MODELING AND IDENTIFICATION OF A NUCLEAR REACTOR WITH TEMPERATURE DEPENDENT REACTIVITY

CSABA FAZEKAS, ATTILA GÁBOR,
KATALIN M. HANGOS

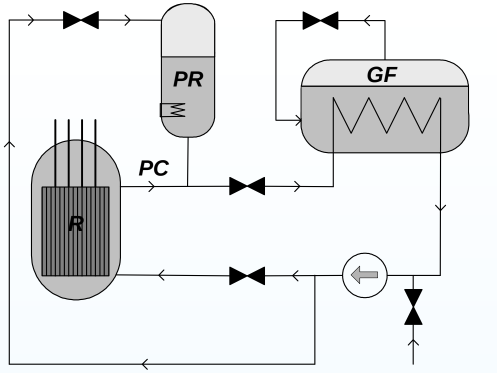
Process Control Research Group
Computer and Automation Research Institute
Budapest, Hungary

Contents

Paks Nuclear Power Plant

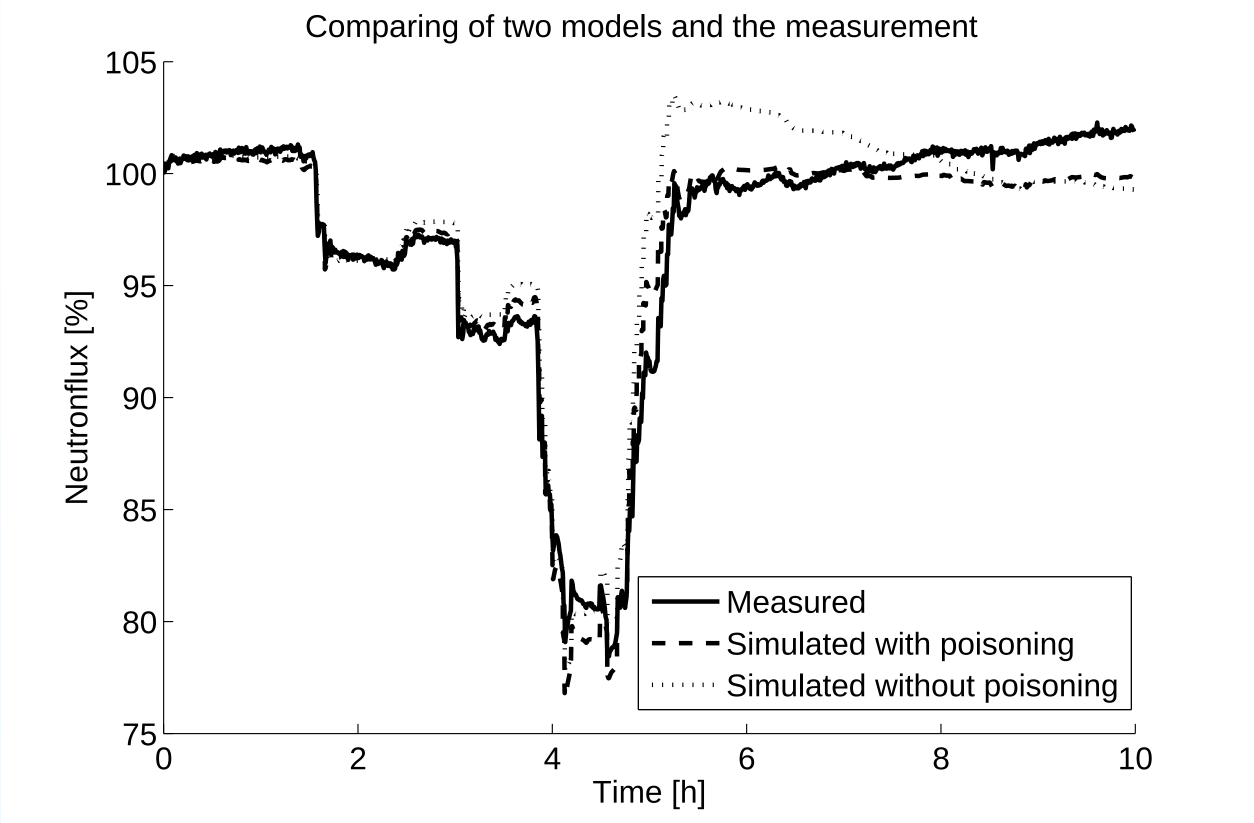
- Founded in 1976, operation started in 1982
- Operates four VVER-440/213 type (pressurized water) reactor units
- Total nominal electrical power: 1860 MWs
- Produces about 40 percent of the electrical energy in Hungary

Motivation and Aim

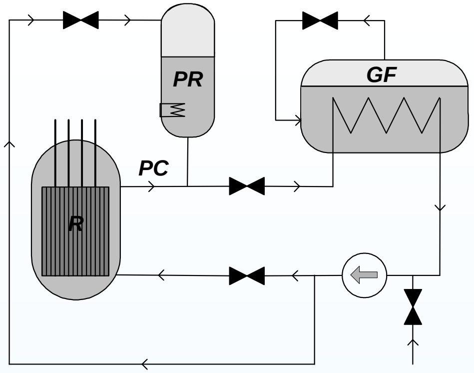


A primitive model

$$\frac{dN}{dt} = \frac{p_1 z^2 + p_2 z + p_3}{\Lambda} N + S$$



Engineering Model Equations - 1



Balances for neutron and for delayed neutron emitting nuclei

$$
\begin{aligned}
\frac{dN}{dt} &= \beta\frac{N}{\Lambda}\left(\rho - 1\right) + C\frac{\beta}{\Lambda} \\
\frac{dC}{dt} &= \lambda_C(N - C)
\end{aligned}
$$

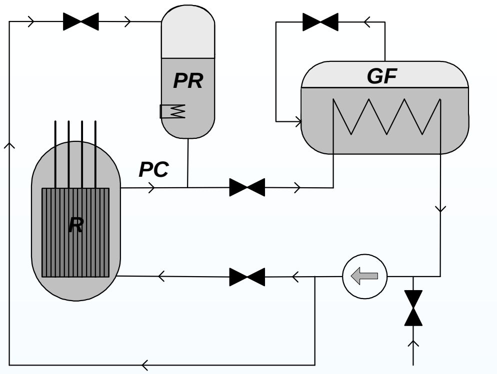
Reactivity equation

$$
\begin{aligned}
\rho &= \alpha_f(T_f - T_{f0}) + \alpha_m(T_m - T_{m0}) \\
&\quad + p_2 z^2 + p_1 z + p_0 + \frac{\sigma_X}{\beta\Sigma_f}(n_X - n_{X0})
\end{aligned}
$$

Energy balances for the fuel and the moderator

$$
\begin{aligned}
\frac{dT_f}{dt} &= -\frac{UA}{M_f c_{pf}}(T_f - T_m) + \frac{F}{M_f c_{pf}}N \\
\frac{dT_m}{dt} &= \frac{UA}{M_m c_{pm}}(T_f - T_m) - \frac{m_p}{M_m}(T_{out} - T_{in})
\end{aligned}
$$

Engineering Model Equations - 2

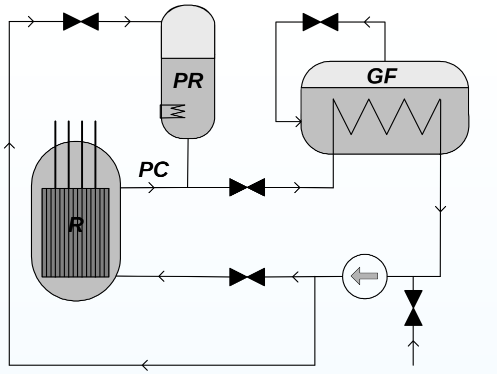


Equations of poisoning

$$\begin{aligned}\frac{dn_I}{dt} &= Y_I \Sigma_f \frac{N}{N_0}\phi_0 - \lambda_I n_I \\ \frac{dn_X}{dt} &= Y_X \Sigma_f \frac{N}{N_0}\phi_0 + \lambda_I n_I - \lambda_X n_X \\ & \quad - \sigma_X n_X \frac{N}{N_0}\phi_0\end{aligned}$$

Measurements available

- neutron flux N
- average temperature of the moderator T_m
- rod position z



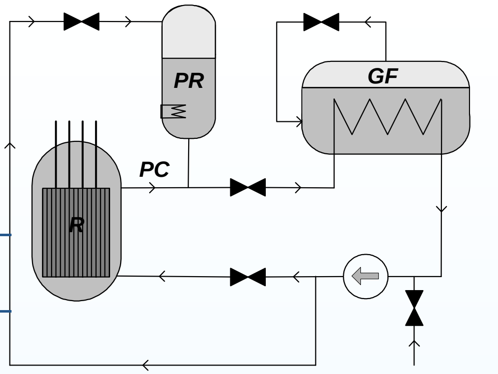
State-space model

State equations

$$
\begin{aligned}
\frac{dN}{dt} &= \beta\frac{N}{\Lambda}\Bigg(\alpha_f(T_f - T_{f0}) + \alpha_m(T_m - T_{m0}) + \\
&\quad + p_2 z^2 + p_1 z + p_0 + \frac{\sigma_X}{\beta}(X - X_0) - 1\Bigg) + C\frac{\beta}{\Lambda} \\
\frac{dC}{dt} &= \lambda_C(N - C) \\
\frac{dT_f}{dt} &= -A_1(T_f - T_m) + A_1\frac{T_{f0} - T_{m0}}{N_0}N \\
\frac{dT_m}{dt} &= A_3(T_f - T_m) - A_3\frac{T_{f0} - T_{m0}}{T_{m0} - T_{in0}}(T_m - T_{in}) \\
\frac{dI}{dt} &= Y_I\frac{N}{N_0}\phi_0 - \lambda_I I \\
\frac{dX}{dt} &= Y_X\frac{N}{N_0}\phi_0 + \lambda_I I - \lambda_X X - \sigma_X X\frac{N}{N_0}\phi_0
\end{aligned}
$$

Output equations

$$
y = [N, T_m]^T
$$

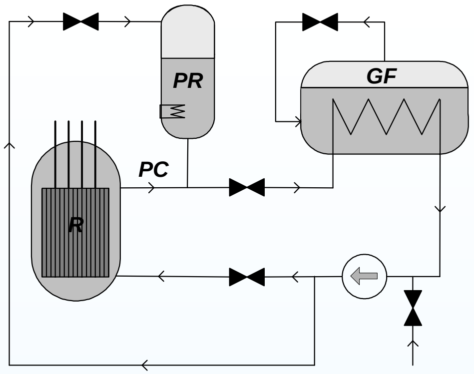


State-space model: parameters

Identifier	Meaning	Domain
ϕ_0	equil. neutron flux	$[10^{13}, 10^{14}]$
σ_X	absorpttion cross section	$[2.8 \cdot 10^{18}, 3.2 \cdot 10^{18}]$
α_f	temp. coefficient, fuel	$[-5.5 \cdot 10^{-3}, -3.8 \cdot 10^{-3}]$
α_m	temp. coefficient, moderator	$[-3.5 \cdot 10^{-2}, -1.8 \cdot 10^{-2}]$
A_1	parameter, energy balances	$[0.1, 1]$
A_3	parameter, energy balances	$[0.1, 1]$
p_0	rod parameter	$[-0.1, 0.1]$
p_1	rod parameter	$[-1, -0.1]$
p_2	rod parameter	$[-1, -0.1]$
Λ	avg. generation time	$[1.5 \cdot 10^{-5}, 3.5 \cdot 10^{-5}]$
λ_I	decay constant of I	$[2.8 \cdot 10^{-5}, 3 \cdot 10^{-5}]$
λ_X	decay constant of Xe	$[2 \cdot 10^{-5}, 2.2 \cdot 10^{-5}]$

The parameters to be estimated **possess physical meaning**

Identification method



The properties of the parameter estimation problem

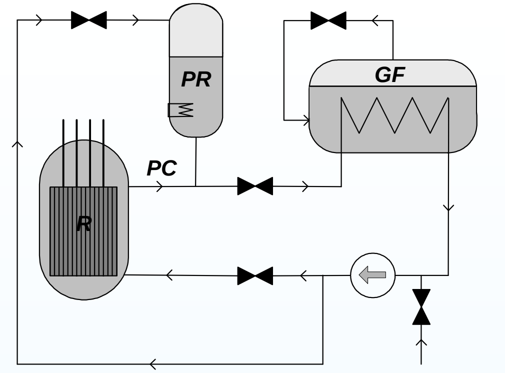
- model equations are **nonlinear in parameters**,
- big difference in the time constants (2 orders of magnitude),
- an optimization-based parameter estimation method, the **Nelder-Mead simplex method** is used with the fit measure

$$f_{obj} = \sqrt{\frac{\int_0^T (\hat{N}(t) - N(t))^2 dt}{\int_0^T N^2(t) dt} + \frac{\int_0^T (\hat{T}_m(t) - T_m(t))^2 dt}{\int_0^T T_m^2(t) dt}}$$

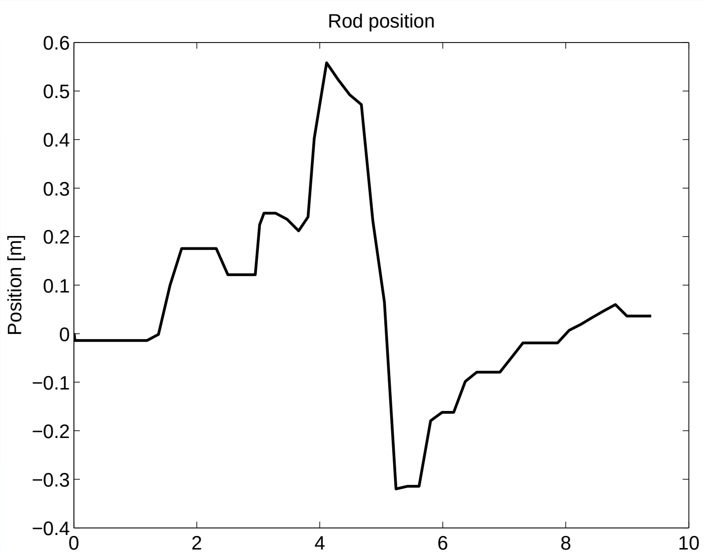
- the **model-predicted output** $\hat{y}(t)$ is computed by simulation ($y = \{N, T_m\}$)

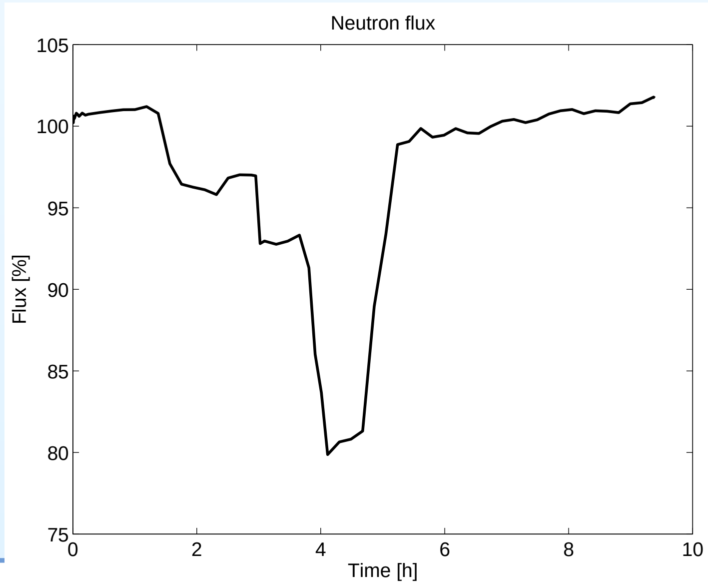
Initial values: engineering tables
Reliability domain could be determined

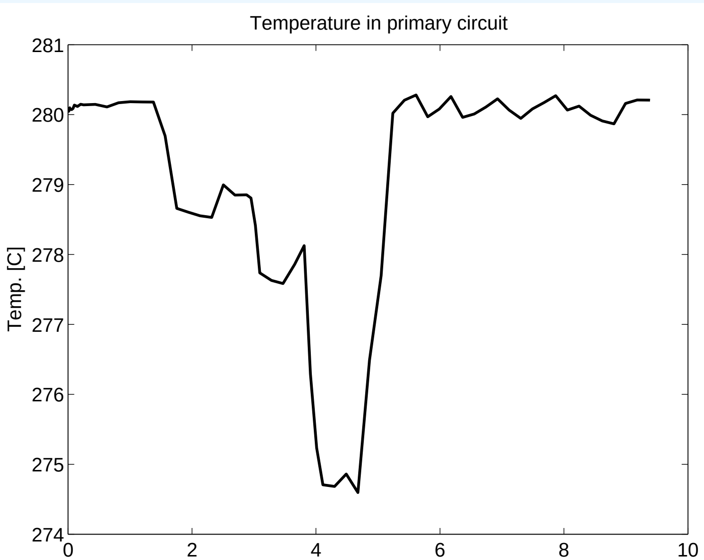
Measured input-output data



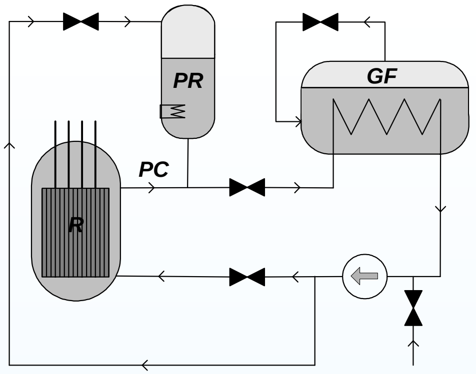
A relatively quick temporal off-loading transient, overall duration 10 hours







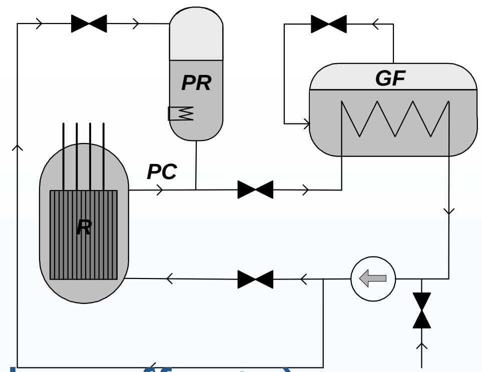
Estimated parameters



Identifier	Value	Domain
ϕ_0	$1.3 \cdot 10^{13}$	$[10^{13}, 10^{14}]$
σ_X	$2.85 \cdot 10^{-18}$	$[2.8 \cdot 10^{18}, 3.2 \cdot 10^{18}]$
α_f	$-5.362 \cdot 10^{-3}$	$[-5.5 \cdot 10^{-3}, -3.8 \cdot 10^{-3}]$
α_m	$-2.018 \cdot 10^{-2}$	$[-3.5 \cdot 10^{-2}, -1.8 \cdot 10^{-2}]$
A_1	0.1056	$[0.1, 1]$
A_3	0.8757	$[0.1, 1]$
p_0	0.0401	$[-0.1, 0.1]$
p_1	-0.44	$[-1, -0.1]$
p_2	-0.976	$[-1, -0.1]$
Λ	$2.18 \cdot 10^{-5}$	$[1.5 \cdot 10^{-5}, 3.5 \cdot 10^{-5}]$
λ_I	$2.9306 \cdot 10^{-5}$	$[2.8 \cdot 10^{-5}, 3 \cdot 10^{-5}]$
λ_X	$2.1066 \cdot 10^{-5}$	$[2 \cdot 10^{-5}, 2.2 \cdot 10^{-5}]$

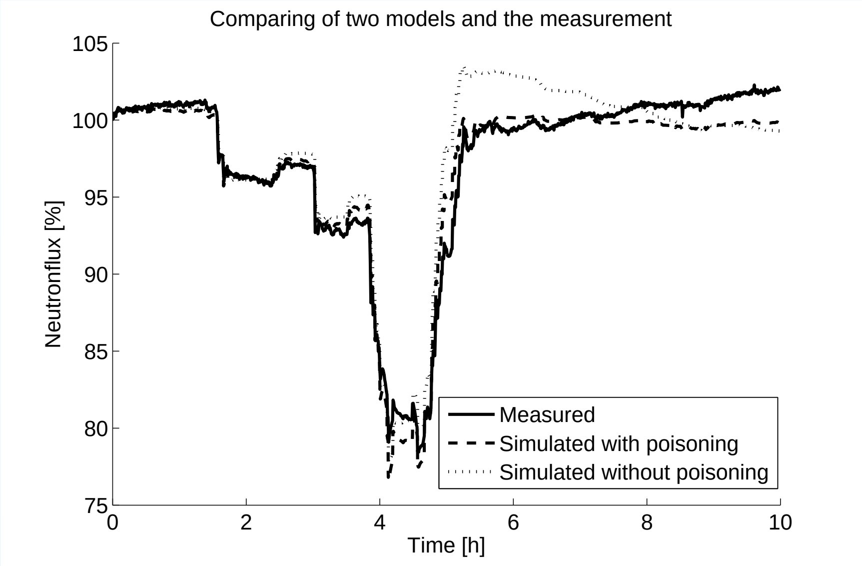
The parameters **are within their realiability domain**

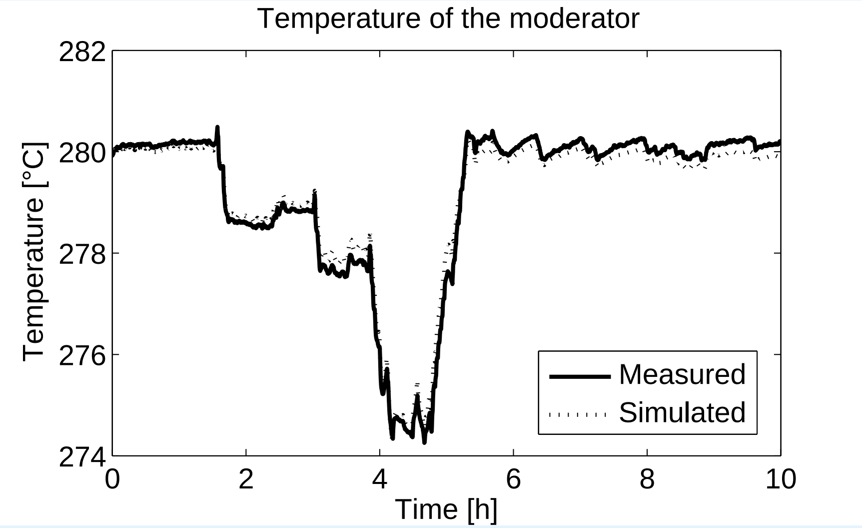
The fit



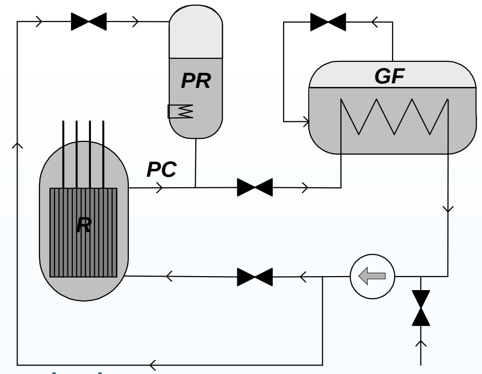
A relatively quick temporal off-loading transient

- duration: 3 hours (small compared to the poisoning effects)
- magnitude: 25 %: large compared to the validity domain of the model





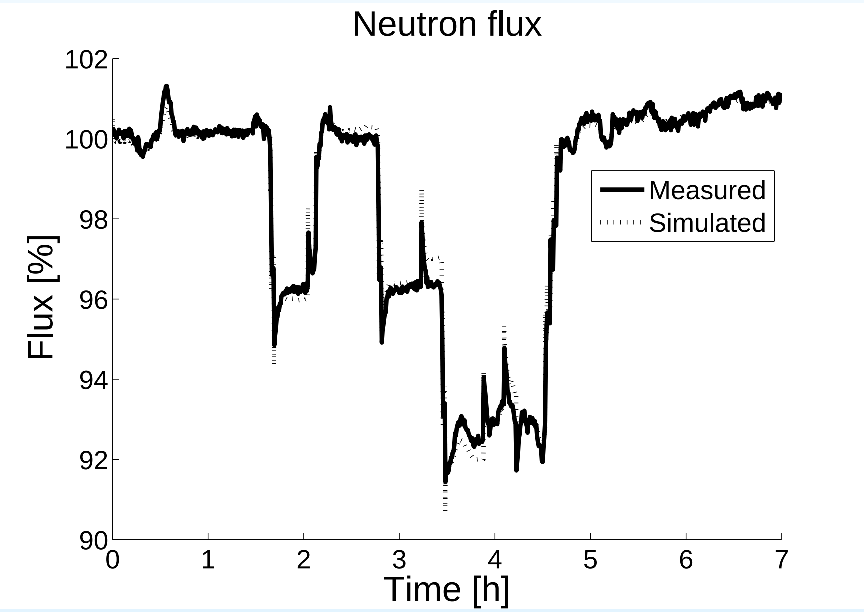
Problems at the end of the transient

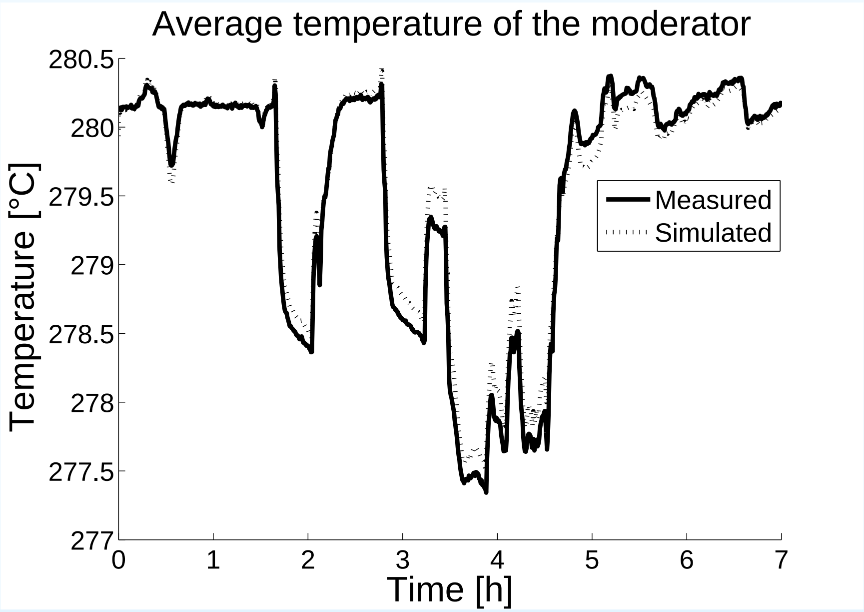


Further improvements

Improved parameter estimation with

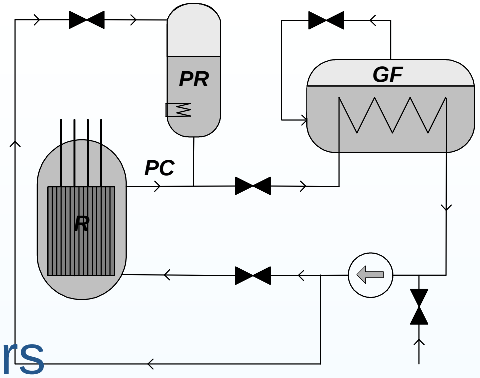
- estimating the initial condition of the non-measurable state variables
- respecting the model validity domain (100-90 %) - another transient





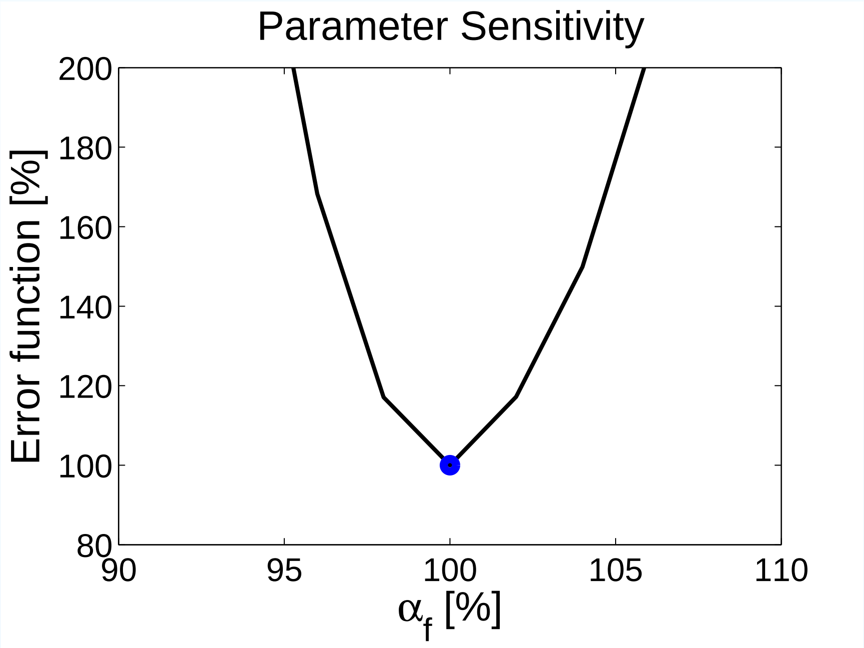
Much better fit

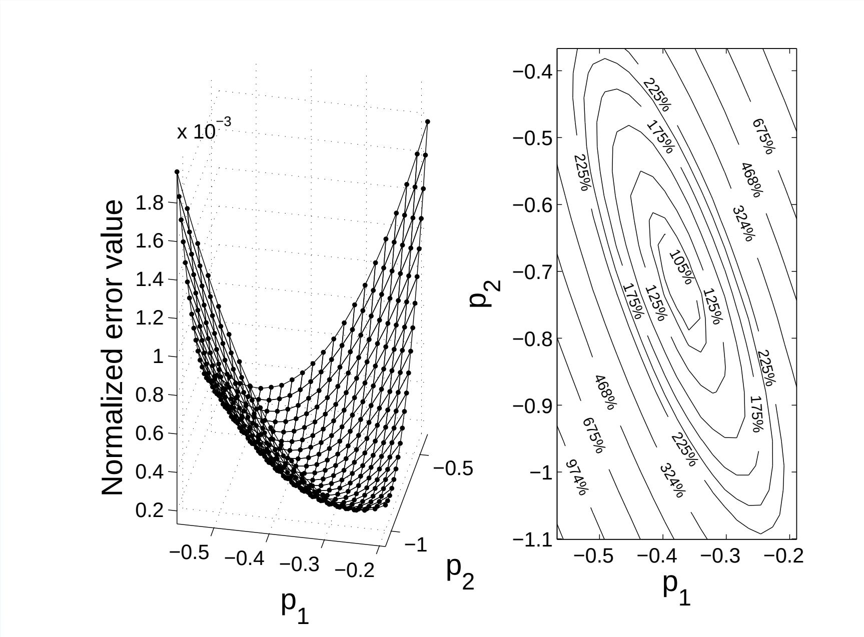
The quality of the estimates

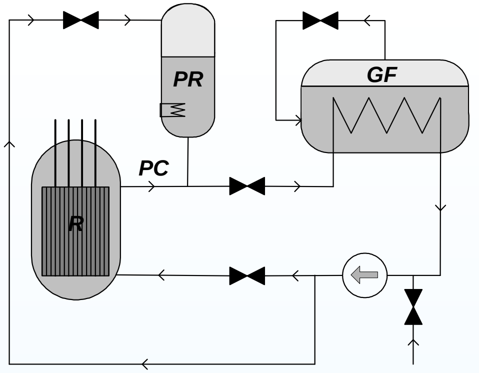


Evaluation

- sensitivity of the loss with respect to single parameters
- sensitivity of the loss with respect to a pair of parameters







Conclusions and future work

Results

- **Simple model in physical coordinates**
  - nonlinear in its variables and parameters
  - parameters with physical meaning
- **Model parameter estimation**
  - Nelder-Mead simplex optimization
- **Refinement by estimating the initial values of the state variables**

Future work

- integrating the reactor model into the primary circuit model
- supervisory controller for coordinating the controllers in the primary circuit