Stability of Direct Spring Operated Pressure Relief Valves – from CFD to spreadsheet

Csaba Hős (BME, Dept. of Hydrodynamic Systems)

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HDR

MŰEGYETEM 1782

... with major contributions by

- Prof. Alan Champneys (University of Bristol, Dept. of Eng. Mathematics)
- Dr. Csaba Bazsó (BME HDS)
- István Erdei (BME HDS)
- Paul Kenneth, Mike McNelly (Pentair, Houston, TX)

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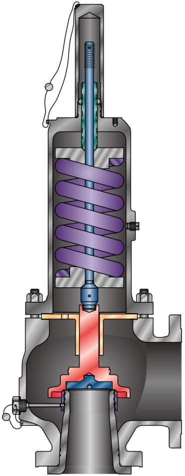
DSOPRV

- Direct Spring Operated Pressure Relief Valves: safety device to limit system pressure - last line of defence.



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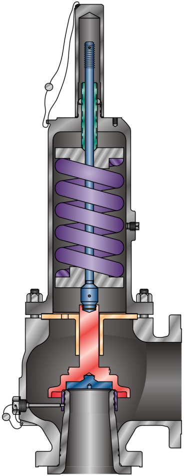

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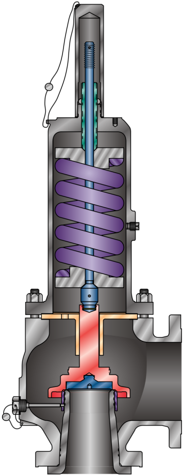

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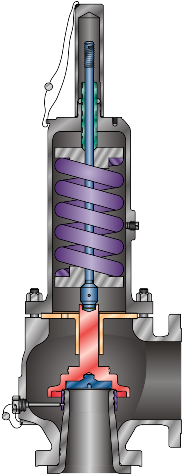

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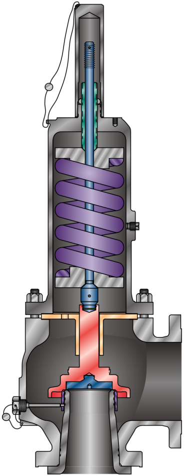

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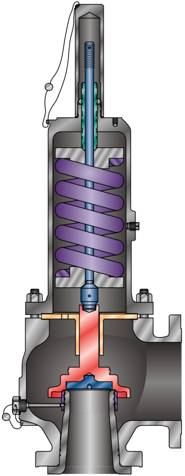

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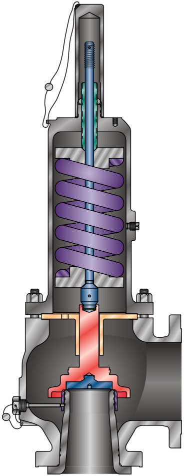

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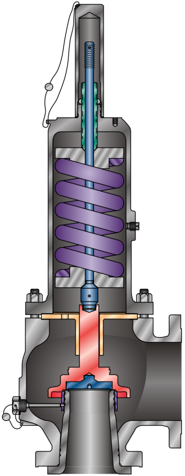
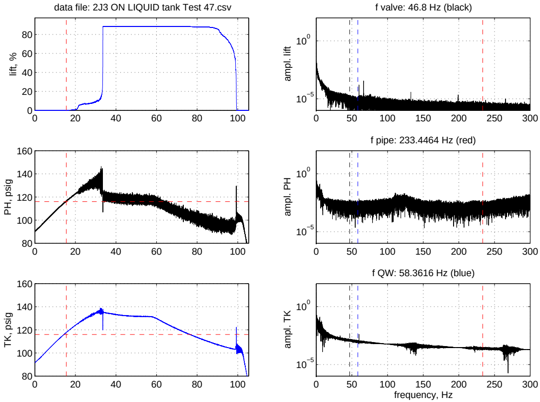
Challenges

- Valve chatter
- API code: 3% rule based on upstream pipe pressure loss – is that sufficient?
- Lack of measurements (in the open domain).



Example of stable opening



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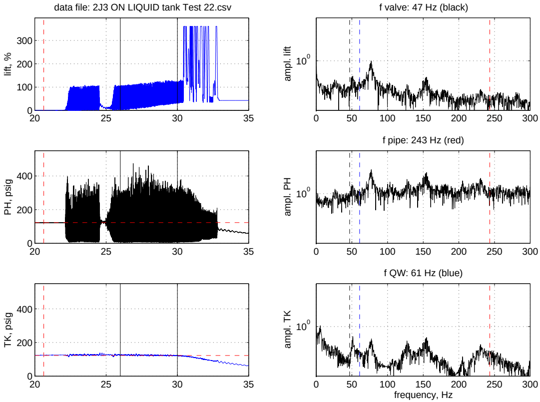

Measurement

video 1

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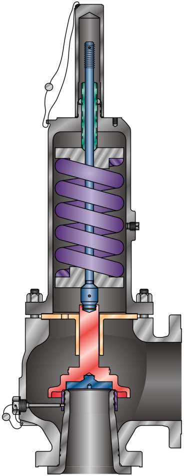
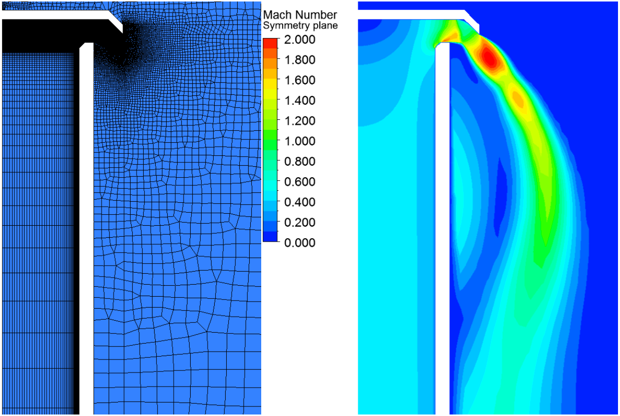
Computational Fluid Dynamics

- ANSYS CFX + Icem
- Deforming mesh + automatic remeshing
- Upstream pipe + simple valve model
- Axisymmetric
- Valve disc as rigid body
- High-resolution, lots of information but slow and no qualitative understanding
- Stable and unstable behaviour reproduced.



CFD

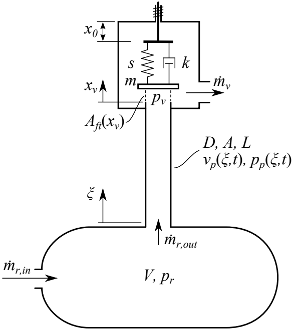

Mach Number
Symmetry plane
2.000
1.800
1.600
1.400
1.200
1.000
0.800
0.600
0.400
0.200
0.000

CFD

video 2

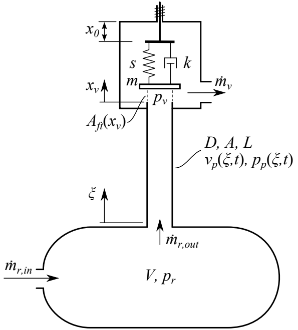
video 3

1D model for liquid service



1D model for liquid service

- valve: 1DoF oscillator
  $m\ddot{x}_v + k\dot{x}_v + s(x_0 + x_v) = F_{\text{lift}},$
  $F_{\text{lift}} = A_{\text{eff}}(x_v)(p_v - p_0)$



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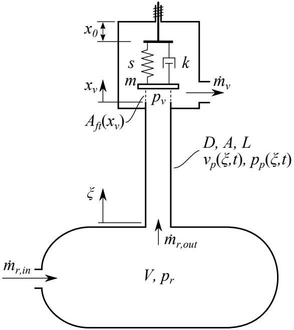
  $\dot{m}_{r,in} - \dot{m}_{r,out} = \frac{V}{a^2}\dot{p}_r$



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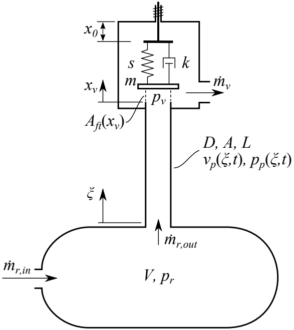
  $\frac{\partial p}{\partial t} + \rho a^2 \frac{\partial v}{\partial \xi} + v\frac{\partial p}{\partial \xi} = 0$
  $\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial \xi} = -\frac{1}{\rho}\frac{\partial p}{\partial \xi} + \frac{\lambda}{2D_{pipe}} v|v|$



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- BC @ res. side:
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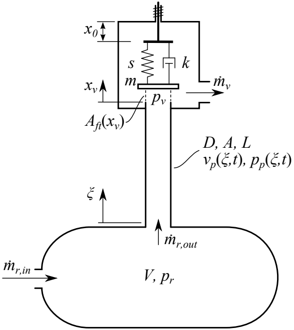

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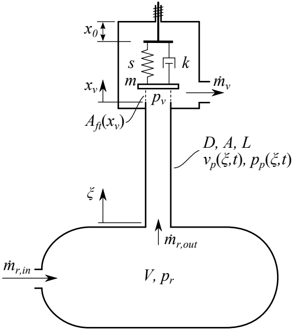
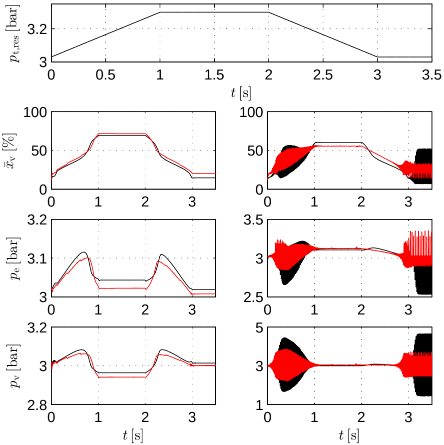
- BC @ valve-end:
  $v(L,t)A_{pipe}\rho = C_d(x_v)A_{ft}(x_v)\sqrt{2\rho\,(p(L,t) - p_0)}$



Simulation results

pipe length: 0.5m
pipe length: 1.1m
pipe length: 1.5m

Simulation results – CFD vs. 1D



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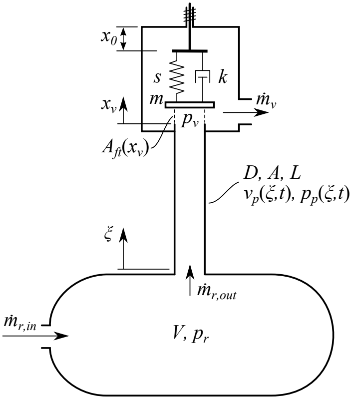
Primary instability types

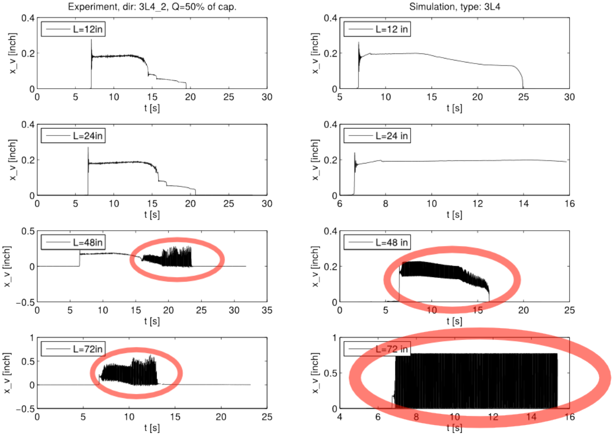
Remember, our model is...

Aim:

Systematically isolate instability types and give design formulae to avoid them.

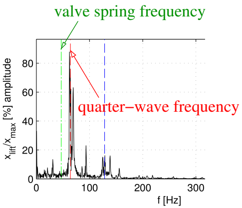


Quarter-wave instability

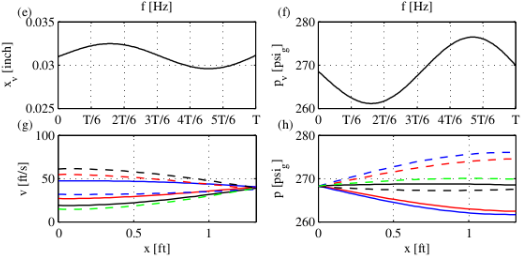


Valve chatter

experiments



theory



- self-excited oscillations despite steady-state BCs
- Quarter-wave frequency of the pipe seems to dominate

The Quarter-wave model (QWM)

Simplest case (liquid, one mode only):

- Aim: replace the PDEs describing the pipeline dynamics to ODEs that allow stability analysis.

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$$v(x,t) = v(L,t) + C(t)\cos\left(2\pi \frac{x}{4L}\right)$$

where $v(L,t) = C_d \frac{A_{ft}(x_v)}{A_{pipe}} \sqrt{\frac{2}{\rho}(p(L,t) - p_0)}$

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- Solve the above equations for $p(L,t)$ and $v(0,t)$.
- Then use 1-point collocation technique (PDE $\rightarrow$ ODE).
- One can perform the same computation for arbitrary wave modes.

The simplest QWM

$$
\begin{aligned}
x_v' &= v_v \\
v_v' &= -\kappa v_v - (x_v + \delta) + \tilde{A}_{eff}(p + B) \\
p' &= \beta\left(q - \mu(v_{end} + C)\right) \\
B' &= \frac{\pi}{2}\frac{\alpha}{\gamma}C - \sqrt{2}p' + \phi\left(\frac{C^2}{\sqrt{2}} + 2Cv_{end} + \sqrt{2}v_{end}^2\right) \\
C' &= -\frac{\pi}{2}\frac{1}{\alpha\gamma}B - \sqrt{2}v_{end}' \\
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... and one can also add pipe friction, convective terms, more modes (however, you might want to use a computer algebra system).

Analytical stability criteria

- Assume large reservoir ($\beta \approx 0$) $\rightarrow$ $y_3 \approx$ konstans.

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- and the valve dynamics is $Y'' = -Y + B - \kappa Y'$.
- Close to the stability boundary: $B(\tau;\tau_2) = A(\tau_2)\cos(\omega_p\tau)$,
- with which the valve displacement becomes:

$$Y = \frac{-1}{1-\omega_p^2}B + \mathcal{O}(\kappa)$$

Analytical stability criteria (cont'd)

- The pipe dynamics is:

$$B'' + \left(\frac{\pi/2}{\gamma}\right)^2 B = -\text{konst.}\frac{\alpha}{\gamma}\sigma \underbrace{\left(\frac{X_0}{2\sqrt{P_0}} - \frac{\sqrt{P_0}}{\omega_p^2 - 1}\right)}_{!>0} B'$$

Analytical stability criteria (cont'd)

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- For small q values, the equilibrium (X_0, P_0) can be expanded into Taylor series and given in closed form.

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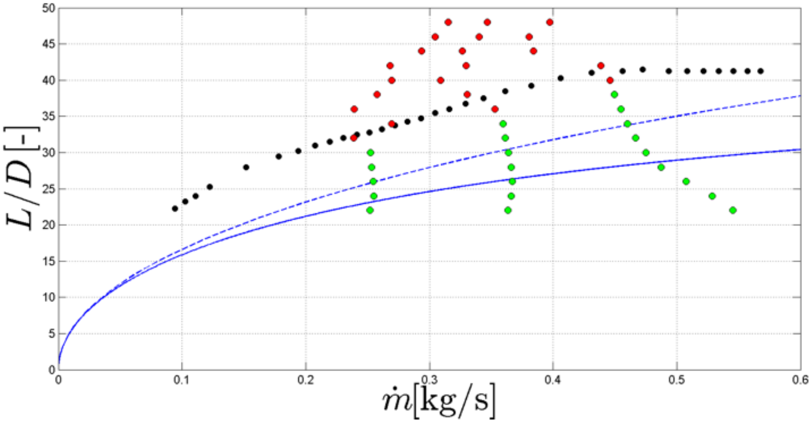
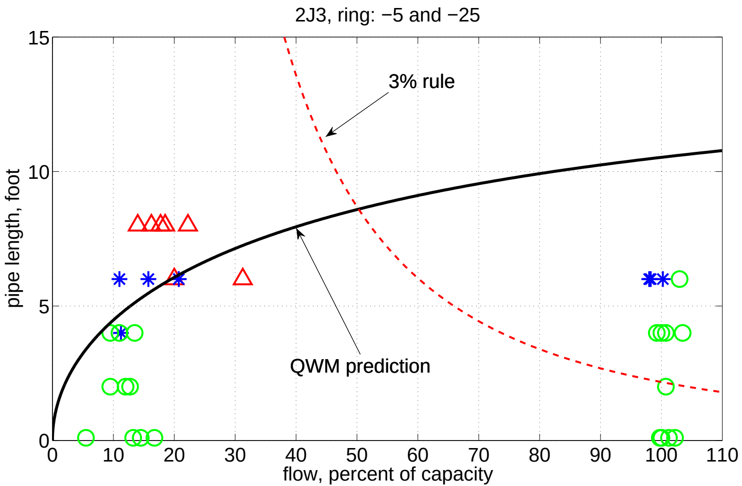
- For small q values, the equilibrium (X_0, P_0) can be expanded into Taylor series and given in closed form.
- After some algebra, one can arrive ar $q \geq 2\frac{\delta^{3/2}}{\mu\sigma((\omega_p(L))^2-1)}$, which is straightforward to implement even in a spreadsheet software.

CFD (red/blue) vs. 1D model (black) vs. QWM analytical (blue)



Stability diagram - QWM vs. meas. 2J3 valve



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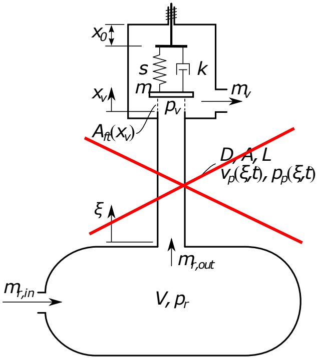
Simplified model without pipe

- Close-coupled valve
- Small reservoir
- The resulting model is:

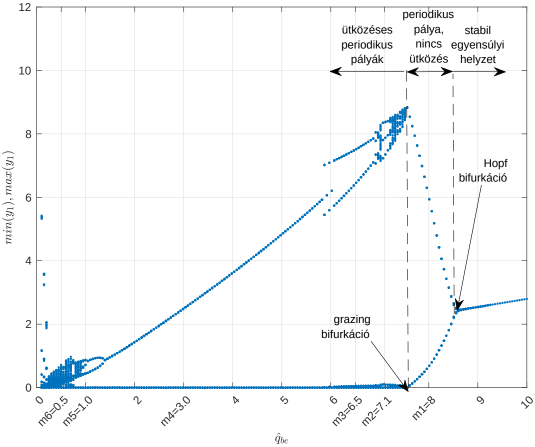
$$
\begin{aligned}
y_1' &= y_2 \\
y_2' &= -\tilde{\kappa} y_2 - (y_1 + \delta) + y_3 \\
y_3' &= \hat{\beta}\left(\hat{q} - y_1\sqrt{y_3}\right)
\end{aligned}
$$

- ... with the impact law at $y_1 = 0$:

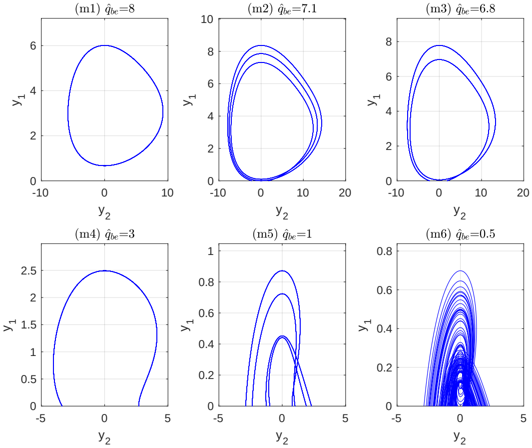
$$(y_1, y_2, y_3)^T \rightarrow (y_1, -r y_2, y_3)^T$$



Bifurcation diagram



Some orbits:


(m1) $\hat{q}_{be}$=8
(m2) $\hat{q}_{be}$=7.1
(m3) $\hat{q}_{be}$=6.8
(m4) $\hat{q}_{be}$=3
(m5) $\hat{q}_{be}$=1
(m6) $\hat{q}_{be}$=0.5
y_1
y_2

Continuation strategy

- Formulate the problem as a BVP, i.e. $y' = TF(y)$ with

$$\begin{aligned} y_1(0) &= 0 \\ y_1(1) &= 0 \\ -ry_2(0) &= y_2(1) \\ y_3(0) &= y_3(1) \end{aligned}$$

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- Use pseudo-arclength cont. to track periodic orbits (+1 BC).

Continuation strategy

- Formulate the problem as a BVP, i.e. $y' = TF(y)$ with

$$\begin{aligned} y_1(0) &= 0 \\ y_1(1) &= 0 \\ -ry_2(0) &= y_2(1) \\ y_3(0) &= y_3(1) \end{aligned}$$

- Use pseudo-arclength cont. to track periodic orbits (+1 BC).
- Stability: solve variational equation to compute monodromy matrix and apply correction at the impact (see Bernardo, M., Budd, C., Champneys, A.R., Kowalczyk, P.: Piecewise-smooth Dynamical Systems: Theory and Applications, Springer, 2008, ISBN 978-1-84628-039-9).

Continuation strategy (cont'd)

- Continuation of grazing points: $y_2(0) = 0$ (+ 4 + 1 BC)

Continuation strategy (cont'd)

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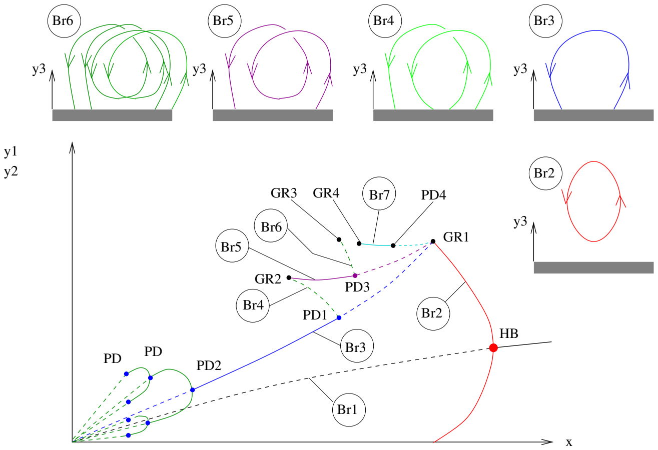
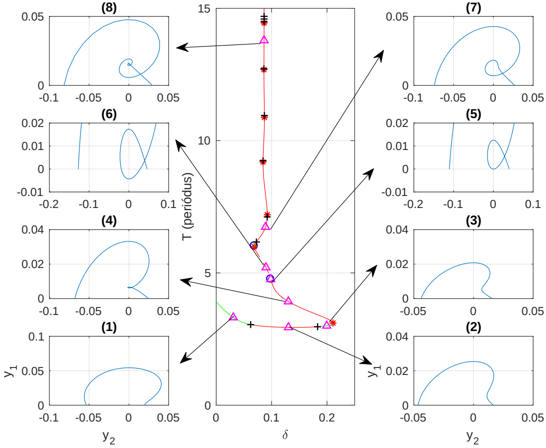
Continuation strategy (cont'd)

- Continuation of grazing points: $y_2(0) = 0$ (+ 4 + 1 BC)
- Continuation of period doublings: one of the characteristic multipliers is -1. Problems with accuracy!
- Implemented in Matlab, using `bvp5c`.

Qualitative bifurcation diagram



Shilnikov-like orbit



Shilnikov-like orbit

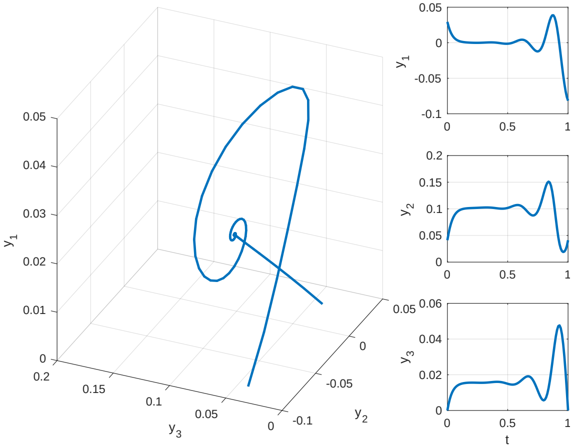


Table of Contents

Summary

Modeling levels

- CFD: few hours $\leftrightarrow$ full 3D, transient
- 1D unsteady model: few minutes $\leftrightarrow$ 1D, transient
- QWM: seconds $\leftrightarrow$ 1D, only close to equilibrium (no large-amplitude oscillations)
- Analytical: ??? $\leftrightarrow$ only around the stability boundary, assumptions need to be checked

Impacting periodic orbits

- Relatively new mathematical results.
- Standard nonlin. dyn. toolkit can be used.
- Surprisingly rich dynamics.

Thank you for you attention!

Effective area

A simple yet accurate estimate for the fluid force is essential:

$$F_{fluid} = \int_{(A)} p(A)\mathrm{dA} + F_{imp}(\dot{m}, \beta)$$

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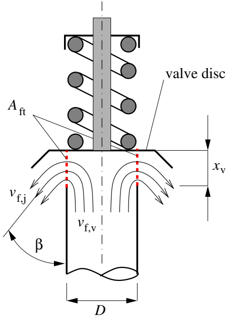
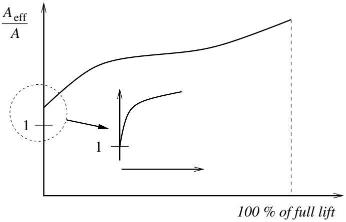
$$F_{fluid} = \int_{(A)} p(A)\mathrm{d}A + F_{imp}(\dot{m}, \beta)$$

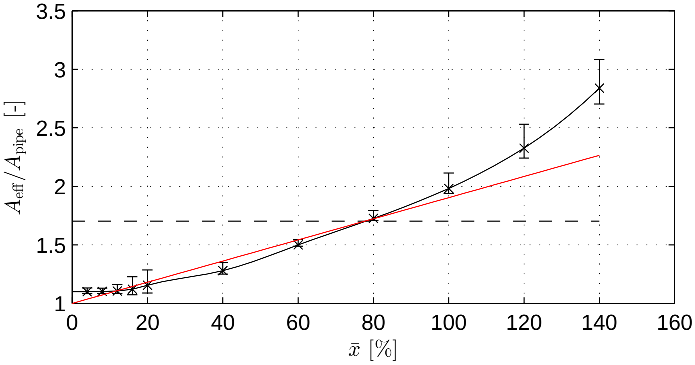
Define effective area as

$$F_{fluid} = A_{eff}(x)\Delta p$$





Effective area – theory vs. CFD



Effective area – two more examples