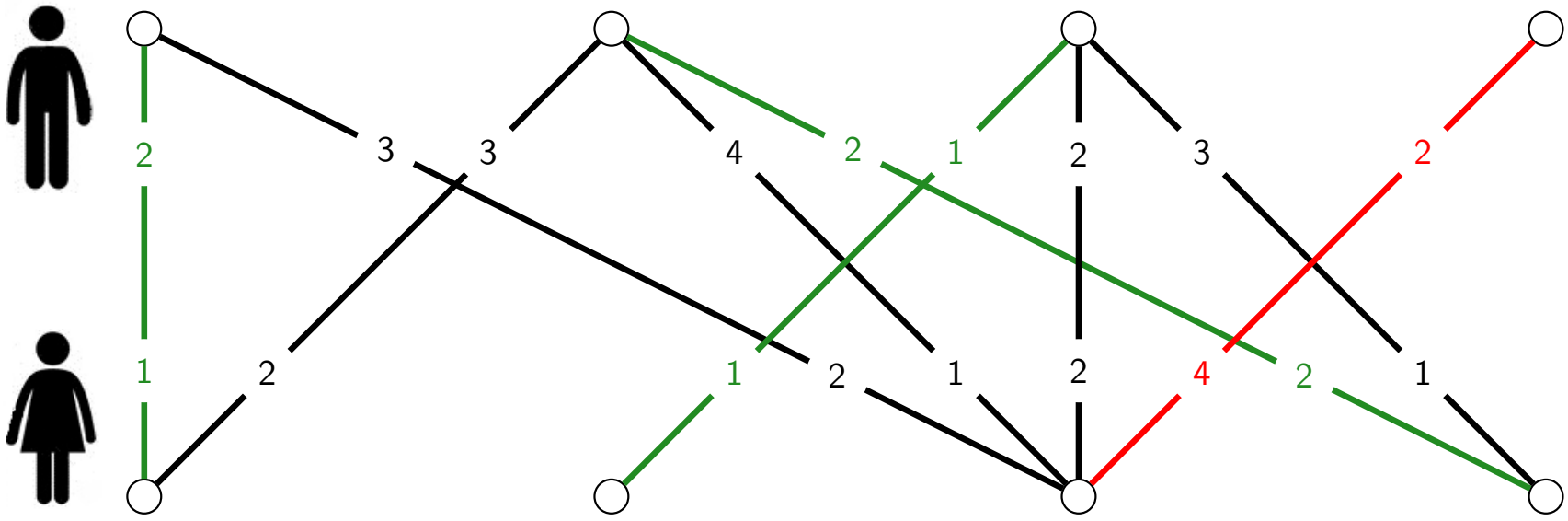

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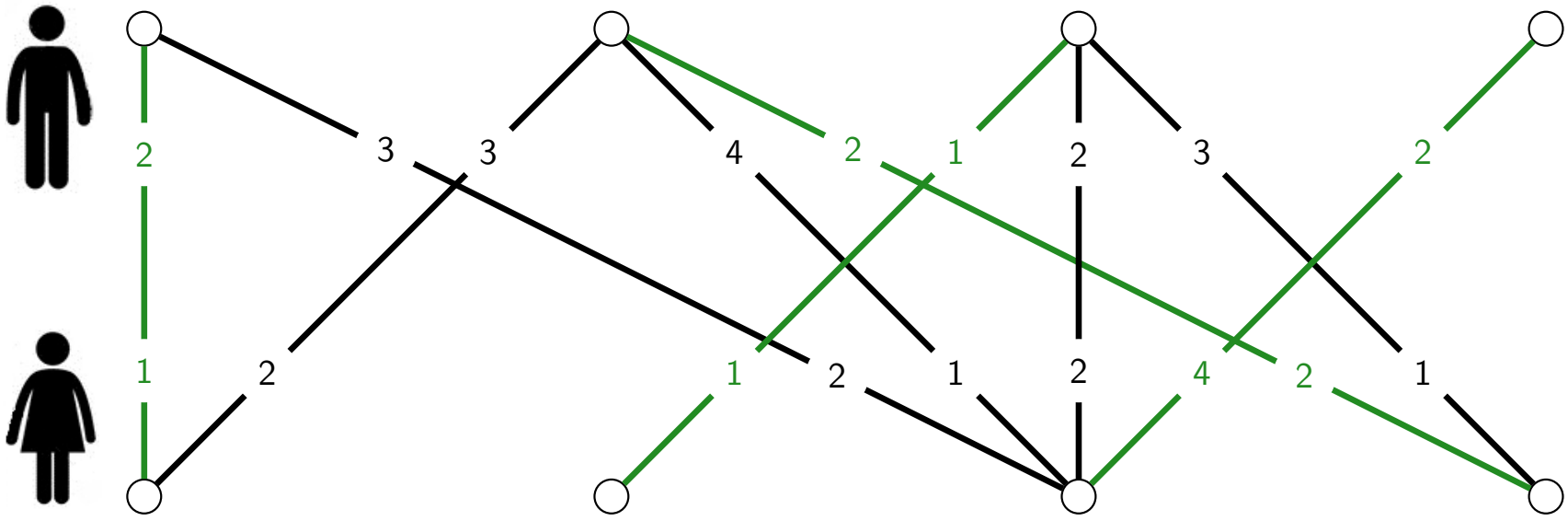

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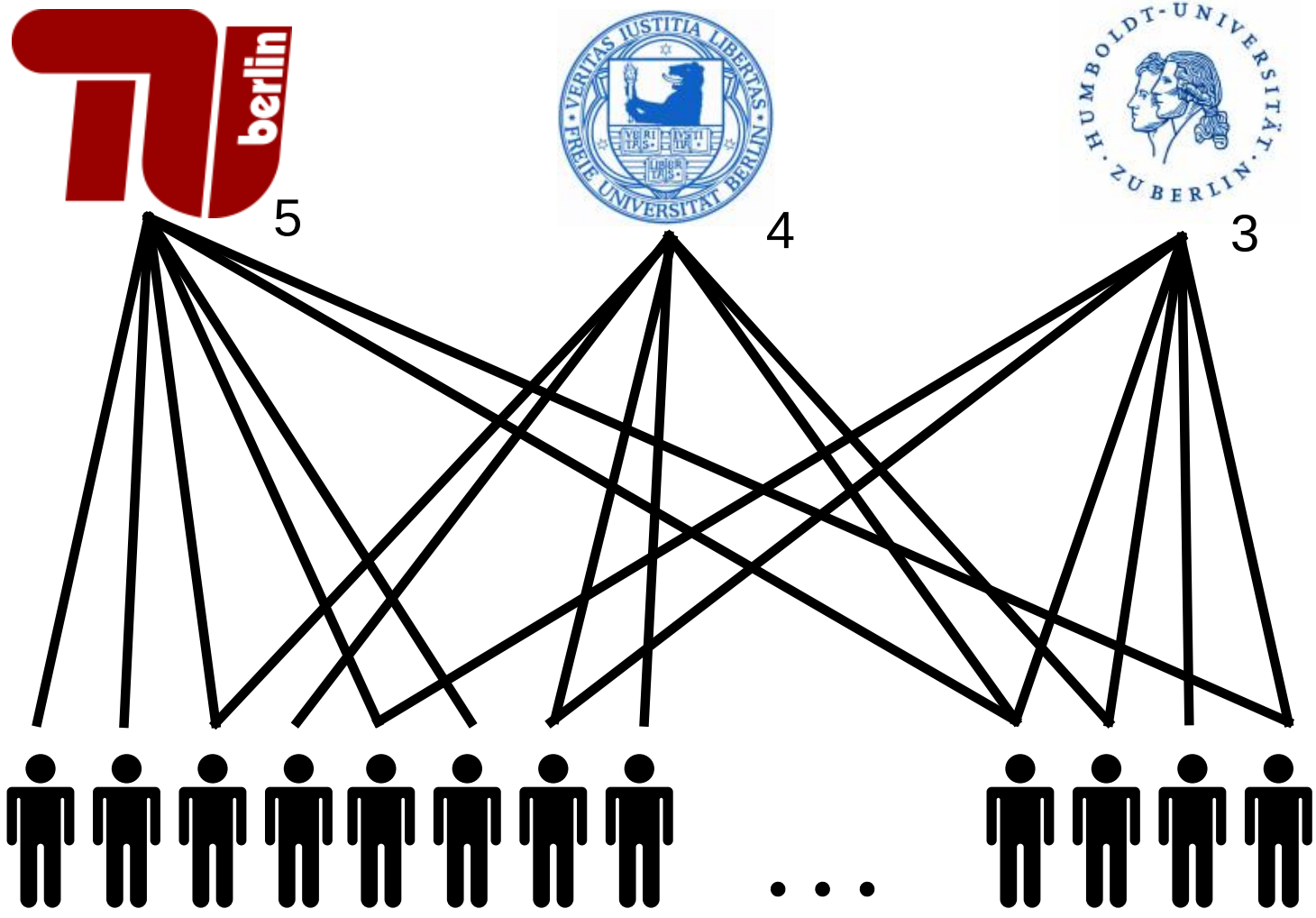
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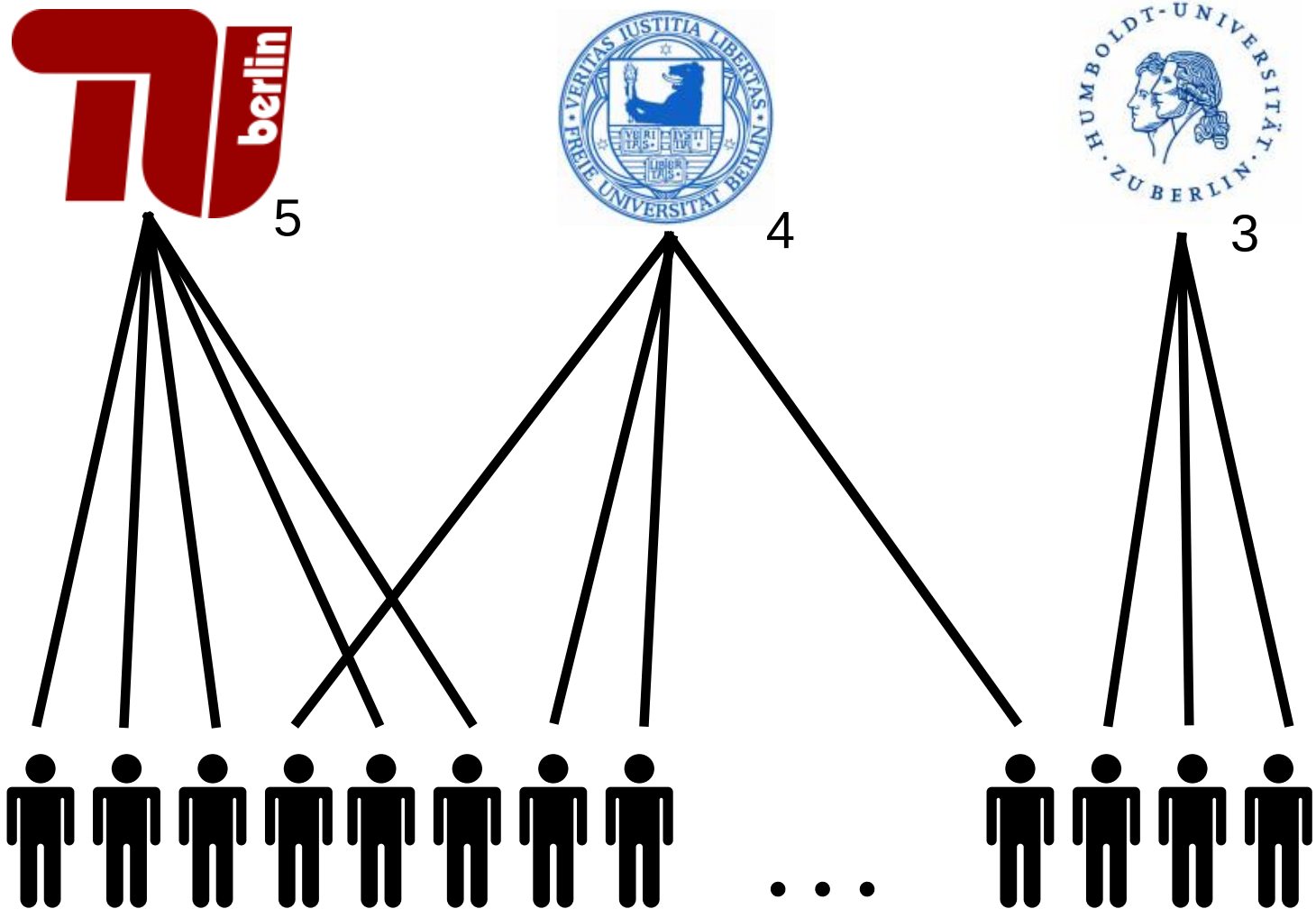
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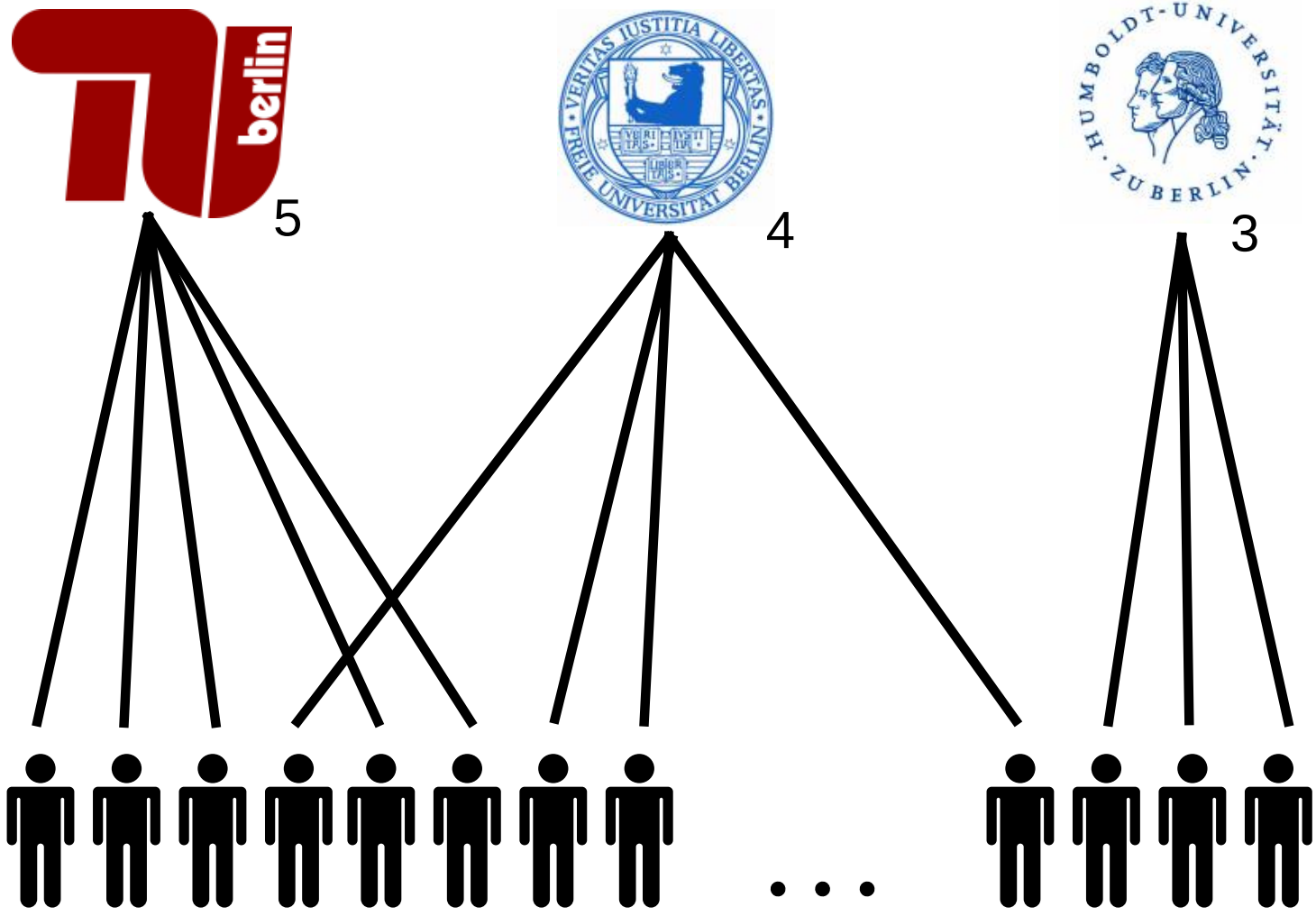

Admission processes in
Germany, Hungary, Turkey, Spain, USA, Singapore

College Admissions as Gale and Shapley (1962) imagined

The solution by the Gale-Shapley mechanism is

- fair: an application is rejected by a college only if its quota is filled with better applicants
- student-optimal: no student could be admitted to a better college in any other fair solution

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The automated procedure based on the Gale-Shapley algorithm is

- fast: the running time is linear in the number of applications (10 seconds in Hungary, would be $\sim$1 minute in the UK and $\sim$15 minutes in China)
- strategy-proof: no student can be better off by giving false preferences

Hungarian higher education matching scheme

Special features:

1. ties
2. lower quotas
3. common quotas
4. paired applications

Theory: Each of the latter three features makes the problem of finding a 'good' solution NP-hard, so heuristics are used.

- P. Biró. Student Admissions in Hungary as Gale and Shapley Envisaged. Technical Report. Dept of Computing Science, University of Glasgow, TR-2008-291.
- P. Biró, T.Fleiner, R.W. Irving and D.F. Manlove. The College Admissions problem with lower and common quotas. Theoretical Computer Science 411, 3136-3153 (2010).
- P. Biró and S. Kiselgof. College admissions with stable score-limits. To appear in Central European Journal of Operations Research (2013).

The Gale–Shapley algorithm in practice

Admission systems in education:

- New York high schools since 2004,
  Boston high schools since 2005
- Higher education admissions in Spain (1998)
- Higher education admissions in Hungary since 1996
- Secondary school admissions in Hungary since 2000
  (Original Gale–Shapley model and algorithm!)

The Gale–Shapley algorithm in practice

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Allocating residents to positions:

- National Resident Matching Program since 1952!
- and many other professions in the US and other countries
  (e.g., Scottish Foundation Allocation Scheme – SFAS)

Hospitals / Residents problem with couples

National Resident Matching Program (since 2009 in SFAS too)
Couples can submit joint preference lists...

Applicants:	**Bill**	**Adam and Eve**
1st choice: 2nd choice:	Queens Memorial	(Memorial, Queens)

the ranking of NY Queens Hospital: Eve, Bill
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Roth (1984): 'Fair' solution may not exist.

Ronn (1990): The related decision problem is NP-complete.

B.-Irving-Schlotter (2011):
NP-complete even for master lists (relevance in SFAS)

Heuristics are used in the applications.

Kidney exchanges

In case of a kidney failure a patient can go for

- dialysis (-)
- transplantation $(+)$, this can be
  - cadaveric (from dead body), but there is a shortage
  - living donation

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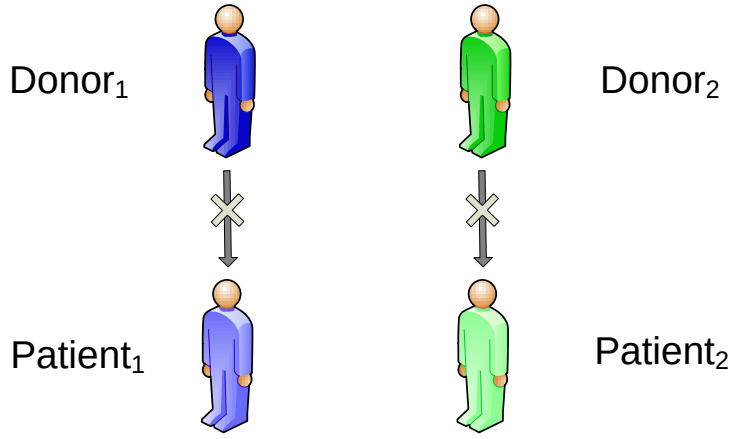
But what if you have a willing donor who is incompatible with you? Perhaps you can exchange kidney with others! Living donor kidney exchange programs in
Netherlands, Spain, United Kingdom, USA, Australia



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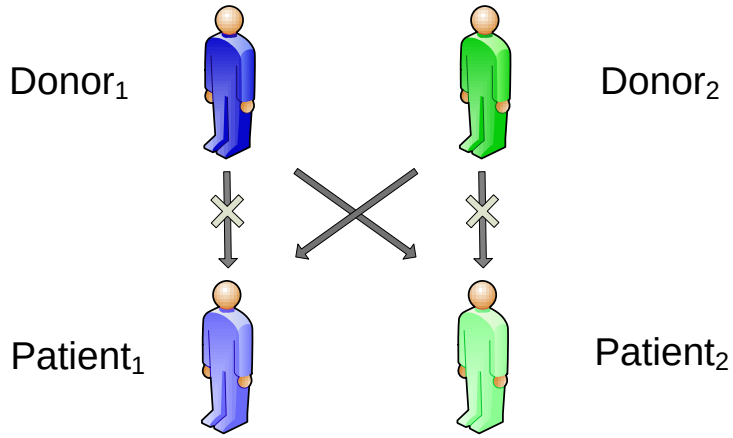

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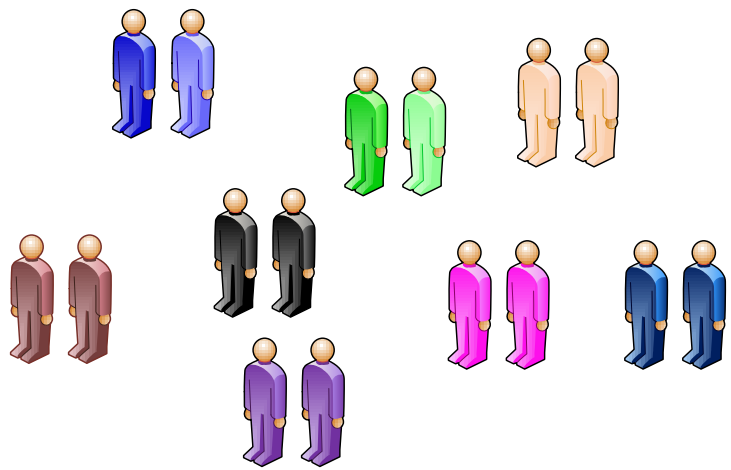

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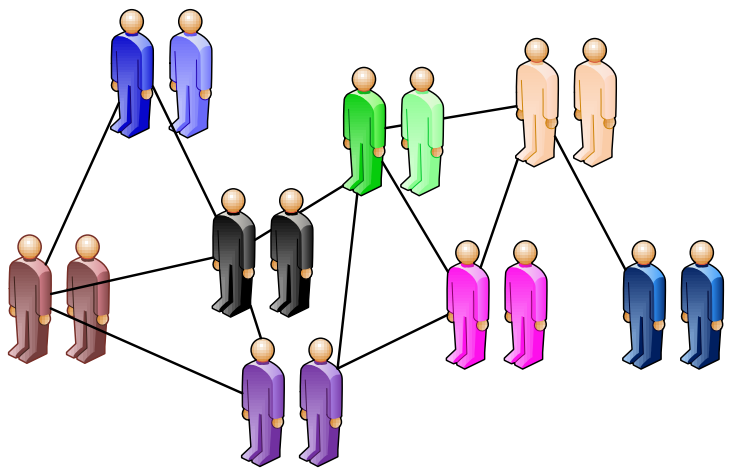

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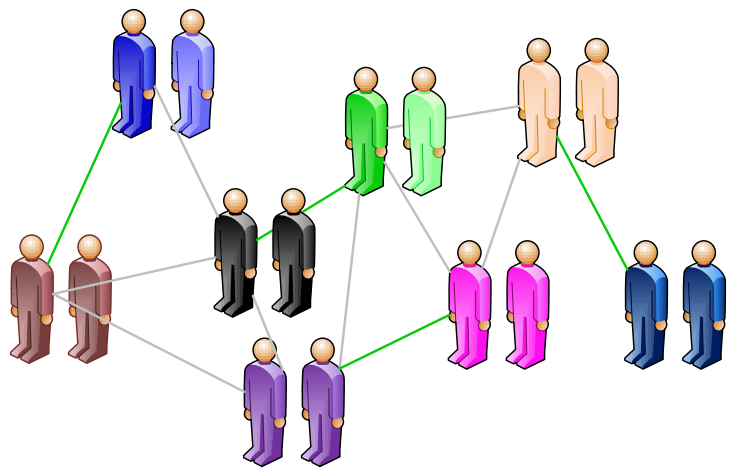

Kidney exchange scheme in the UK

- matching runs every three months with 150 pairs in the pool
- 2-way and 3-way exchanges allowed in the solution
- 'optimality' means that the number of donations is maximal

Theory: The related problem is NP-complete. We implemented an exact algorithm and an IP heuristic and both worked fine!
Led to 134 successful transplants between 2007 and 2011, including twenty-two 3-way exchanges...

- R.J. Johnson, J.E. Allen, S.V. Fuggle, J.A. Bradley and C. Rudge; on behalf of the Kidney Advisory Group. Early Experience of Paired Living Kidney Donation in the United Kingdom. Transplantation, 86(12) : 1672-1677, 2008.
- P. Biró, D.F. Manlove and Romeo Rizzi. Maximum weight cycle packing in directed graphs, with application to kidney exchange programs. Discrete Mathematics, Algorithms and Applications, 1 (4) : 499-517, 2009.
- D.F. Manlove and G. O'Malley. Paired and altruistic kidney donation in the UK: Algorithms and experimentation. In Proceedings of SEA 2012, vol. 7276 of LNCS, pp 271-282.

Daily Mail, Thursday, December 6, 2007

The transplant pact

Two saved as families exchange kidneys

By **Luke Salkeld**

THEY were both in desperate need of a kidney donor, and both had relatives who were willing to sacrifice an organ.

But without a family match, strangers Donald Planner and Margaret Wearn instead entered into an extraordinary pact.

Mr Planner's daughter donated her kidney to Mrs Wearn, whose husband gave his kidney to Mr Planner.

The operations took place 170 miles apart in synchronised procedures with the organs transported by ambulances travelling in opposite directions between the two hospitals

'Completely amazing': Donald Planner with his daughter Suzanne

Margaret and Roger Wearn: 'No different to a direct donation'

organ or he would die. His ... lly reliant on the dialysis

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Three-way kidney transplant success

By Graham Satchell
BBC News Breakfast reporter

Step back to nine in the morning on 4 December 2009.

Six patients are ready for surgery at three different hospitals across the UK.

It is the culmination of months of preparation and a remarkable event in the history of live organ donation in this country.

This is a three-way kidney swap between couples who've never met.

Chris Brent with his sister Lisa Burton

In Aberdeen, 54-year-old Andrea Mullen suffered sudden kidney failure three years ago.

It had a devastating impact on her life. She had to have dialysis three

> **"It's a threefold thing really so it's a real good feelgood factor all round"**
>
> Lisa Burton, who donated a kidney

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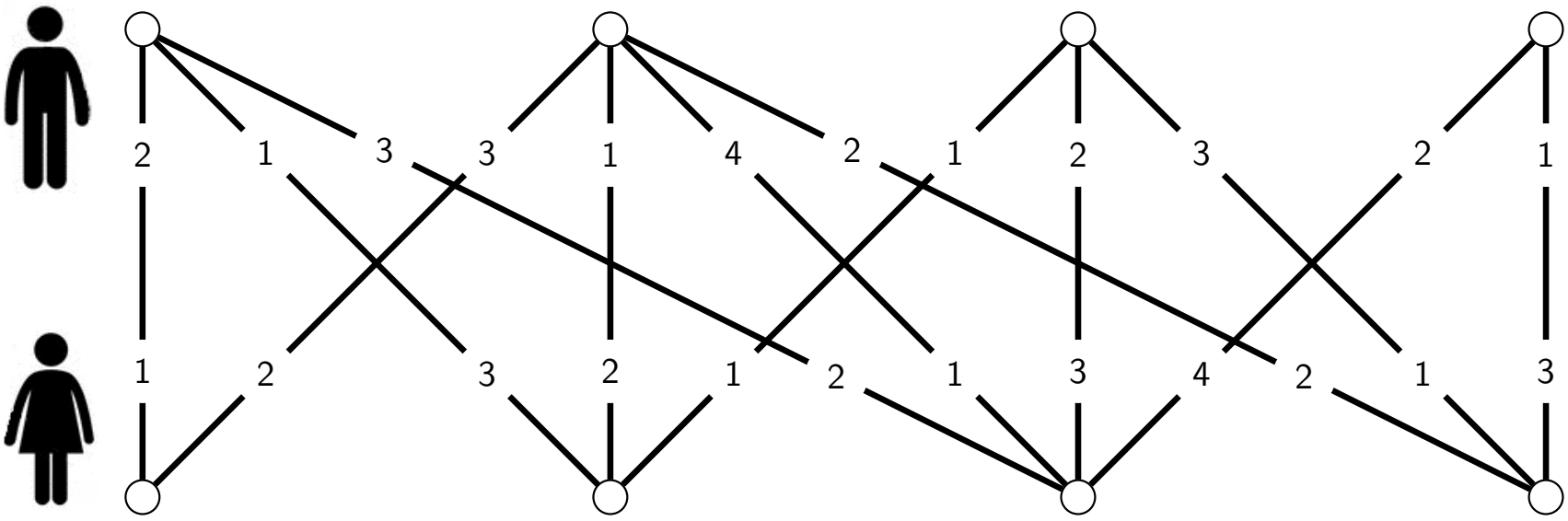

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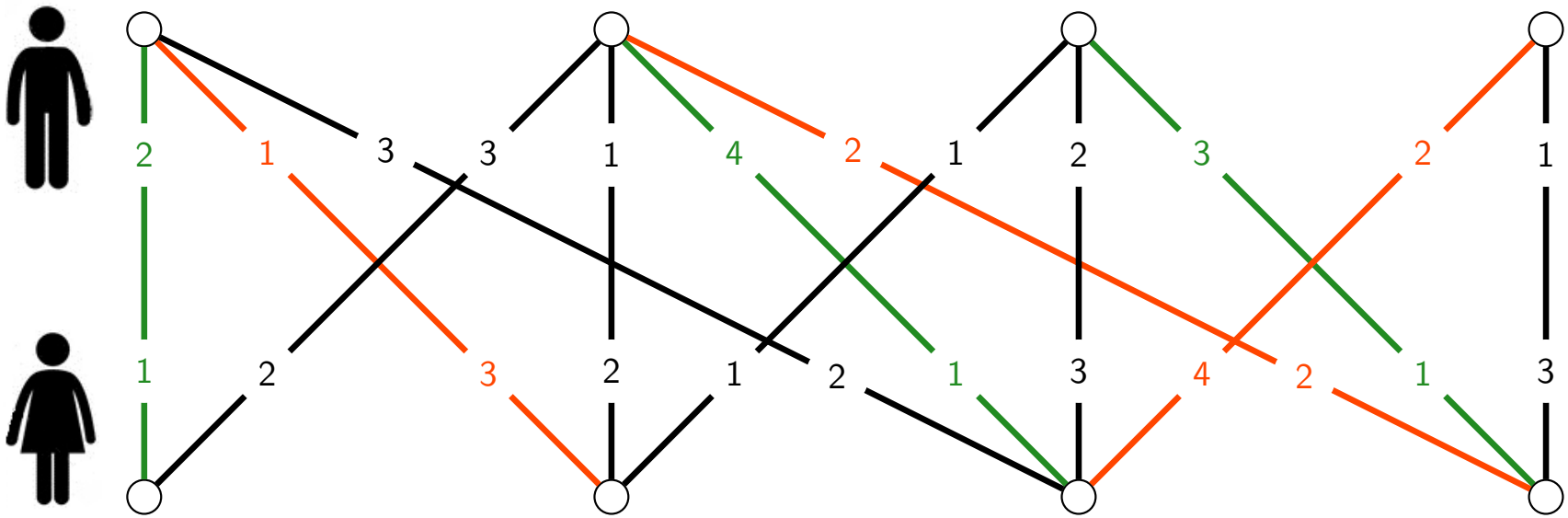

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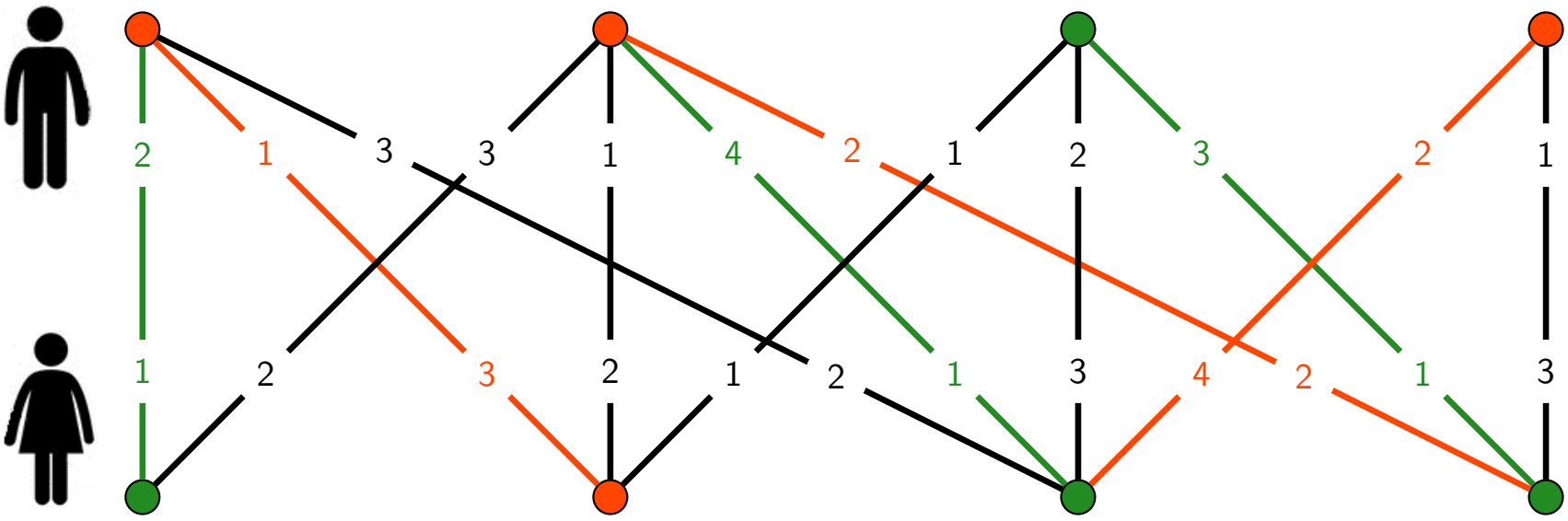

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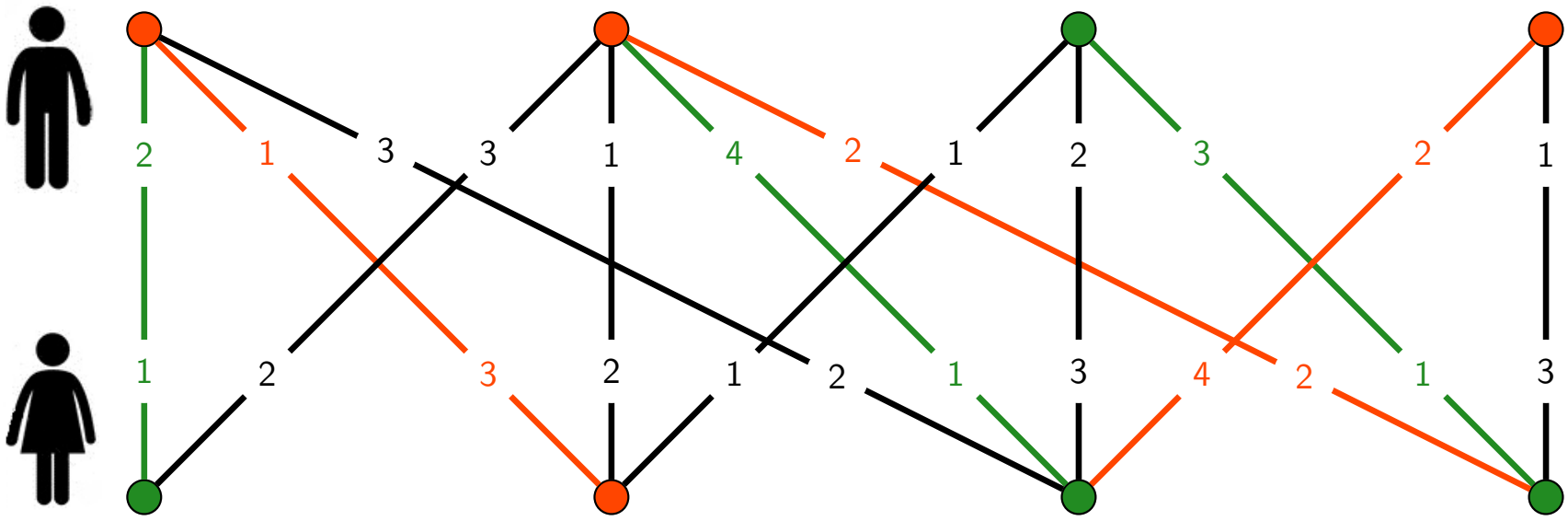
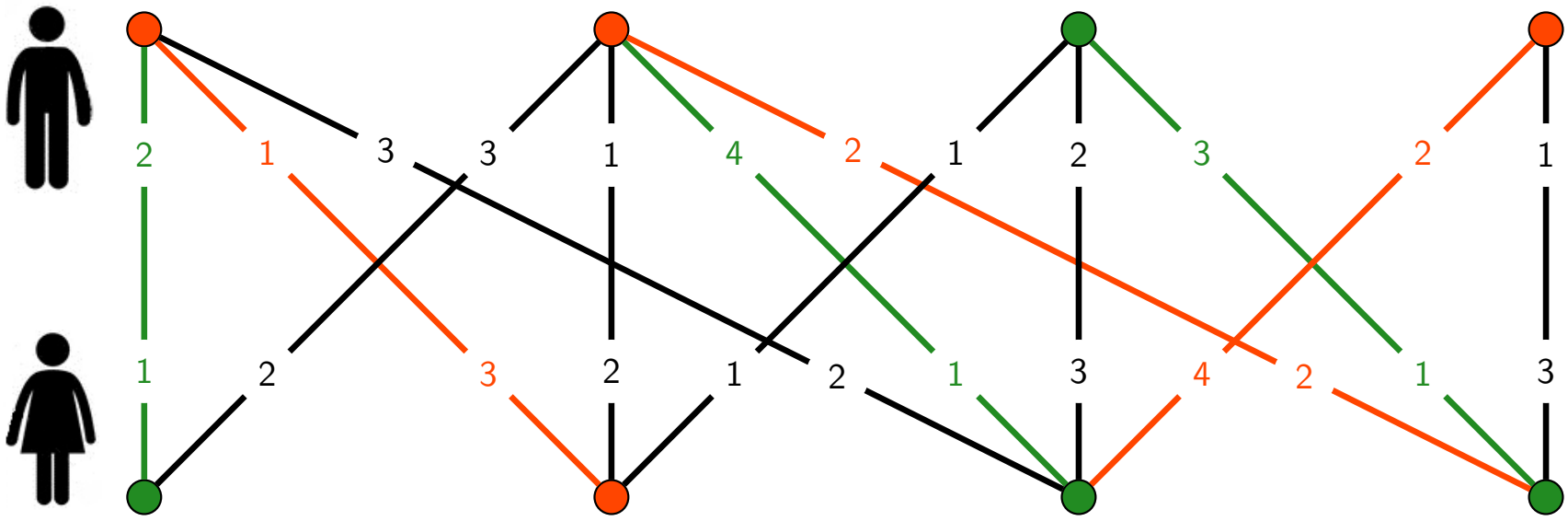


Definition

M is popular, if it is at least as popular as any other matching.

Popular matchings



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Theorem (Gärdenfors 1975)

A popular matching always exists.

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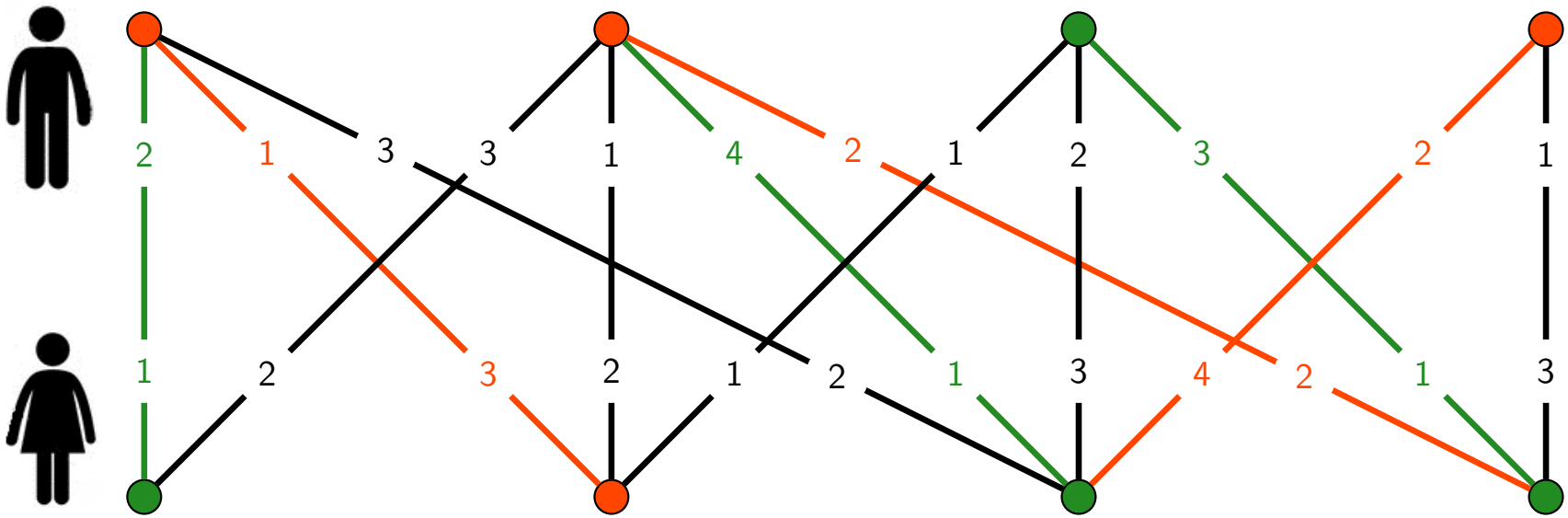

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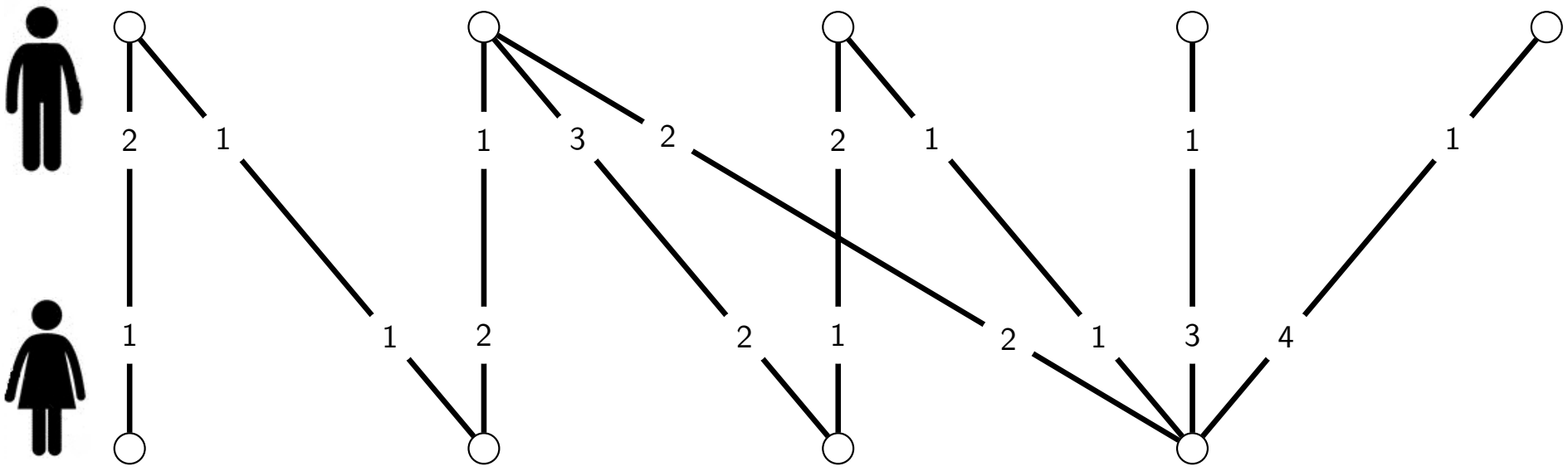
Theorem (Gärdenfors 1975)

A popular matching always exists.

Theorem (Biró, Irving, Manlove 2010)

Stable matchings are minimum size popular matchings.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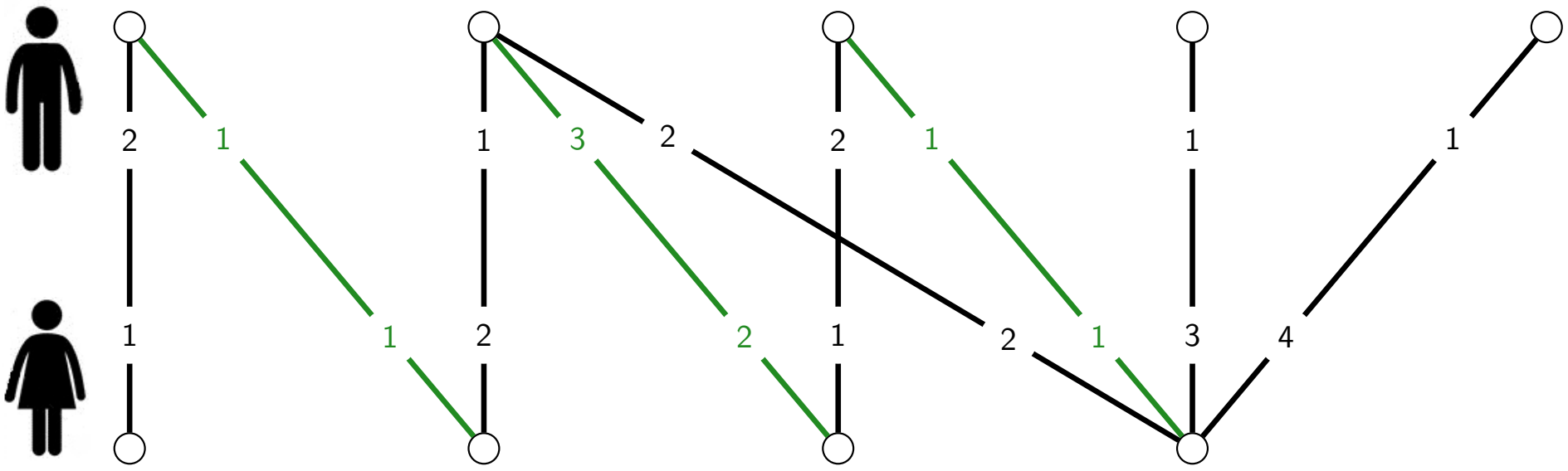
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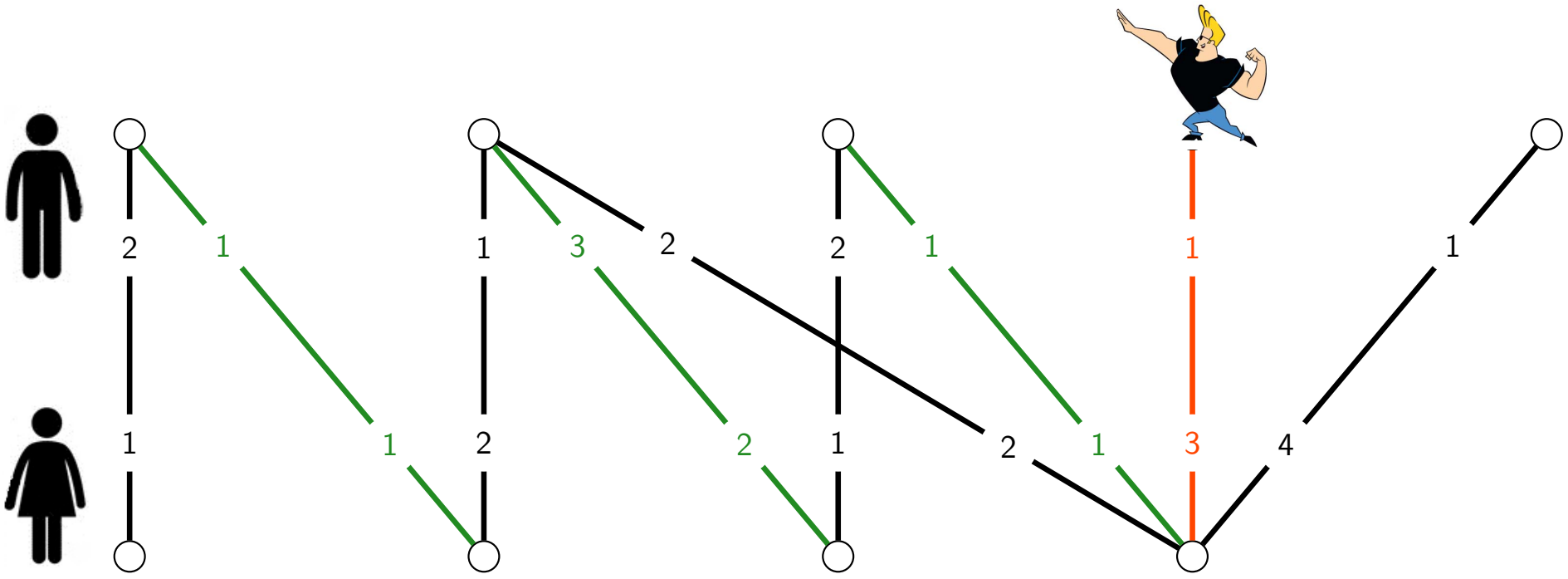
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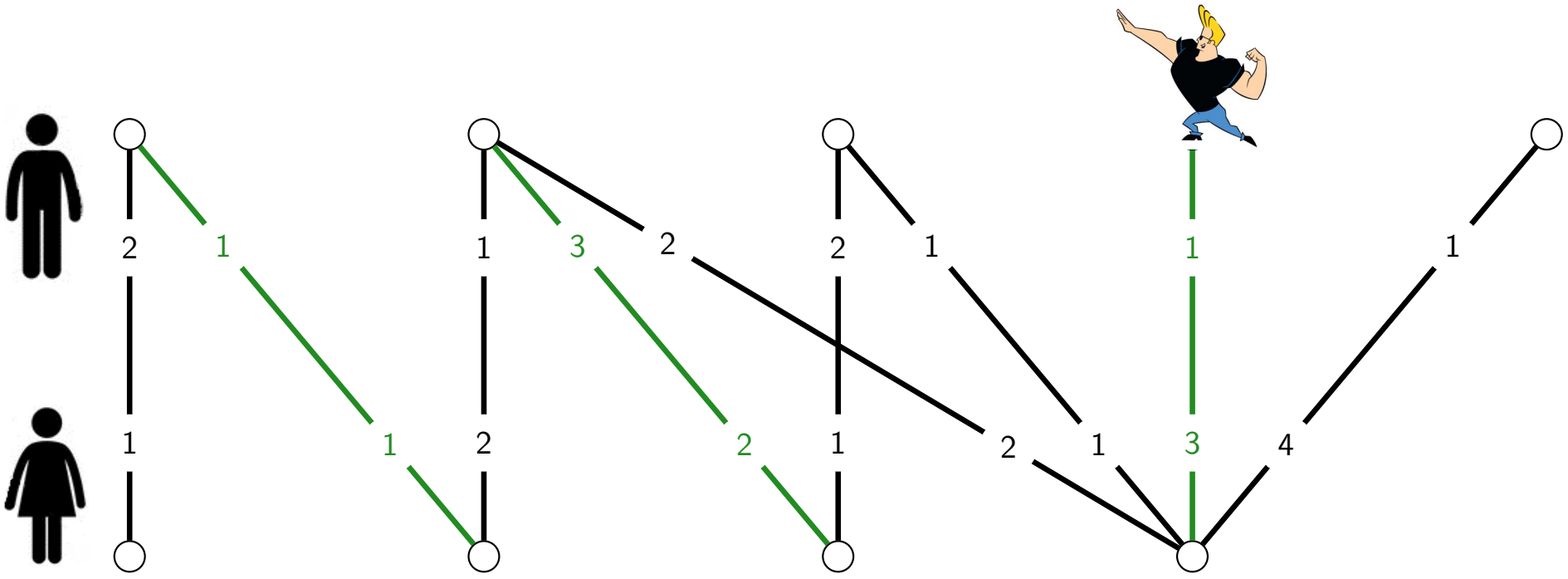
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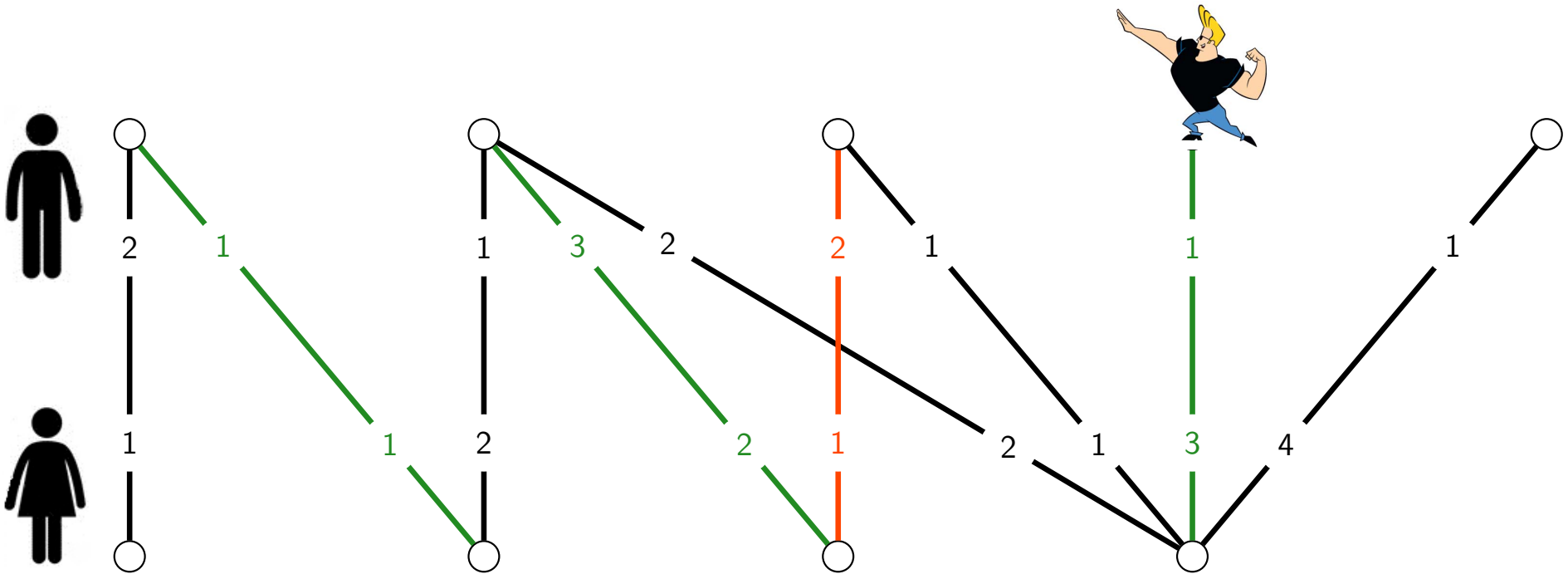
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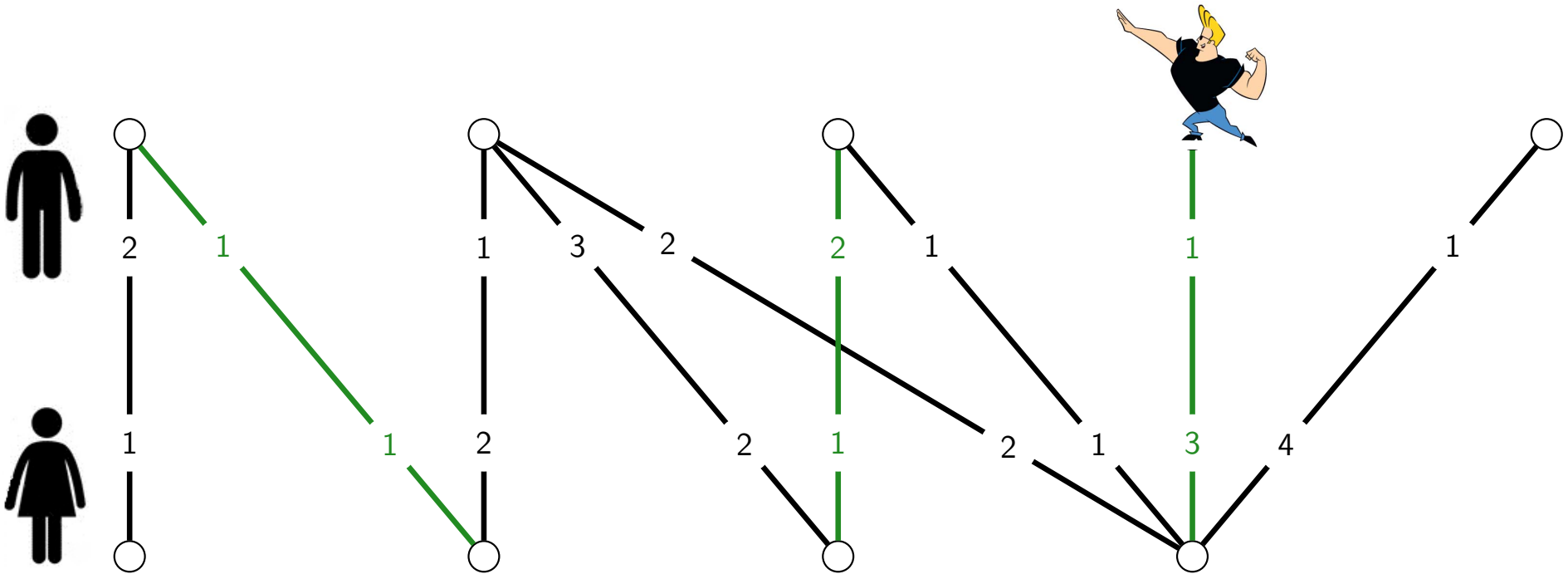
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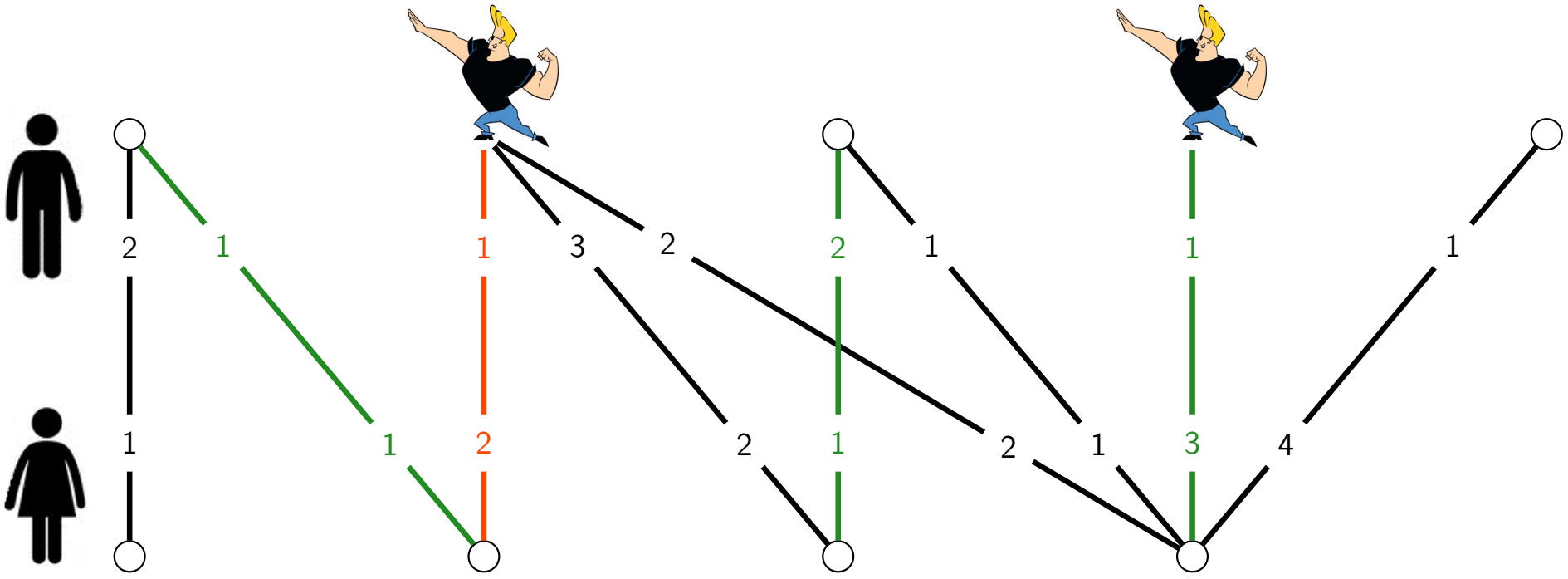
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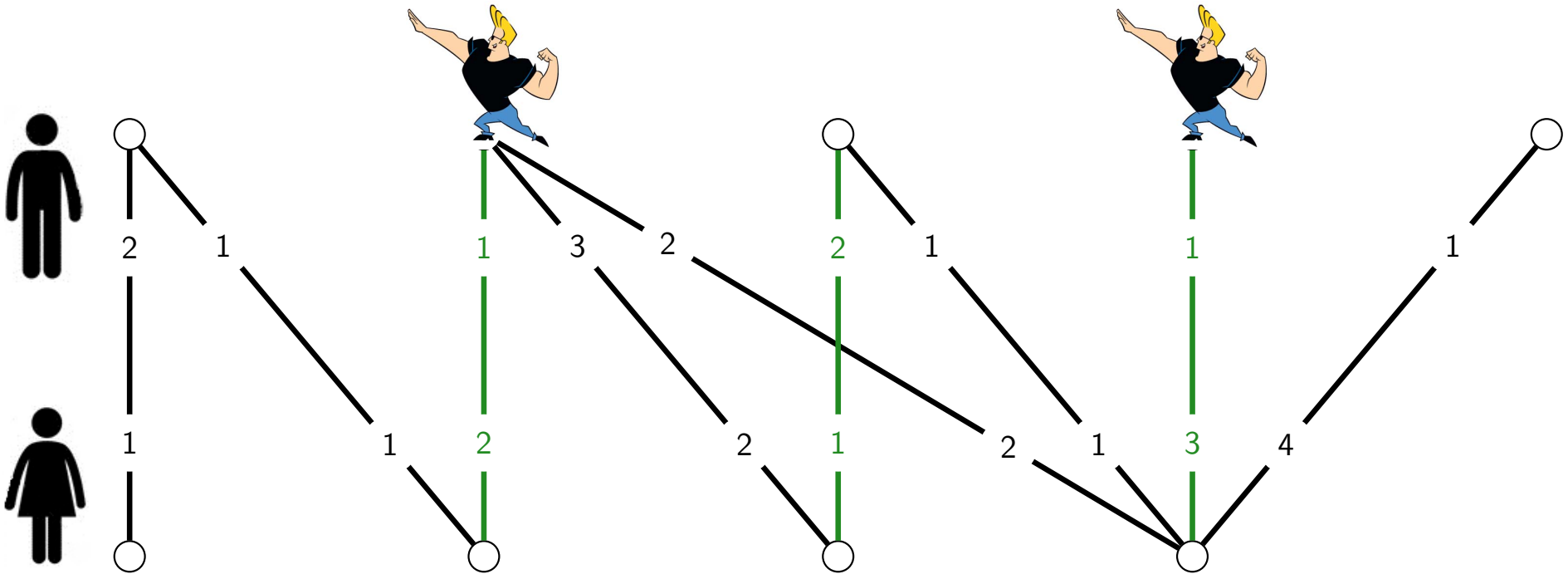
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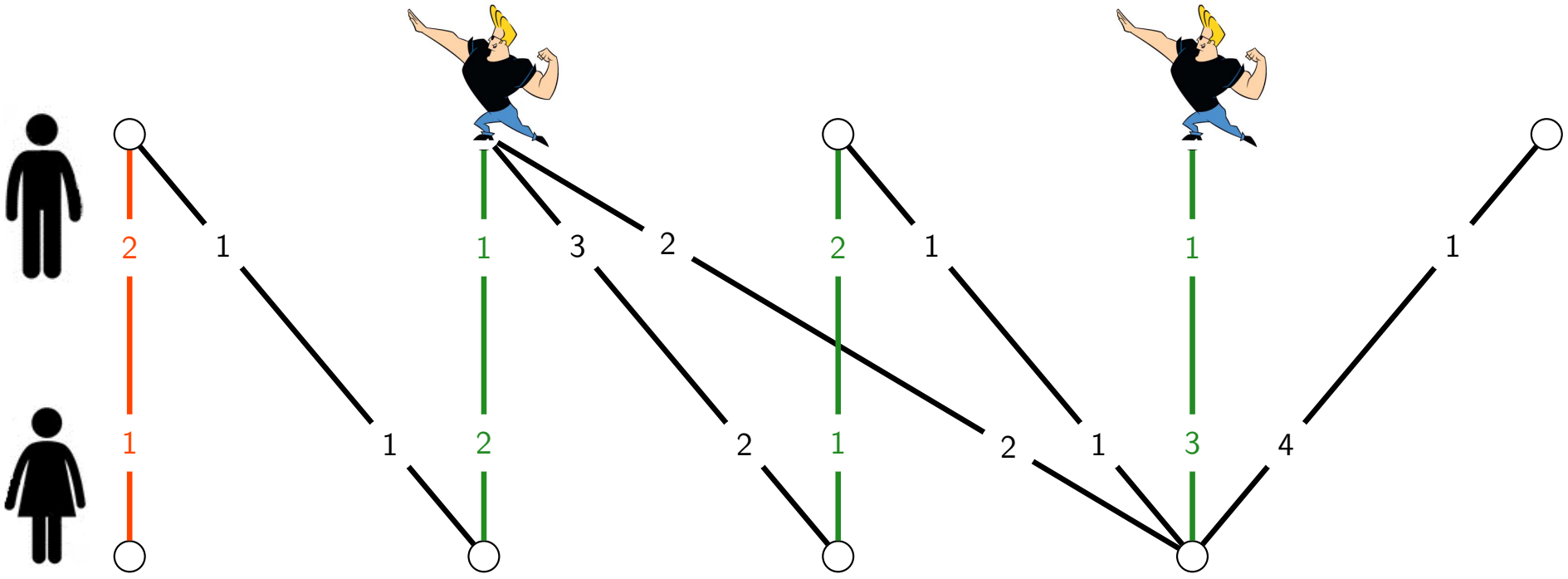
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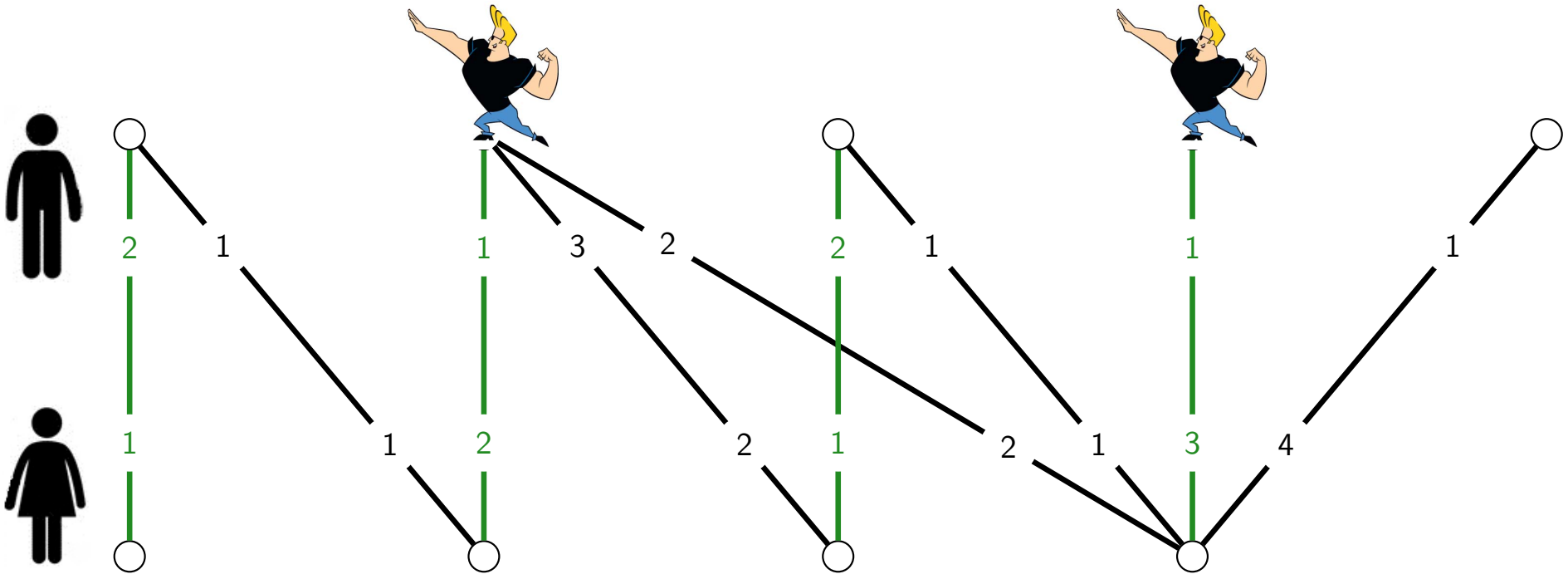
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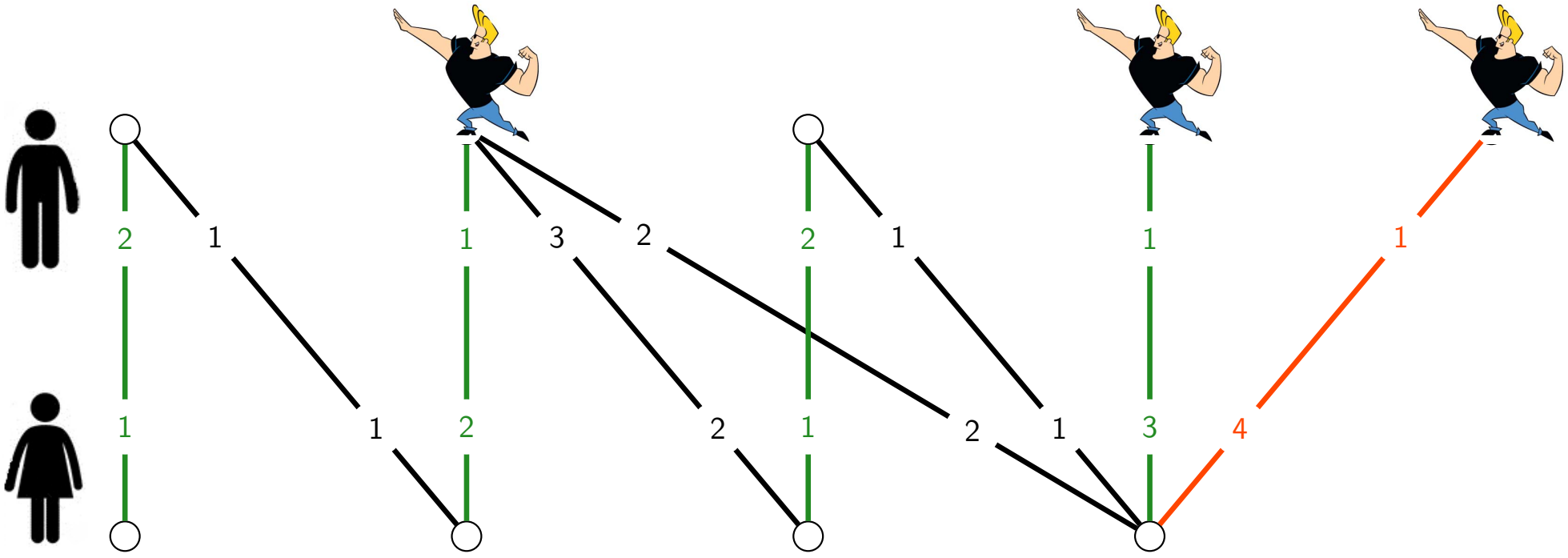
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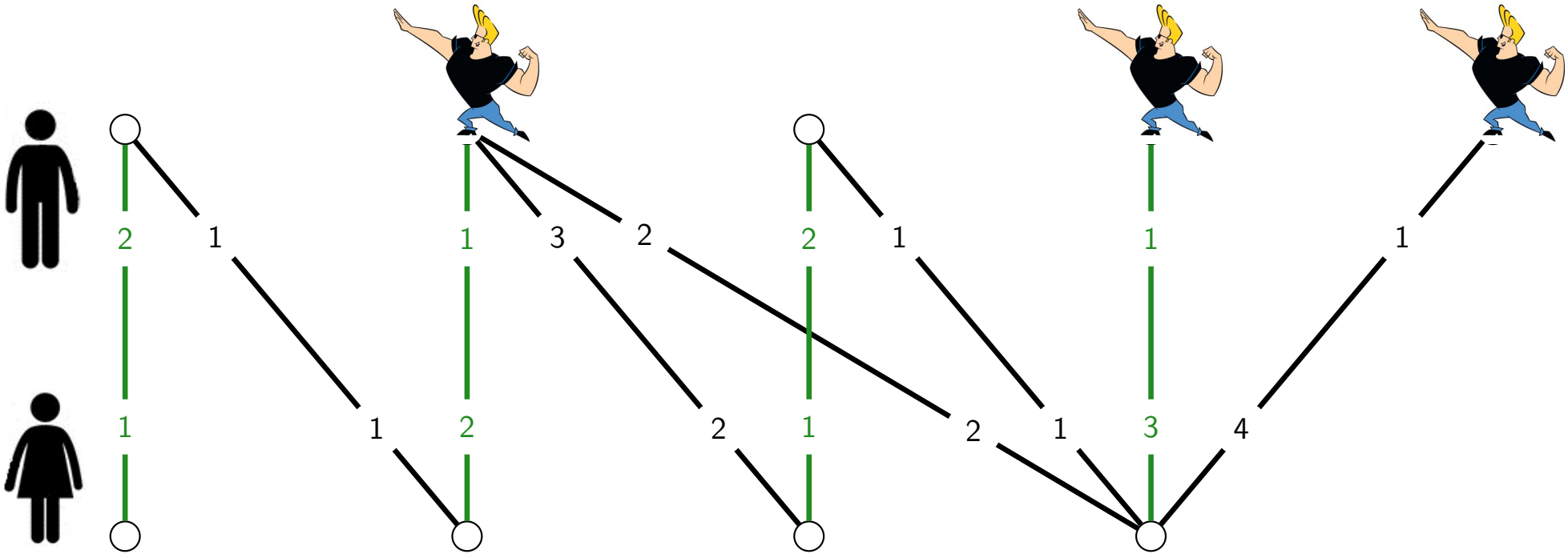
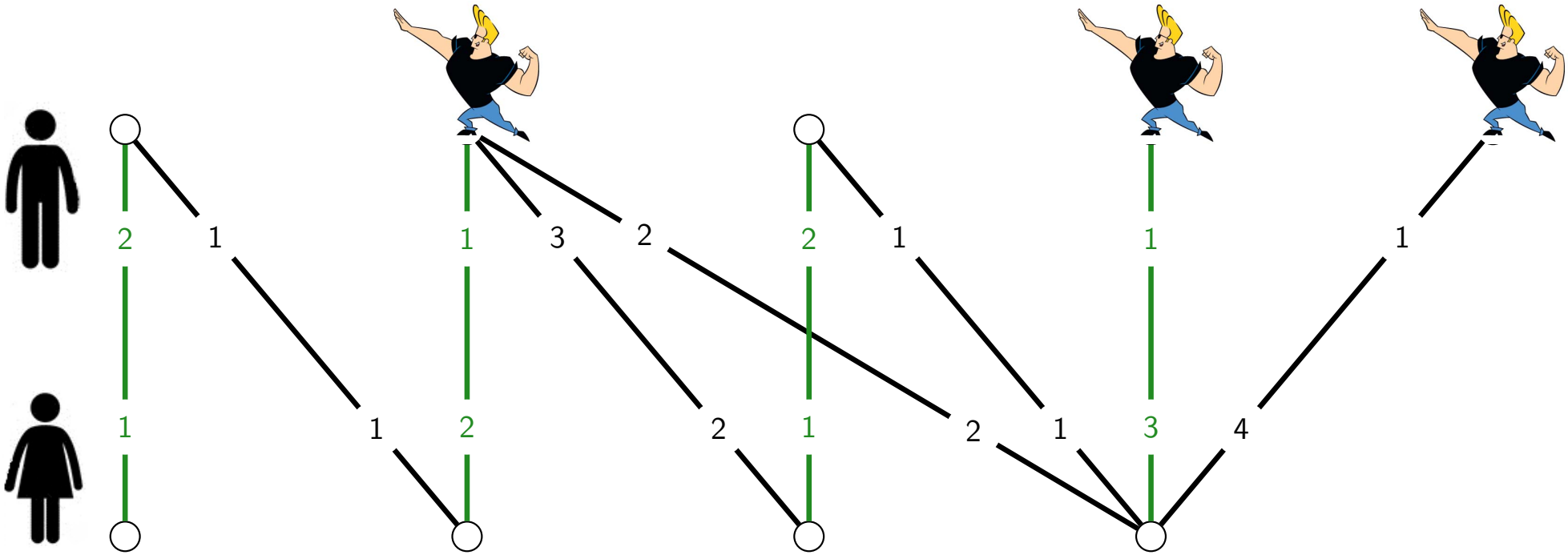


Theorem (Kavitha 2012)

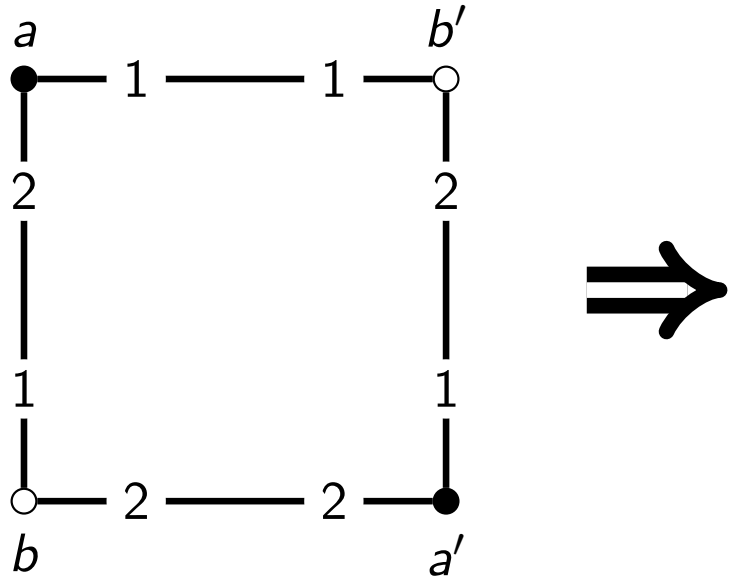
A max size popular matching can be computed in linear time.



Theorem (Kavitha 2012)

A max size popular matching can be computed in linear time.

Question: Given an edge e^*, can we decide efficiently whether there is a popular matching that contains e^*?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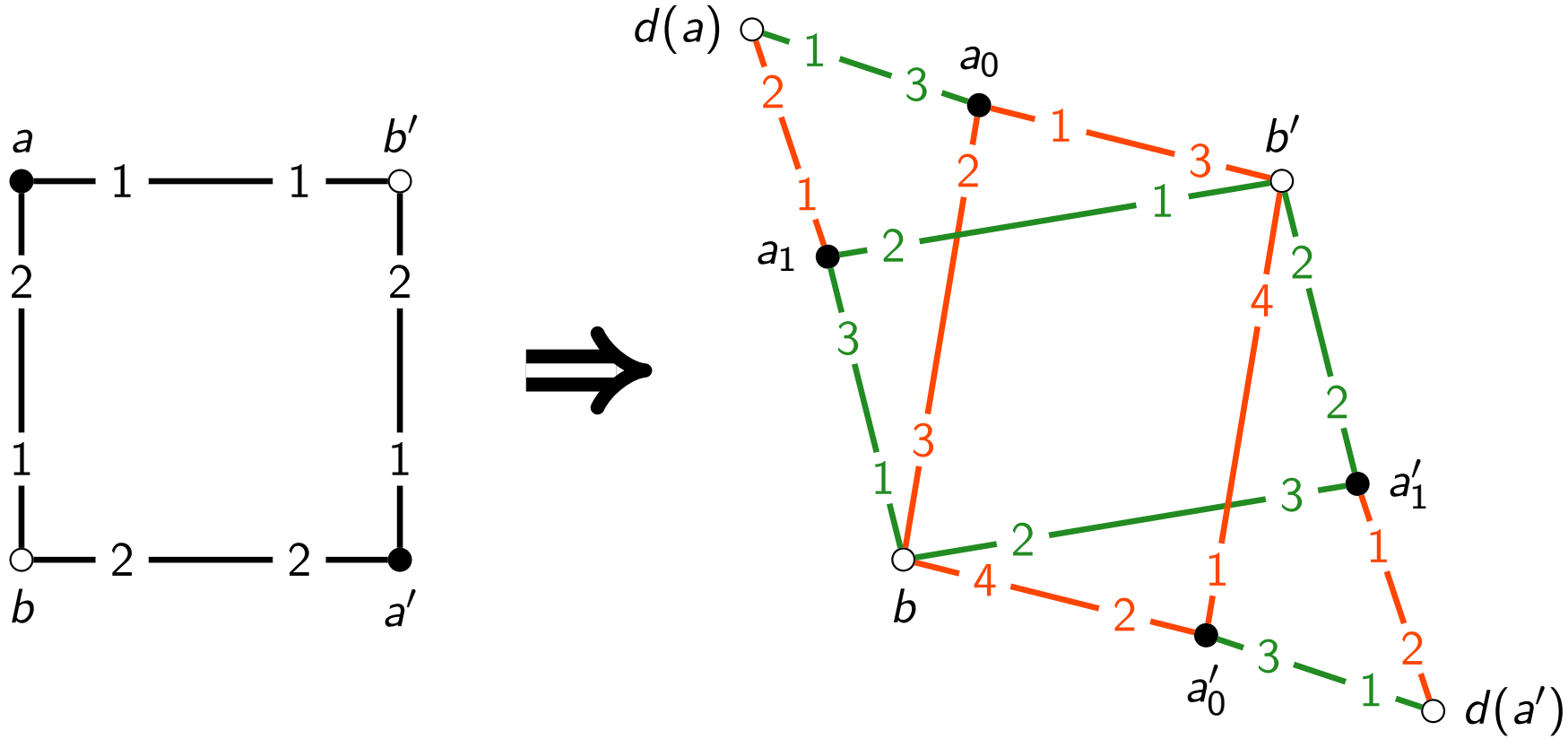
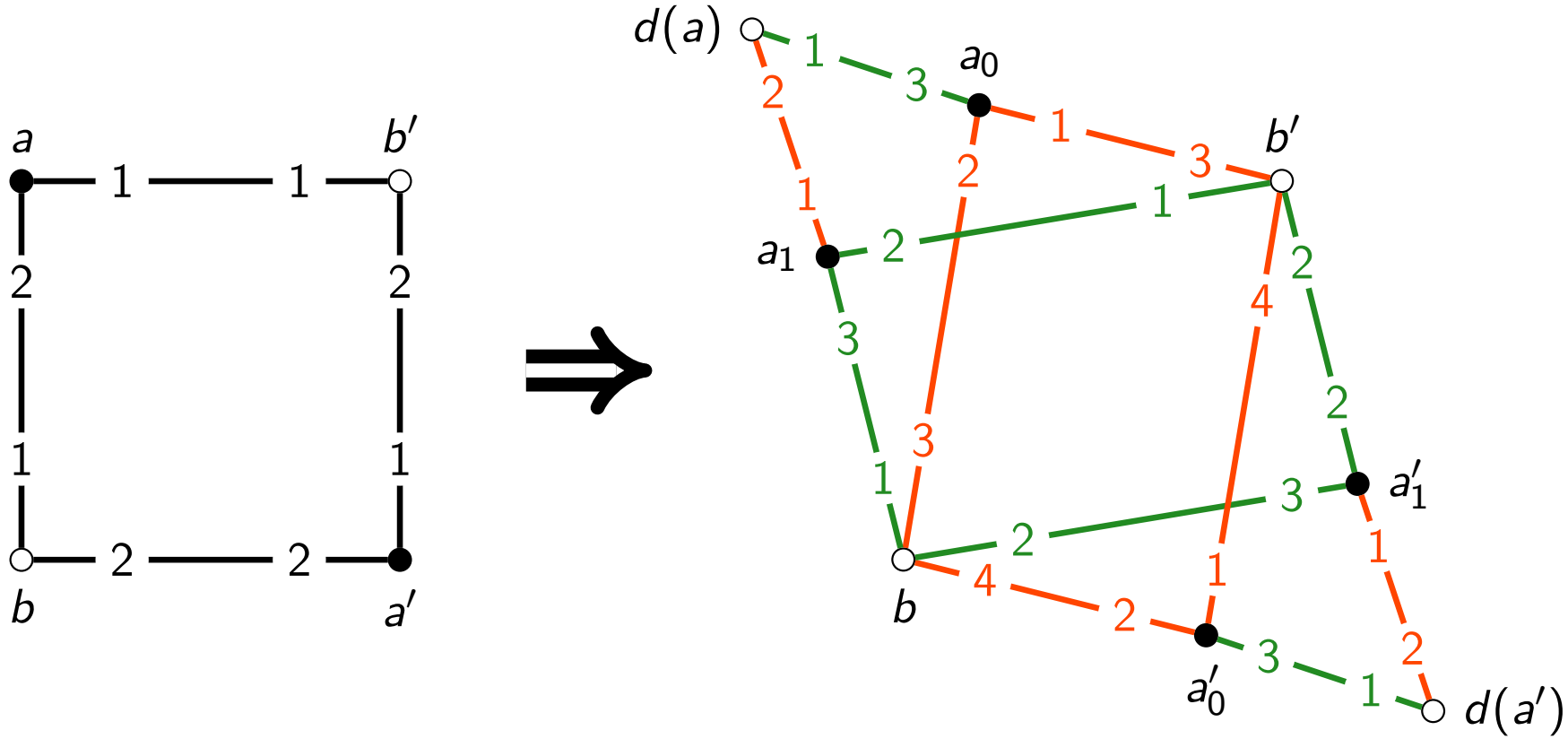
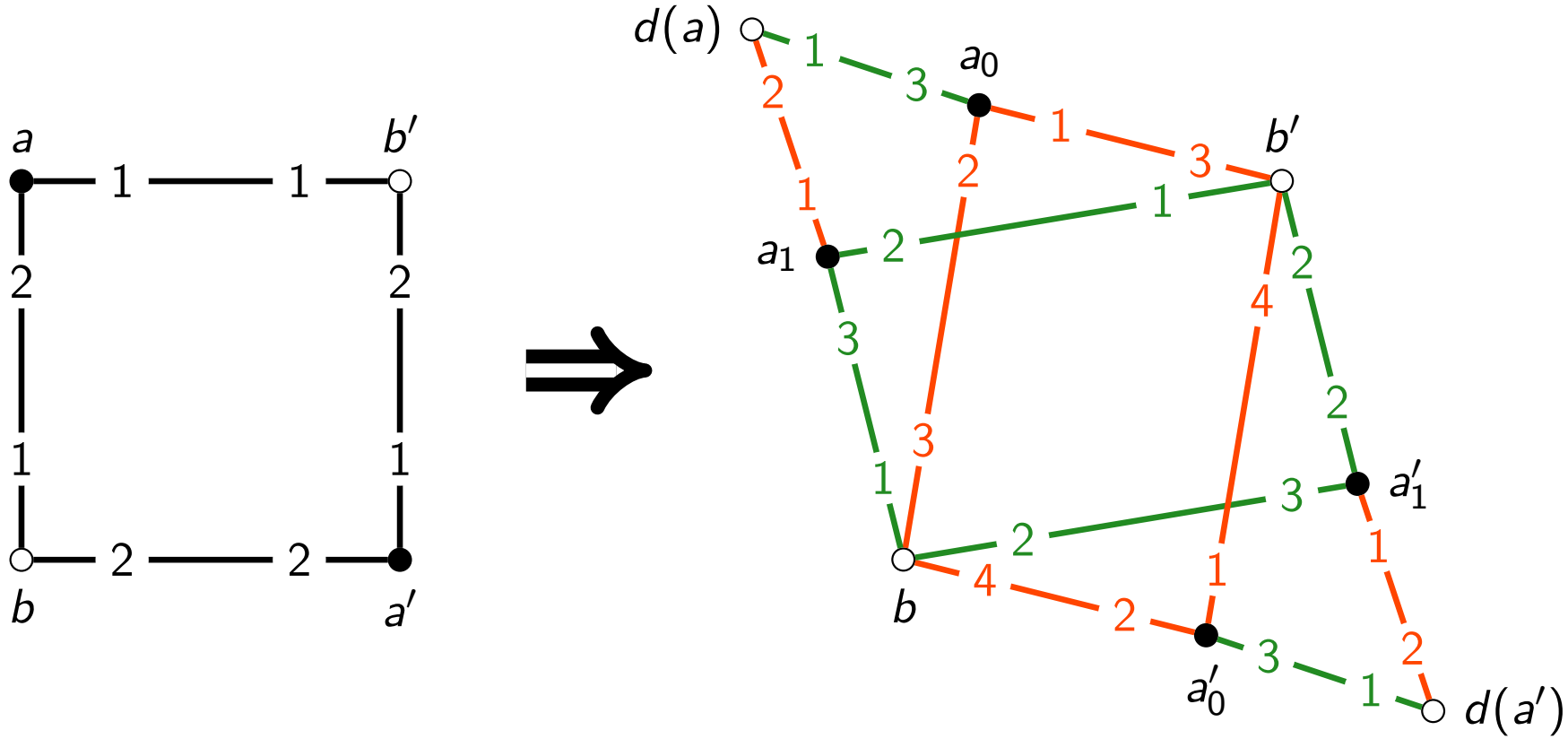
b′
1
1
2
2
1
1
2
2
b
a′


a
b′
b
a′
d(a)
a0
a1
b′
b
a′1
a′0
d(a′)



dominant matching $\leftrightarrow$ stable matching



dominant matching $\leftrightarrow$ stable matching

Theorem (Cs., Kavitha 2016)

There is a popular matching M such that $e^* \in M \Leftrightarrow$

- *there is a stable matching M_1 such that $e^* \in M_1$ or*
- *there is a dominant matching M_2 such that $e^* \in M_2$.*

Future challenges

Theoretical research:

- Studying computational and strategic issues (computer science, game theory)
- What are the benefits of having a centralized scheme? e.g., compare student admissions practices in Poland and Hungary (economics, social sciences)

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New applications?

- **Hungary:** school choice (nursery, kindergarten, primary schools), resident allocation, kidney exchanges
- **Europe:** united schemes for higher education admissions and kidney exchanges
- Running project: students-to-course allocation in high schools

Munkatársat keresünk

Feladat

- Párosítási algoritmusok leprogramozása választott nyelven.
- Alkalmazásokhoz kapcsolódó weboldalak megtervezése, programozása. Cél egy olyan webes alkalmazás létrehozása, mint amilyet a hazai középiskolai és egyetemi felvételikben is használnak.
- A szimulációs és valódi futtatások eredményeinek tudományos elemzése a témavezető segítségével.
- Alkalmazások: bölcsődei, óvodai, általános iskolai felvételik, rezidensek allokációja, egyetemi felvételi duális képzésekre, gyakornokok allokálása cégekhez, felvételi szakkollégiumokban, vesecsere-program, sakk-párosító algoritmus, tantárgyválasztási eljárás, kollégiumi férőhelyek elosztása, élelmiszerbank allokációja.

Munkatársat keresünk

Kit keresünk?

- Követelmény: befejezett BSc, némi programozási tapasztalat
- jelentkezés nov. 23-ig

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Mit nyújtunk?

- teljes állás 8 hónapra, fizetéssel
- rugalmas munkaidő
- TDK/diplomamunka téma, akár doktori téma is
- Stabil csoport Magyarországon, kiváló kapcsolatok külföldre

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Várjuk a kérdéseiteket!