DESIGNING OPTIMAL BENEFIT RULES FOR FLEXIBLE RETIREMENT

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1. Introduction

Macroformula for pensions:

\[
\text{benefit} = \frac{\text{lifetime contribution}}{\text{remaining lifespan}}
\]

i.e.

\[
b = \frac{\tau R w}{t - R}
\]

where
\[
w = \text{total wage}
\]
\[
\tau = \text{contribution rate}
\]
\[
b = \text{benefit}
\]
\[
R = \text{adult retirement age}
\]
\[
t = \text{adult life span}
\]
Problem of 21st century

because life span increases,
- benefit/net wage decreases
- contribution rate increases
- retirement age increases (LECTURE)

How to deal with heterogeneity in LEXP
Flexible retirement

benefit depends on age at retirement
Naive proposal (NDC): 

\[ b = \frac{\tau Rw}{m - R} \]

where \( m = Et = \) average adult life span

but asymmetric information undermines the optimality

longer employment \( \rightarrow \) longer lifespan

Memo: miner vs. professor

Table 0. Comparison of solutions

Benefits in terms of total wage
(1.25 net wage)
Retirement ages – 20 years
Yield–variance trade-off
Table 0. Summary results

<table>
<thead>
<tr>
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<th>NEUT</th>
<th>EST</th>
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<tbody>
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<td>0.64</td>
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<td>0.50</td>
<td>0.72</td>
</tr>
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<td>1.19</td>
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</tr>
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<td>42.3</td>
<td>34.4</td>
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NDC unfair
NEUT low yield–no variance
EST optimal
Plan

1. Introduction
2. Theoretical background
3. Model
4. Redistribution vs. neutrality
5. First-best optimum
6. Second-best optimum
7. Numerical results
8. Conclusions
2. Theoretical Background

Optimum mechanism design

Two examples

• Optimal income tax (Mirrlees 1971)
  makes people interested in working and paying taxes
asymmetric information calls for incentive compatibility:
Nobel-prize
  ⇒ second best solution
• Insurance design (Rothschild–Stiglitz, 1976)
  low risk must prove his type by accepting partial insurance
neutrality may be Pareto-inferior to redistribution
Diamond–Mirrlees (1978): opt. disability retirement

Now: Opt. old-age retirement incentives

Table 1. Models and assumptions

<table>
<thead>
<tr>
<th>Author</th>
<th>Lifespan</th>
<th>Labor disutility</th>
<th>Retirement age</th>
<th>Benefit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( t )</td>
<td>( \varepsilon )</td>
<td>( R )</td>
<td>( b )</td>
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<tr>
<td>Fabel</td>
<td>2</td>
<td>fixed</td>
<td>continu.</td>
<td>arbitrary</td>
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<tr>
<td>Diamond</td>
<td>contin.</td>
<td>( \varepsilon(t) )</td>
<td>continu. ( R_m, R_M )</td>
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<tr>
<td>Simon.</td>
<td>contin.</td>
<td>arbitrary</td>
<td>continu.</td>
<td>linear</td>
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<tr>
<td>EST</td>
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<td>fixed</td>
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3. Model

population: lifespan \( t = S, \ldots, T \)

mandatory pension system with life annuity

Table 2. Stages

<table>
<thead>
<tr>
<th>Stage</th>
<th>worker</th>
<th>pensioner</th>
</tr>
</thead>
<tbody>
<tr>
<td>consumption span</td>
<td>1 (-\tau)</td>
<td>(b_t)</td>
</tr>
<tr>
<td>span</td>
<td>(R_t)</td>
<td>(t - R_t)</td>
</tr>
<tr>
<td>inst. utility</td>
<td>(u)</td>
<td>(w)</td>
</tr>
</tbody>
</table>
\[ u(x) = w(x) - \varepsilon, \ \text{fixed, } u = u(1 - \tau) \]

individual lifetime utility

\[ v_t = R_t u + (t - R_t) w(b_t). \]

Lifetime balance: \( z_t = \tau R_t - (t - R_t)b_t \)

Budget constraint:

\[ Z = \sum_{t=S}^{T} z_t f_t = 0 \]

where \( f_t = \) frequency of type \( t \).
4. Redistribution vs. neutrality

Neutral: $z_t \equiv 0$:

$$R^N_t = \frac{b_t}{\tau + b_t}t, \quad t = S, \ldots, T.$$  

Inevitability of redistribution

Innocent assumption: increasing benefits: $b_{t+1} \geq b_t$

Additional plausible assumption

Moderate sensitivity to life expectancy:

$$R_{t+1} - R_t < \frac{b_t}{\tau + b_t} < 1$$
Strong form of redistribution

Decreasing balances:

\[ z_S > \cdots > z_t > z_{t+1} > \cdots > z_T. \]

Theorem. Increasing benefits and moderate sensitivity \(\rightarrow\) decreasing balances

Corollary: \( z_S > 0 > z_T \)
5. First-best optimum

- no asymmetric information

Two objective functions:
- maximizing the average utility
- and minimizing the variance of balances

unification with penalty coeff. \( \delta \geq 0 \)

- benevolent dictator

\[
\max_{(b_t,R_t)} \sum_{t=S}^{T} [v_t - \delta z_t^2] f_t
\]

subject to

\[
\begin{align*}
v_t &= [u - w(b_t)] R_t + w(b_t) t, \\
z_t &= (\tau + b_t) R_t - t b_t, \\
\sum_{t=1}^{T} z_t f_t &= 0
\end{align*}
\]
Theorem. First-best benefit: \( b_t^* \equiv b^* : \)

\[
F(b^*) = 0.
\]

First-best retirement age:

a) no penalty

\[
R_t^* = R^* = \frac{b^*}{\tau + b^*} m.
\]

b) penalty

\[
R_t^N = \frac{b^*}{\tau + b^*} t.
\]
$R_t^N$ also no-penalty first-best but is not second-best, because
the govt. does not know $t$ →
individuals underreport $t$
6. Second-best optimum

First-best problem

with incentive compatibility (IC) constraints:

(D) type \( t + 1 \) prefers \((b_{t+1}, R_{t+1})\) to \((b_t, R_t)\),

(U) type \( t \) prefers \((b_t, R_t)\) to \((b_{t+1}, R_{t+1})\),

Corollary [Monotonicity]:

\[
\begin{align*}
    b_t &\leq b_{t+1}, \\
    R_t &\leq R_{t+1}.
\end{align*}
\]

(U) dropped and (D) reduced to (D=)
Simple optimal isoperimetric control problem:

\[ v_{t+1} = v_t + w(b_t), \quad t = S, \ldots, T - 1 \]

\[ v = \text{state}, \ b = \text{control} \]

Two cases are distinguished:

(i) No penalty for redistribution (Utilitarian)
(ii) Penalty for redistribution
(i) No penalty

Theorem. Inflexible plan:

\[ \hat{b}_t = b^* \quad \text{and} \quad \hat{R}_t = R^* \]

and second-best is first-best

(ii) Penalty

now first-best is not socially optimal:

too much redistribution from the short-lived to the long-lived

Corollary: \( b_T = b^* \) (first-best).

Typical for second-best
Lagrange multipliers: $\lambda$ to $Z$ and $\mu_t$ to $IC_t$

Theorem. Necessary First-Order Conditions
7. Numerical results

NFOC easy to solve numerically but this problem has many stationary solutions for $n > 3$

Run 1. Three types: $S = 50$ and $T = 60$, uniform.

No penalty: $R^* = 44$ years, $b^* = 0.8$

Run 2. Equivalent solutions, no penalty

### Table 3. Equivalence

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<td>50.3</td>
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<td>31.9</td>
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<td>50.2</td>
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From now on positive penalty

- **Run 3 Comparisons**

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- analysis of imperfect benefit formulas
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- more analytical results are needed