

# Credit Default Risk & Correlated Defaults

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BME

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# Agenda

Credit Risk

Merton's Model for Credit Risk

Basel Rules - Incremental Risk Charge

Multivariate Factor Models

Correlated Defaults

Empirical Evidences

# Summary

We briefly present a multi-factor Gaussian copula portfolio model for default risk. The model assumes three types of systematic factors driving the asset returns, hence the value of each company. These factors represent the state of the global economy and the economic conditions of different geographical regions and industries. The corresponding factor loadings play a key role in the model, as they capture the correlation structure between the asset returns of different companies and therefore influence the joint probabilities of default. Higher correlation between the returns of different companies in a portfolio increases the likelihood that multiple companies will default simultaneously, thus increasing the likelihood of extreme losses in the portfolio. Hence, accurately measuring these correlations is essential for the identification of portfolio risk.

We describe a possible methodology for measuring the correlations between asset returns of different companies, which can be used for calibrating the corresponding factor loadings. The approach relies upon single-name CDS spread data. We will also present the structure of correlations obtained using this methodology.

# Credit Risk Modelling

- Credit Risk
- Credit Ratings (S&P, Moody's, Fitch ...)
- Probability of Default (*PD*)

|                  |   | Rating Category | PD lower bound (in bps) | PD upper bound (in bps) |
|------------------|---|-----------------|-------------------------|-------------------------|
| Investment Grade | { | AAA             | 0                       | 0.16                    |
|                  |   | AA              | 0.16                    | 2.39                    |
|                  |   | A               | 2.39                    | 9.28                    |
|                  |   | BBB             | 9.28                    | 35.95                   |
| Non Investment   | { | BB              | 35.95                   | 139                     |
|                  |   | B               | 139                     | 539                     |
|                  |   | CCC / CC / C    | 539                     | 10,000                  |

- Recovery Rate (*R*) & Loss Given Default (*LGD*)
- Expected Loss =  $PD * LGD$
- Credit Spread: price paid for the risk
  - Bonds: risk-free rate + credit spread
  - CDS (Credit Default Swaps): credit spread

# Credit Risk Modelling

Hazard rate ( $\lambda$ ):

$$P(\text{default between } T \text{ and } T + \Delta t \mid \text{no default until } T) = \lambda * \Delta t$$

Probability of default:

$$PD[0; t] = 1 - e^{-\int_0^t \lambda(\tau) d\tau}$$

Credit spread:

$$\lambda(t) * T * (1 - R) = s(T) * T$$

Hazard rate:

$$\lambda(t) = \frac{s(T)}{1 - R}$$

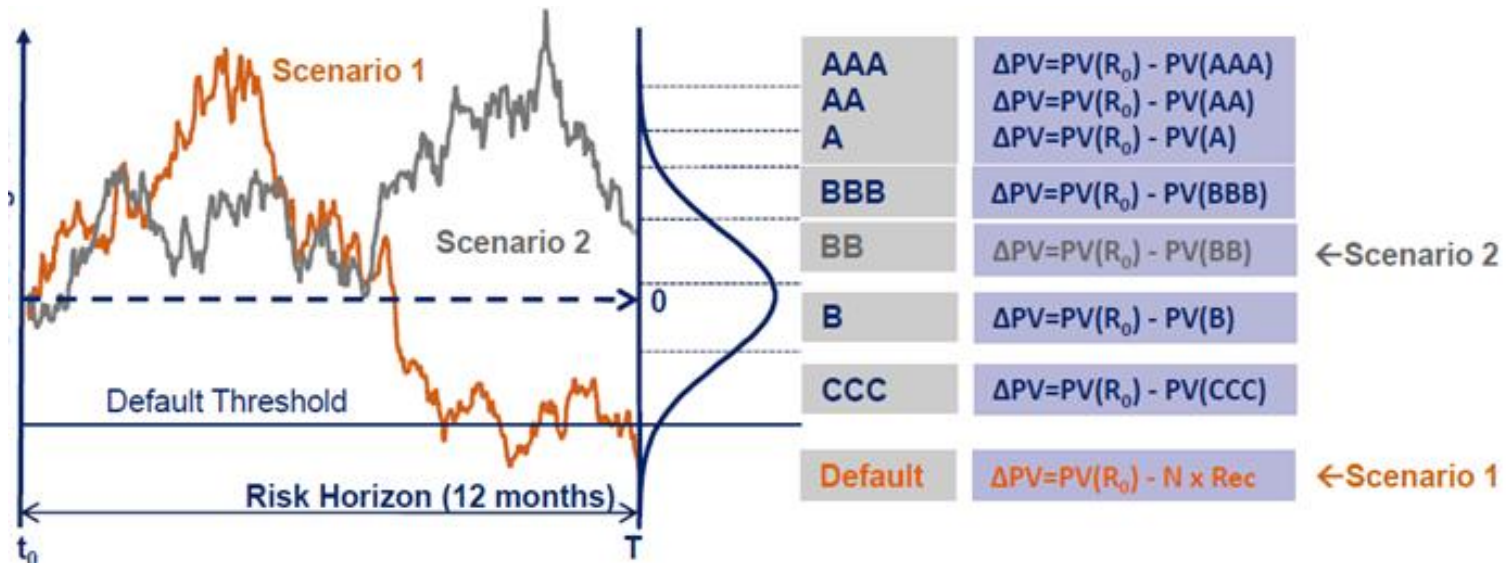
Market:  $s(T), R \Rightarrow$  Calculate  $\lambda, PD$

# Merton Model – Single Names

- Modelling Probability of Default & Rating Migrations
- Modelling Single Issuers
- Company (Asset) Value: Stochastic Process (Brownian Motion)

$$dV(t) = (\alpha * V(t) - C)dt + \sigma * V(t) * dz$$

- Thresholds for default and rating migration



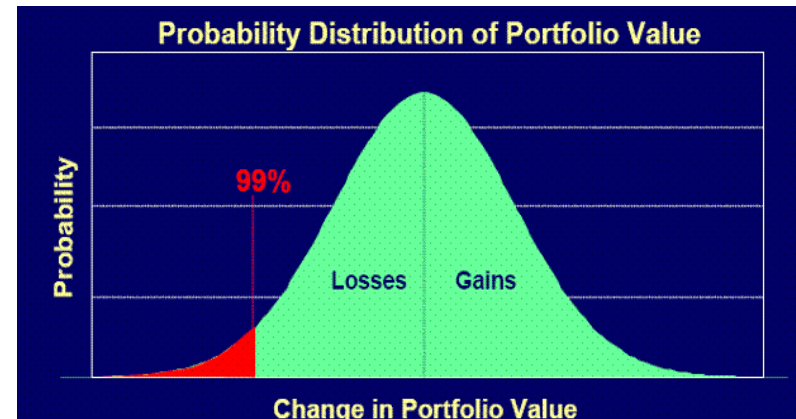
# Merton Model – Portfolios

- Multiple Asset Return Processes
- Multiple Rating Migrations & Defaults
- Correlation Structure

# Incremental Risk Charge

## Regulatory Requirements (Basel 2.5, 2013)

- Risk scope
  - Defaults
  - Rating migrations
- VaR-type model at 99.9% confidence level (max loss we can suffer...)
- Capital Horizon: 1 year
- Correlations between default and credit migration events among obligors (may depend on industry or region)
- Capital Requirement



## Modelling Framework

- Portfolio-level Merton Models
- Factor Models (correlation structure)
- Monte Carlo Simulation



# Factor Models – Correlations

Normalized log return of the company's asset value:

$$r_i = \underbrace{R X_i}_{\substack{\text{Systematic} \\ \text{Risk} \\ \text{MARKET}}} + \underbrace{\sqrt{1 - R^2} \varepsilon_i}_{\substack{\text{Idiosyncratic} \\ \text{Risk} \\ \text{ISSUER}}} = R \sum \beta_{i,k} \overset{\substack{\text{Systematic} \\ \text{Factors}}}{\downarrow} F^k + \sqrt{1 - R^2} \varepsilon_i \sim N(0,1)$$

$\uparrow$   
 Issuer-specific  
 factor loadings

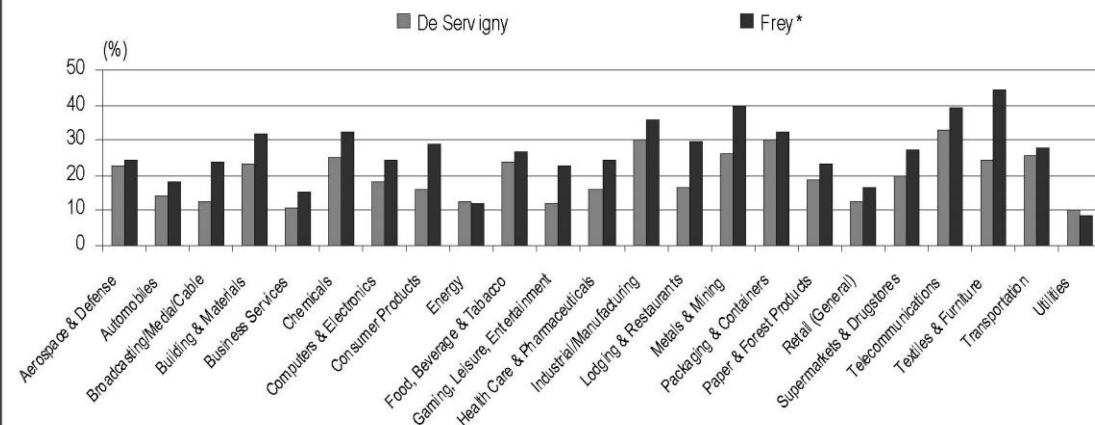
- Impact of systematic vs. idiosyncratic risk driven by parameter  $R$
- Systematic factors ( $F$ ): e.g. global economy, regional or industry effects
- Correlations of default and migration events are driven by correlation matrix of systematic factors and the factor loadings of the issuers
- Higher correlation → more simultaneous defaults  
     → more extreme losses in the portfolio → HIGHER RISK

# Estimating Asset Correlations

- From historical **default** information
  - Long history
  - Defaults are rare events
- From historical **rating migration** information
  - more information
  - Ratings are more subjective, not always consistent, lagging
- From **equity** information
  - Wide coverage of industry and geography
  - Liquid markets, high frequency data
  - Equity prices are contaminated by information that is unrelated to credit risk
- From **credit spread** information
  - Readily available
  - Universe of liquid credit is smaller than equity universe
  - Shorter history for CDS

# Estimating Asset Correlations – EQ data

**Intra-Industry Asset Correlation Comparison**

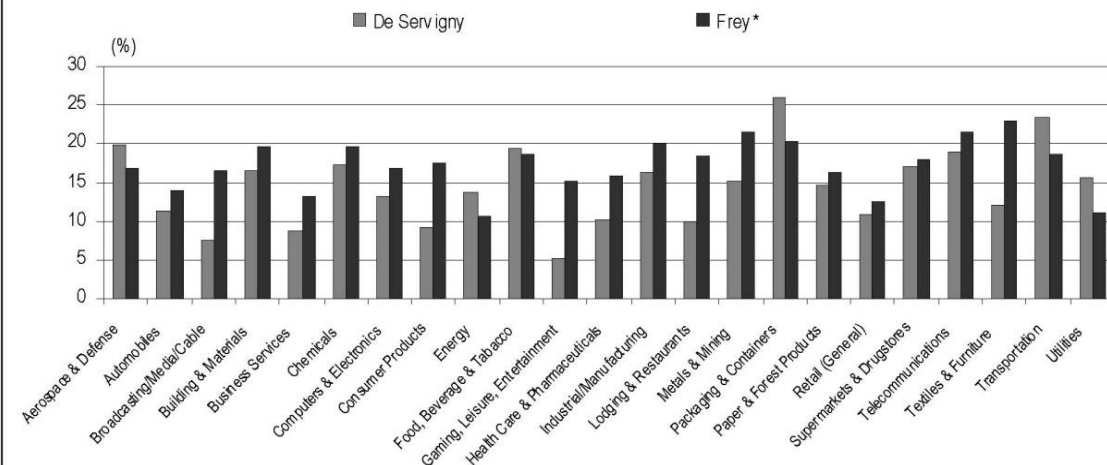


\*Adjusted.

For companies in the same industry: correlations range between 10% - 45%

For companies in different industries: correlations range between 5% - 25%

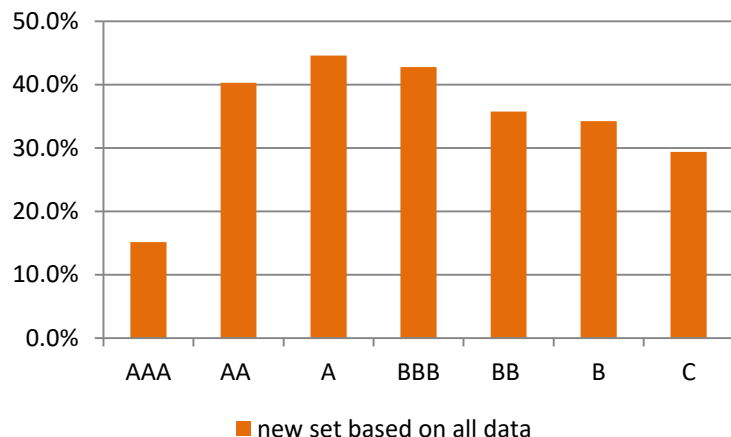
**Inter-Industry Asset Correlation Comparison**



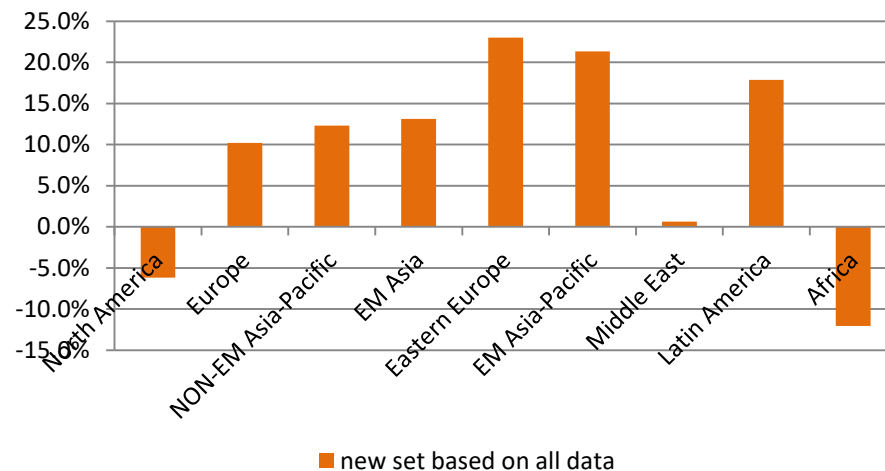
\*Adjusted.

# Estimating Asset Correlations – CDS data

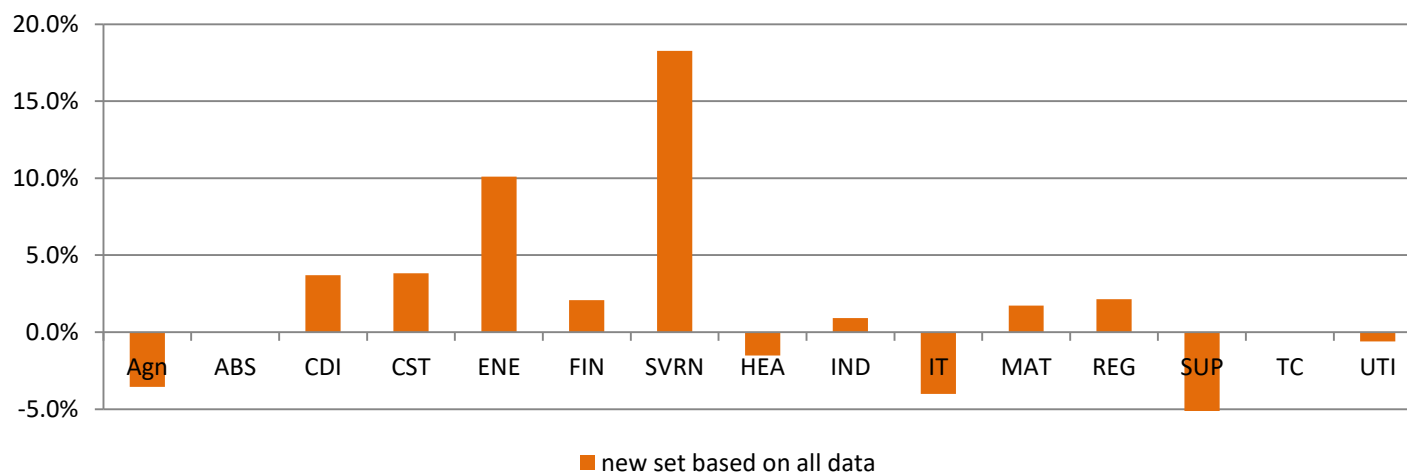
## Global correlations



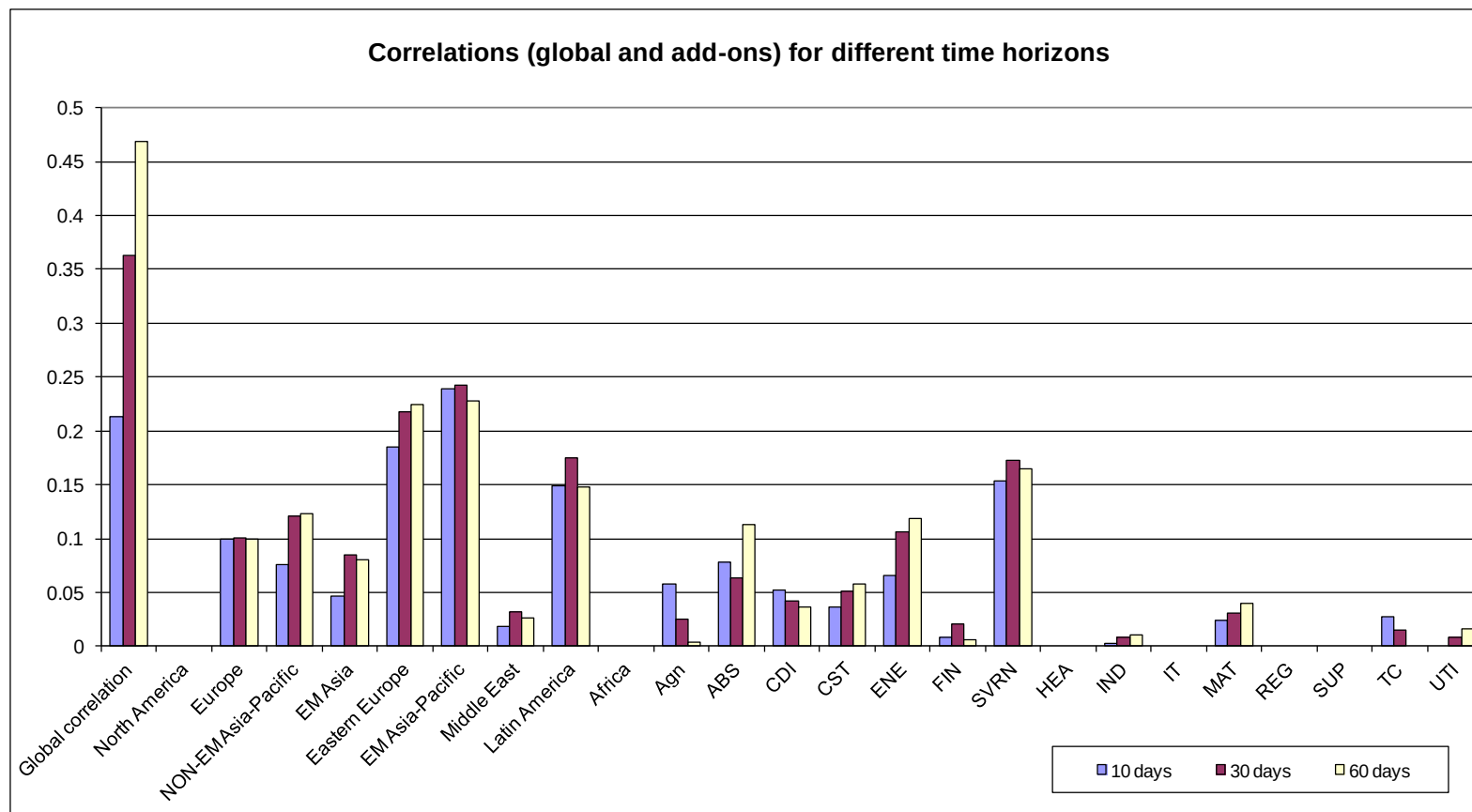
## Regional extra correlations



## Industry extra correlations



# Estimating Asset Correlations – CDS data



## References

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