Generating Mid Prices with the Kalman Filter

Mathematical Modeling Seminar

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Agenda

Introduction

Markets Quantitative Analysis

- Markets Quantitative Analysis have responsibility for developing and supporting the financial models used for the pricing of securities and for the risk management of the Firm's positions globally
- MQA have teams of quantitative analysts vertically aligned to the Trading desks according to asset class, e.g. Equities, G10 Rates, FX, Credit, Commodities
- MQA also have teams which are horizontally aligned and whose work span multiple asset classes. In areas such as Algorithmic trading, Risk Management, Investment Strategy Modelling, Trade analysis, CVA and regulatory requirements (Dodd-Frank, Basel II / III)
- MQA's key business partners are Trading, Structuring and Sales Desks and the various Risk functions, including Model Validation

We do a lot of stuff globally, with not a lot of people

The Linear Kalman Filter

The Linear Kalman Filter

- Predict an unknown state of a dynamic system
- Optimally (BLUE) combine noisy measurements and a priori estimates
- State accumulates all previous information – light on memory
- In-built modeling of external control

Basic assumptions

- Linear model of state
  - Easier to interpret and control
  - Linear system theory is more complete and practical
  - Nonlinearities can usually be ‘linearized’ away
- System and measurement noise are both white (no autocorrelation, equal power in all frequencies)
  - Wideband noise is white within system bandpass
  - Autocorrelation can be introduced if needed
- System and measurement noise are both Gaussian
  - Sum of IIDs
  - Usually only Mean and Variance are the available statistics
  - Even without Gaussianness, the Kalman Filter is BLU filter (minimum error variance)

The Kalman Filter Equations

Prediction Step

$$\hat{\mathbf{x}}_k = \mathbf{F}_k\hat{\mathbf{x}}_{k-1} + \mathbf{B}_k\overrightarrow{\mathbf{u}_k}$$
$$\mathbf{P}_k = \mathbf{F_k}\mathbf{P}_{k-1}\mathbf{F}_k^T + \mathbf{Q}_k$$

Update Step

$$\hat{\mathbf{x}}'_k = \hat{\mathbf{x}}_k + \mathbf{K}'(\overrightarrow{\mathbf{z}_k} - \mathbf{H}_k\hat{\mathbf{x}}_k)$$
$$\mathbf{P}'_k = \mathbf{P}_k - \mathbf{K}'\mathbf{H}_k\mathbf{P}_k$$

$$\mathbf{K}' = \mathbf{P}_k\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_k\mathbf{H}_k^T + \mathbf{R}_k)^{-1}$$

Pros and Cons
Extensions and Complications

Using the Kalman Filter for Price Prediction?

Pros

- Easy to interpret
- Optimal estimator of hidden states in a noisy environment
- Low memory consumption – tick output possible
- Easy to expand with additional signals of any type
- Does not require frequent calibration
- Has manual override built in

Cons

- Multiple Pros rely on good implementation
- Data heavy
- Correlation matrix can get unwieldy
- Hard to backtest
- Model assumptions are usually not satisfied
- Inherent lag
- Observation noise is hard to estimate

Extensions and Complications

- Large correlation matrix
  – combine multiple smaller hierarchical filters
- Data heavy
  – use fast but expensive database technologies
- Hard to backtest
  – build circuit breakers around it, add human oversight via control variables
- Model assumptions are usually not satisfied
  – classify regimes in state and observation domain based on performance
  – switch between models if regime changes
- Inherent lag

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