Inflation Modelling

Seminar Talk

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Models

- Two-factor Hull-White model:

Models

- Two-factor Hull-White model:
    - inflation

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- Two-factor Hull-White model:
  - inflation
  - nominal interest rate

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- Jarrow-Yildirim model:

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  - inflation
  - nominal interest rate
- Jarrow-Yildirim model:
  - forward nominal interest rate

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  - inflation
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- Jarrow-Yildirim model:
  - forward nominal interest rate
  - forward real interest rate

Models

- Two-factor Hull-White model:
    - inflation
    - nominal interest rate
- Jarrow-Yildirim model:
    - forward nominal interest rate
    - forward real interest rate
    - price index

Two-factor Hull-White model

- Inflation:

$$d_t i\left(t\right) = \kappa_i \left(\theta_i\left(t\right) - i\left(t\right)\right) dt + \sigma_i d_t W_i,$$

Wiener-processes W_i and W_r have a constant ρ correlation;

Two-factor Hull-White model

- Inflation:

$$d_t i\left(t\right) = \kappa_i \left(\theta_i \left(t\right) - i\left(t\right)\right) dt + \sigma_i d_t W_i,$$

- Interest rate:

$$d_t r\left(t\right) = \kappa_r \left(\theta_r \left(t\right) - r\left(t\right)\right) dt + \sigma_r d_t W_r,$$

Wiener-processes W_i and W_r have a constant ρ correlation;

- Forward nominal interest rate:

$$d_t f_n(t, T) = \mu_n(t, T)\, dt + \sigma_n(t, T)\, d_t W^P,$$

W^P is a multi-dimensional Wiener-process;

Jarrow-Yildirim model

FX-analogy

- Forward nominal interest rate:

$$d_t f_n(t,T) = \mu_n(t,T)\,dt + \sigma_n(t,T)\,d_t W^P,$$

- Forward real interest rate:

$$d_t f_r(t,T) = \mu_r(t,T)\,dt + \sigma_r(t,T)\,d_t W^P,$$

W^P is a multi-dimensional Wiener-process;

Jarrow-Yildirim model

FX-analogy

- Forward nominal interest rate:

$$d_t f_n(t,T) = \mu_n(t,T)\,dt + \sigma_n(t,T)\,d_t W^P,$$

- Forward real interest rate:

$$d_t f_r(t,T) = \mu_r(t,T)\,dt + \sigma_r(t,T)\,d_t W^P,$$

- Index:

$$\frac{d_t I(t)}{I(t)} = \mu_I(t)\,dt + \sigma_I(t)\,d_t W^P,$$

W^P is a multi-dimensional Wiener-process;

- Forward nominal interest rate:

$$d_t f_n(t,T) = \mu_n(t,T)\,dt + \sigma_n(t,T)\,d_t W^P,$$

- Forward real interest rate:

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- Index:

$$\frac{d_t I(t)}{I(t)} = \mu_I(t)\,dt + \sigma_I(t)\,d_t W^P,$$

-

$$\begin{aligned}\sigma_n(t,T) &= \sigma_n \exp\left(a_n(T-t)\right),\\ \sigma_r(t,T) &= \sigma_r \exp\left(a_r(T-t)\right),\\ \sigma_I(t) &= \sigma_I,\end{aligned}$$

W^P is a multi-dimensional Wiener-process;

Jarrow-Yildirim model

Implied dynamics 1.

- $n(t)$ denotes the **instantaneous nominal interest rate**:

$$n(t) = f_n(t, t),$$

Jarrow-Yildirim model

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- $r(t)$ denotes the **instantaneous real interest rate**:

$$r(t) = f_r(t, t),$$

- $B(t)$ is the **nominal money market account** (the numeraire of the spot measure Q):

$$B(t) = \exp\left(\int_0^t n(s)\, ds\right),$$

Jarrow-Yildirim model

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- $r(t)$ denotes the **instantaneous real interest rate**:

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$$B(t) = \exp\left(\int_0^t n(s)\, ds\right),$$

- $B_r(t)$ denotes the **real money market account** in real units:

$$B_r(t) = \exp\left(\int_0^t r(s)\, ds\right),$$

Jarrow-Yildirim model

Implied dynamics 2.

- $P_n(t, T)$ stands for the price of a **bond paying** 1 **at time** T:

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$$P_r(t,T) = \exp\left(-\int_t^T f_r(t,u)\,du\right),$$

- $P_I(t,T)$ is the price of an **index-linked bond** paying the value $I(T)$ at time T:

$$P_I(t,T) = E_t\left[I(T)\cdot\frac{B(t)}{B(T)}\right],$$

Jarrow-Yildirim model

Implied dynamics 2.

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- $P_I(t, T)$ is the price of an **index-linked bond** paying the value $I(T)$ at time T:

$$P_I(t, T) = E_t\left[I(T) \cdot \frac{B(t)}{B(T)}\right],$$

- $\hat{I}(t, T)$ denotes the **forward index** for time T:

$$\hat{I}(t, T) = \frac{P_I(t, T)}{P_n(t, T)}.$$

- $$\begin{aligned} Z^{(n)}(t) &= \frac{P_n(0,t)}{B_n(t)} \\ &= \exp\left(-\int_0^t n(s)\,ds - \int_t^T f_n(t,s)\,ds\right) \end{aligned}$$

  needs to be a martingale.

-
$$Z^{(n)}(t) = \frac{P_n(0,t)}{B_n(t)} = \exp\left(-\int_0^t n(s)\,ds - \int_t^T f_n(t,s)\,ds\right)$$
  needs to be a martingale.
- So

$$\frac{dP_n(t,T)}{P_n(t,T)} = \left(n(t) + \frac{1}{2}\left(\int_t^T \sigma_n(t,u)\,du\right)^2 - \int_t^T \mu_n(t,u)\,du\right) dt - \left(\int_t^T \sigma_n(t,u)\,du\right) d_t W.$$

- Whence

$$\mu_n(t,T) = \sigma_n(t,T) \cdot \int_t^T \sigma_n(t,u)\, du.$$

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- Thus
$$d_t f_n(t,T) = \left(\sigma_n(t,T) \int_t^T \sigma_n(t,u)\, du\right) dt + \sigma_n(t,T)\, d_t W$$

and
$$\frac{dP_n(t,T)}{P_n(t,T)} = n(t)\, dt - \left(\int_t^T \sigma_n(t,u)\, du\right) d_t W.$$

Jarrow-Yildirim model

Nominal dynamics 3.

- $$f_n(t,T) = f_n(t_0,T) + \int_{t_0}^{T} \left(\sigma_n(s,T) \int_s^T \sigma_n(s,u)\,du \right) ds + \int_{t_0}^{T} \sigma_n(s,T)\, d_s W,$$

Jarrow-Yildirim model

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- hence
$$n(t) = f_n(t_0,t) + \int_{t_0}^{t}\left(\sigma_n(s,t)\int_s^t \sigma_n(s,u)\,du\right)ds + \int_{t_0}^{t}\sigma_n(s,t)\,d_sW.$$

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- Thus
$$\begin{aligned} d_t n(t) &= a_n\left[\left(\frac{\partial_2 f_n(0,t)}{a_n} + f_n(0,t) + \frac{\sigma_n^2}{2a_n^2}\left(1-e^{-2a_nt}\right)\right) - n(t)\right]d \\ &\quad + \sigma_n d_t W_n. \end{aligned}$$

- $$\begin{aligned} Z^{(I)}(t) &= \frac{I(t) \cdot B_r(t)}{B_n(t)} \\ &= I(t) \cdot \exp\left(\int_0^t r(s)\,ds - \int_0^t n(s)\,ds\right) \end{aligned}$$

  needs to be a martingale. Hence

  $$\frac{d_t I(t)}{I(t)} = (n(t) - r(t))\,dt + \sigma_I d_t W_I.$$

Jarrow-Yildirim model

Real dynamics 1.

- $$\begin{aligned} Z^{(r)}(t) &= \frac{I(t) \cdot P_r(0,t)}{B_r(t)} \\ &= I(t) \cdot \exp\left(-\int_t^T f_r(t,s)\, ds - \int_0^t r(s)\, ds \right). \end{aligned}$$

  needs to be a martingale.

- $$
\begin{aligned}
Z^{(r)}(t) &= \frac{I(t) \cdot P_r(0,t)}{B_r(t)} \\
&= I(t) \cdot \exp\left(- \int_t^T f_r(t,s)\, ds - \int_0^t r(s)\, ds \right).
\end{aligned}
$$

  needs to be a martingale.
- Similarly as in the nominal case, we derive

$$
\begin{aligned}
\frac{dP_r(t,T)}{P_r(t,T)} &= \left(r(t) + \frac{1}{2} \left(\int_t^T \sigma_r(t,u)\, du \right)^2 - \int_t^T \mu_r(t,u)\, du \right) dt \\
&\quad - \left(\int_t^T \sigma_r(t,u)\, du \right) d_t W_r,
\end{aligned}
$$

Jarrow-Yildirim model

Real dynamics 2.

- $$\mu_r(t,T) = \sigma_r(t,T)\left(\int_t^T \sigma_r(t,u)\,du - \sigma_I(t)\right),$$

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- $$d_t f_r(t,T) = \sigma_r(t,T)\left(\int_t^T \sigma_r(t,u)\,du - \sigma_I(t)\right)dt + \sigma_r(t,T)\,d_t W_r.$$

Jarrow-Yildirim model

Real dynamics 2.

- $$\mu_r(t,T) = \sigma_r(t,T)\left(\int_t^T \sigma_r(t,u)\,du - \sigma_I(t)\right),$$

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- $$\begin{aligned}\frac{dP_r(t,T)}{P_r(t,T)} &= \left(r(t) + \sigma_I(t)\int_t^T \sigma_r(t,u)\,du\right)dt \\ &\quad - \left(\int_t^T \sigma_r(t,u)\,du\right)d_t W_r,\end{aligned}$$

Jarrow-Yildirim model

Real dynamics 3.

- $$\begin{aligned} r(t) &= f_r(t_0, t) + \int_{t_0}^{t} \left(\sigma_r(s,t) \int_s^t \sigma_r(s,u)\,du - \sigma_I(s) \right) ds \\ &\quad + \int_{t_0}^{t} \sigma_r(s,t)\, d_s W_r. \end{aligned}$$

- $$r(t) = f_r(t_0, t) + \int_{t_0}^{t} \left(\sigma_r(s,t) \int_s^t \sigma_r(s,u)\, du - \sigma_I(s) \right) ds + \int_{t_0}^{t} \sigma_r(s,t)\, d_s W_r.$$

- Thus

$$d_t r(t) = \left[\left(\begin{array}{c} \frac{\partial_2 f_r(0,t)}{a_r} + f_r(0,t) \\ + \frac{\sigma_r^2 \left(1 - e^{-2a_r t}\right)}{2a_r^2} - \rho_{r,I}\sigma_I\sigma_r \end{array} \right) - r(t) \right] dt + \sigma_r d_t W_r.$$

$$
\begin{aligned}
& V^{\text{inf}}_{ZCS}\left(t; T, T_{pay}\right) \\
= \quad & E_t\left[\left(\frac{I\left(T\right)}{I\left(T_0\right)} - 1\right) \cdot \frac{B\left(t\right)}{B\left(T_{pay}\right)}\right] \\
= \quad & \frac{B\left(t\right)}{I\left(T_0\right)} E_t\left[e^{\log \frac{I(T)}{B(T)} + \log \frac{1}{B\left(T_{pay}\right)} - \log \frac{1}{B(T)}}\right] - P_n\left(t, T_{pay}\right) \\
= \quad & P_n\left(t, T_{pay}\right) \cdot \frac{\hat{I}\left(t, T\right)}{I\left(T_0\right)} \\
& \cdot \exp\left[\int_t^T \left(\sigma_{\hat{I}(t,T)}\left(s\right) \int_T^{T_{pay}} \sigma_n\left(s, u\right) du\right) ds\right] \\
& - P_n\left(t, T_{pay}\right).
\end{aligned}
$$

Jarrow-Yildirim model

Monte Carlo simulation for the index 1.

- Let $a = \frac{Cov_t(\log I(T)\ ,\ \log B(T))}{D_t[\log I(T)]}$ and

$$R_{I(T),B(T)} = \begin{pmatrix} D_t\left[\log I\left(T\right)\right] & 0 \\ a & \sqrt{D_t^2\left[\log B\left(T\right)\right] - a^2} \end{pmatrix}.$$

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- Let ξ be 2-dimensional standard normal random variable, and let X and Y be random variables such that

$$\begin{pmatrix} X \\ Y \end{pmatrix} = R_{I(T),B(T)} \cdot \xi.$$

Jarrow-Yildirim model

Monte Carlo simulation for the index 2.

$$
\begin{aligned}
\frac{B(t)}{B(T)} \cdot I(T) &\sim \exp(X - Y) \cdot P_I(t, T) \cdot \exp\left(-\frac{D_t^2\left[\log I(T)\right]}{2}\right) \\
&\quad \cdot \exp\left(-\frac{D_t^2\left[\log B(T)\right]}{2} + Cov_t\left(\log I(T), \log B(T)\right)\right), \\
\frac{B(t)}{B(T)} &\sim \exp(-Y) \cdot P_n(t, T) \\
&\quad \cdot \exp\left(-\frac{D_t^2\left[\log B(T)\right]}{2}\right), \\
I(T) &\sim \exp(X) \cdot \hat{I}(t, T) \\
&\quad \cdot \exp\left(-\frac{D_t^2\left[\log I(T)\right]}{2} + Cov_t\left(\log I(T), \log B(T)\right)\right).
\end{aligned}
$$

$$D_t^2\left[\log I\left(T\right)\right] = \int_t^T\left(\int_s^T\left(\sigma_n\left(s,u\right)-\sigma_r\left(s,u\right)\right)du+\sigma_I\left(s\right)\right)^2 ds,$$

$$D_t^2\left[\log B\left(T\right)\right] = \int_t^T\left(\int_s^T\sigma_n\left(s,u\right)du\right)^2 ds,$$

$$Cov_t\left(\log I\left(T\right)\ ,\ \log B\left(T\right)\right)$$

$$= \int_t^T\left(\int_s^T\sigma_n\left(s,u\right)du-\int_s^T\sigma_r\left(s,u\right)du+\sigma_I\left(s\right)\right)\left(\int_s^T\sigma_n\left(s,u\right)du\right)$$

2-factor model

Compounding effect 1.

- $$d_t i\left(t\right) = \kappa_i\left(\theta_i\left(t\right) - i\left(t\right)\right)dt + \sigma_i d_t W_i,$$

2-factor model

Compounding effect 1.

- $$d_t i(t) = \kappa_i \left(\theta_i(t) - i(t) \right) dt + \sigma_i d_t W_i,$$
- $$d_t r(t) = \kappa_r \left(\theta_r(t) - r(t) \right) dt + \sigma_r d_t W_r.$$

2-factor model

Compounding effect 1.

- $$d_t i(t) = \kappa_i (\theta_i(t) - i(t)) \, dt + \sigma_i d_t W_i,$$
- $$d_t r(t) = \kappa_r (\theta_r(t) - r(t)) \, dt + \sigma_r d_t W_r.$$
- Now $\log B(T) = \int_0^t r(s)\,ds$ and $\log I(T) = \int_0^t i(s)\,ds$ are normally distributed, and

$$\begin{aligned}
\log E_t\left[\frac{I(T)}{I(t)}\right] &= -\log I(t) + E_t[\log I(T)] + \frac{D_t^2[\log I(T)]}{2} \\
&= -\log I(t) + E_t\left[\log \frac{I(T)}{B(T)}\right] \\
&\quad - E_t\left[\log \frac{1}{B(T)}\right] + \frac{D_t^2[\log I(T)]}{2}
\end{aligned}$$

2-factor model

Compounding effect 2.

$$
\begin{aligned}
= \quad & -\log I(t) + \left(\log E_t\left[\frac{I(T)}{B(T)}\right] - \frac{D_t^2\left[\log \frac{I(T)}{B(T)}\right]}{2} \right) \\
& - \left(\log E_t\left[\frac{1}{B(T)}\right] - \frac{D_t^2\left[\log \frac{1}{B(T)}\right]}{2} \right) \\
& + \frac{D_t^2\left[\log I(T)\right]}{2} \\
= \quad & -\log I(t) + \log P_I(t, T) \\
& - \log P(t, T) + Cov_t\left(\log I(T), \log B(T)\right)
\end{aligned}
$$

2-factor model

Compounding effect 3.

$$
\begin{aligned}
&= \log \hat{I}(t,T) - \log I(t) + Cov_t\left(\log I(T), \log B(T)\right) \\
&= \log \hat{I}(t,T) - \log I(t) \\
&\quad + \frac{\rho\sigma_i\sigma_r}{\kappa_i\kappa_r}\left(\begin{array}{c} T - t + \frac{1-e^{-(\kappa_i+\kappa_r)(T-t)}}{\kappa_i+\kappa_r} \\ -\frac{1-e^{-\kappa_i(T-t)}}{\kappa_i} - \frac{1-e^{-\kappa_r(T-t)}}{\kappa_r} \end{array} \right).
\end{aligned}
$$

Thank you for your attention

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