Causalitty or something else

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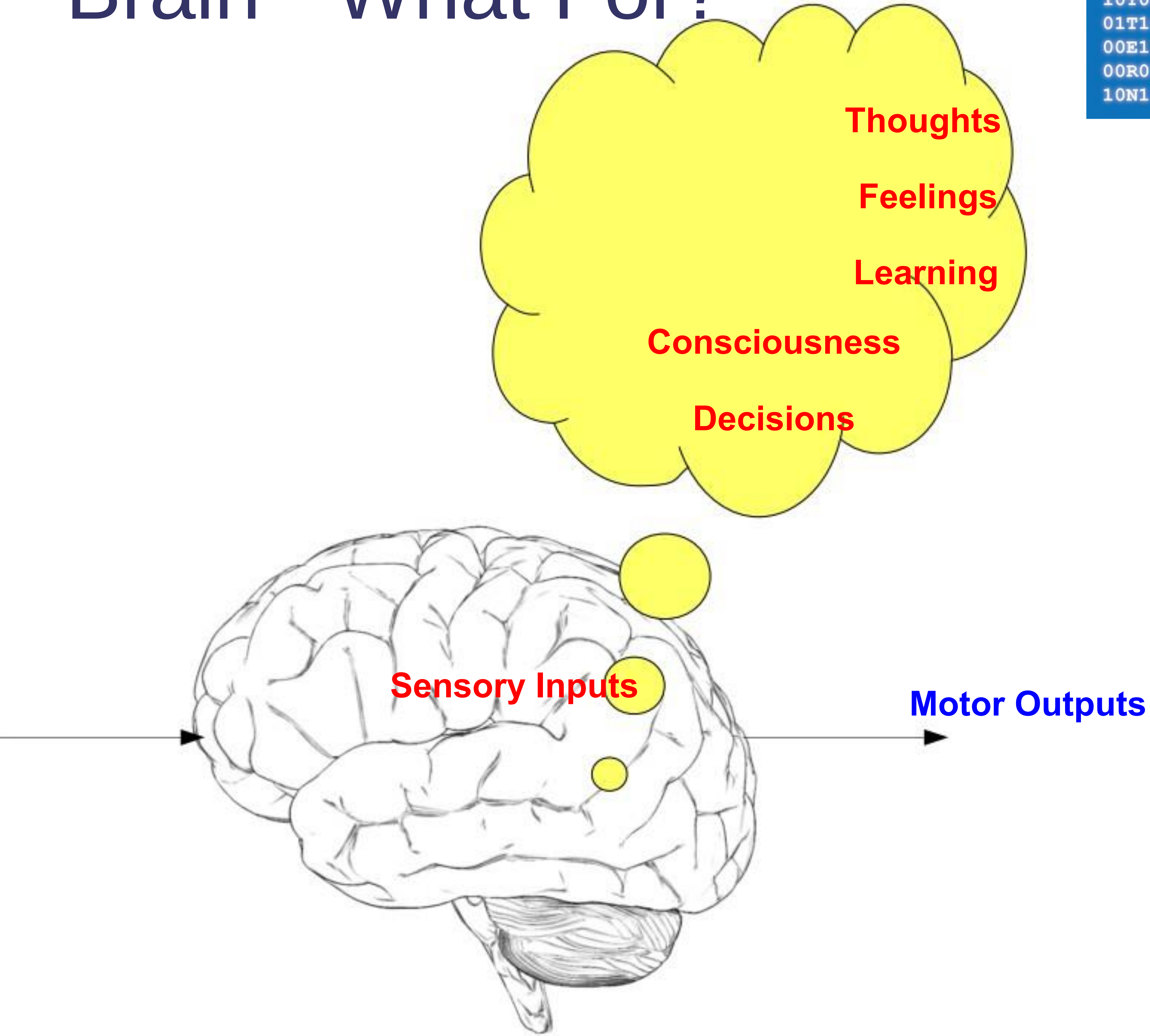
HUNGARIAN BRAIN RESEARCH PROGRAM
NEMZETI AGYKUTATÁSI PROGRAM

NAP-B PATTERN (2015-2017)
Population Activity Research Unit
MTA WIGNER RESEARCH CENTRE FOR PHYSICS

Intro

- Stat Phys
- Random walks

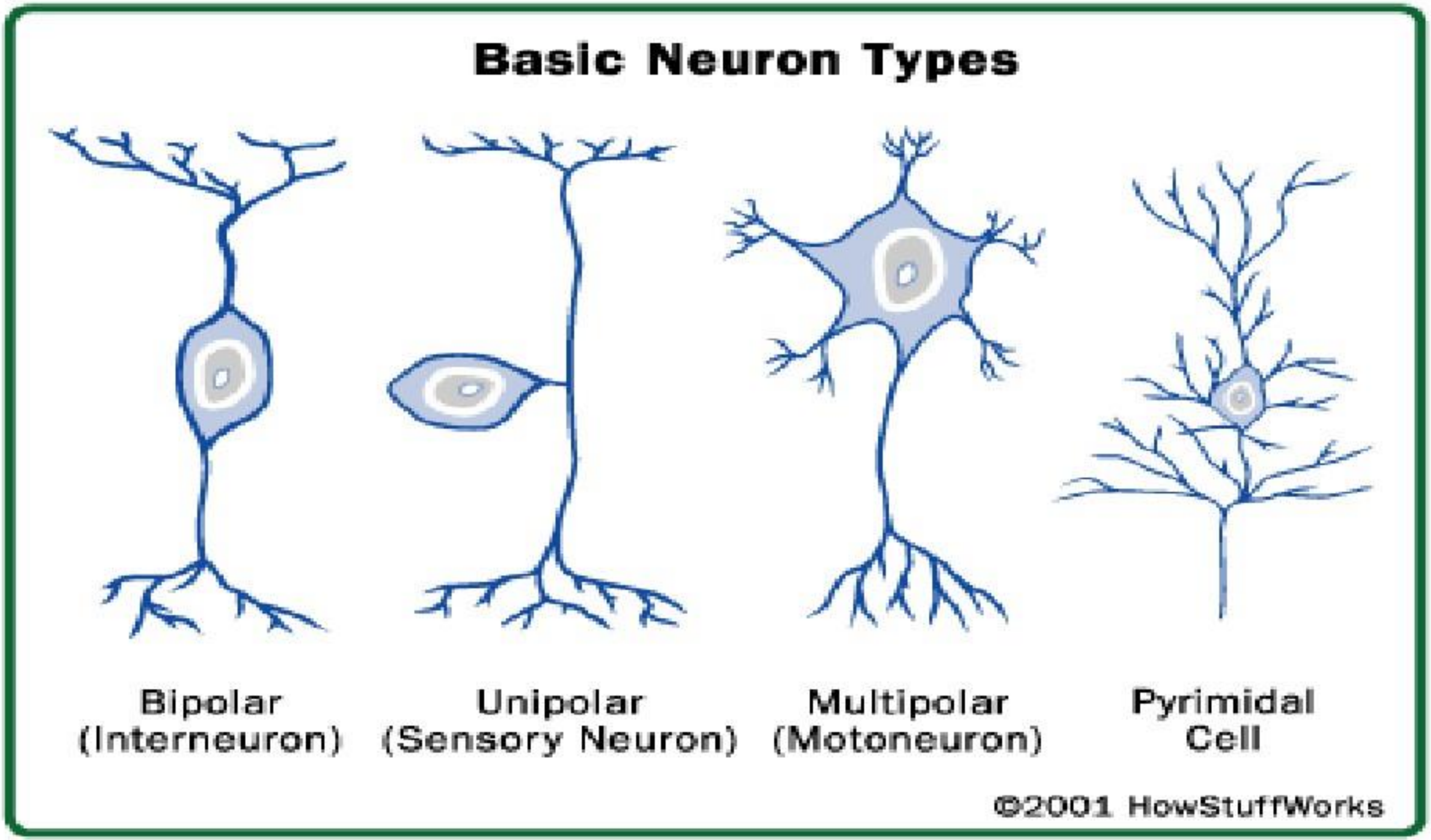
Brain - What For?



The brain

In the human brain there are:

~ 10^{12} (trillion) Neurons
~ 10^{15} (quadrillion) Synapses
~ 10^{5} Neurons/mm^3
~ 10^{9} Synapse/mm^3
~ 4 Km Axon/mm^3
~ 500 million dendrites /mm^3
~ 10^{4} Input Synapses / neuron


Basic Neuron Types
Bipolar
(Interneuron)
Unipolar
(Sensory Neuron)
Multipolar
(Motoneuron)
Pyrimidal
Cell
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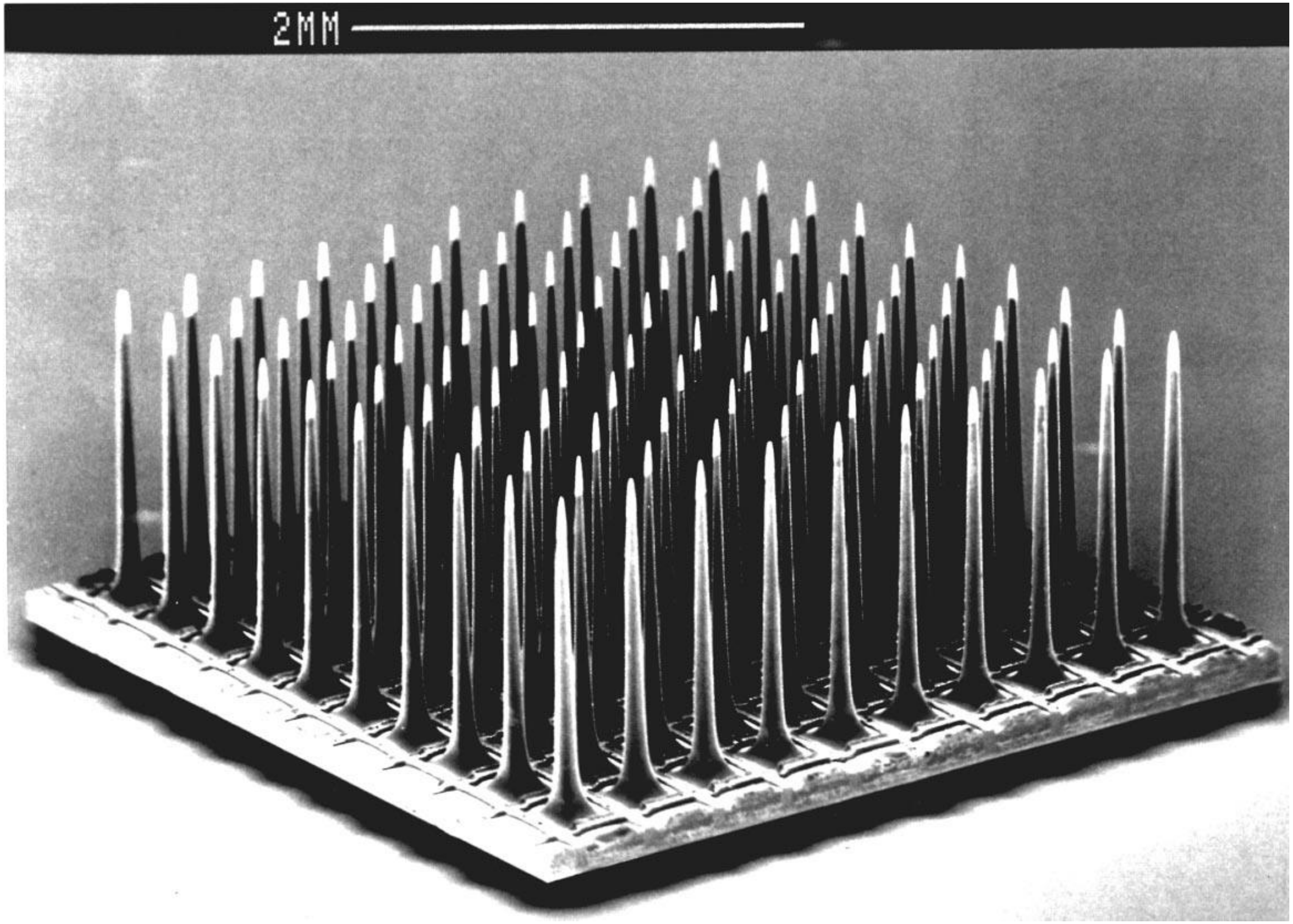
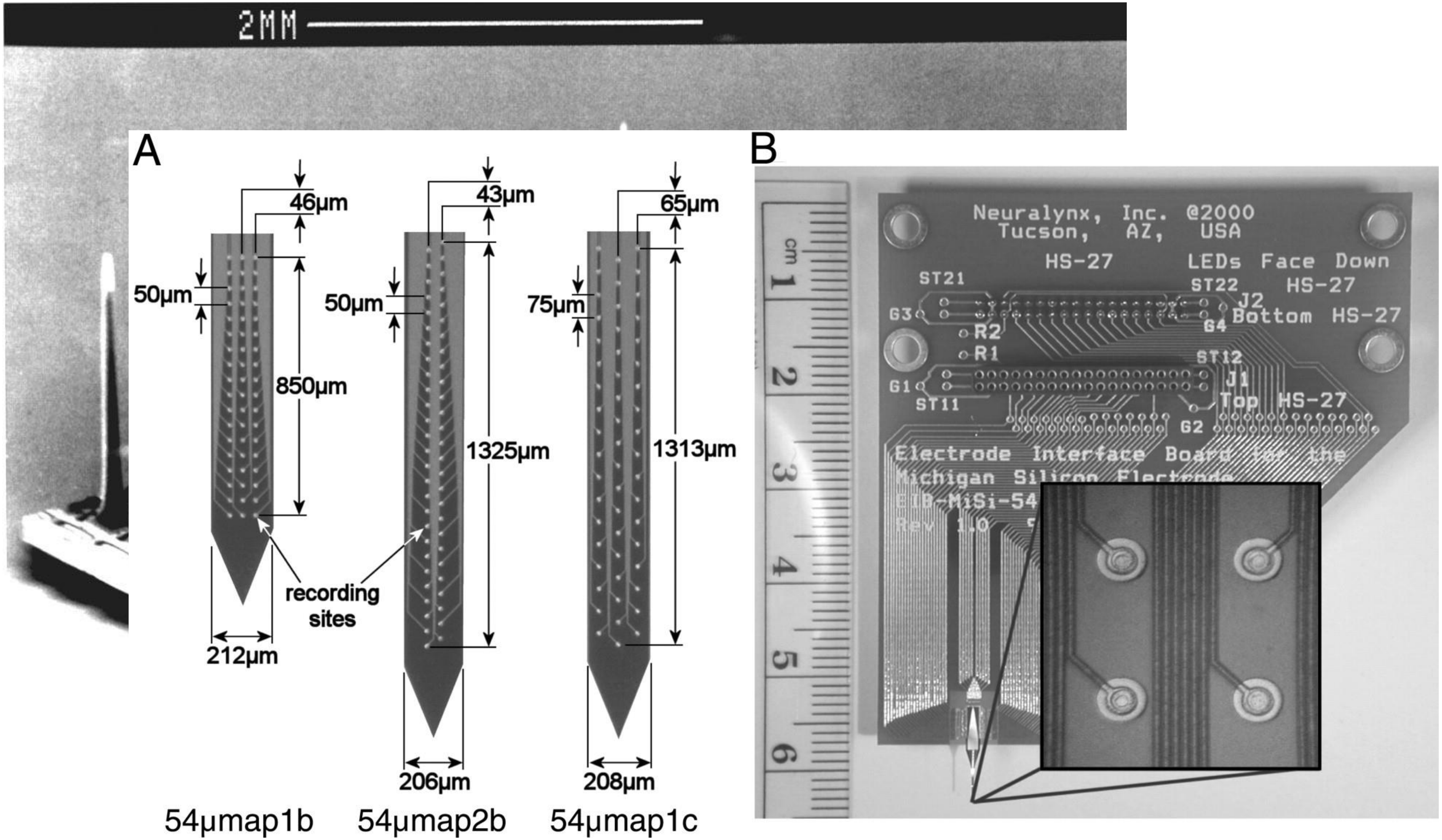

2MM


2MM
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46µm
43µm
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Neuralynx, Inc. @2000
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HS-27
LEDs Face Down
ST21
ST22
HS-27
J2
G3
Bottom HS-27
G4
R2
R1
ST12
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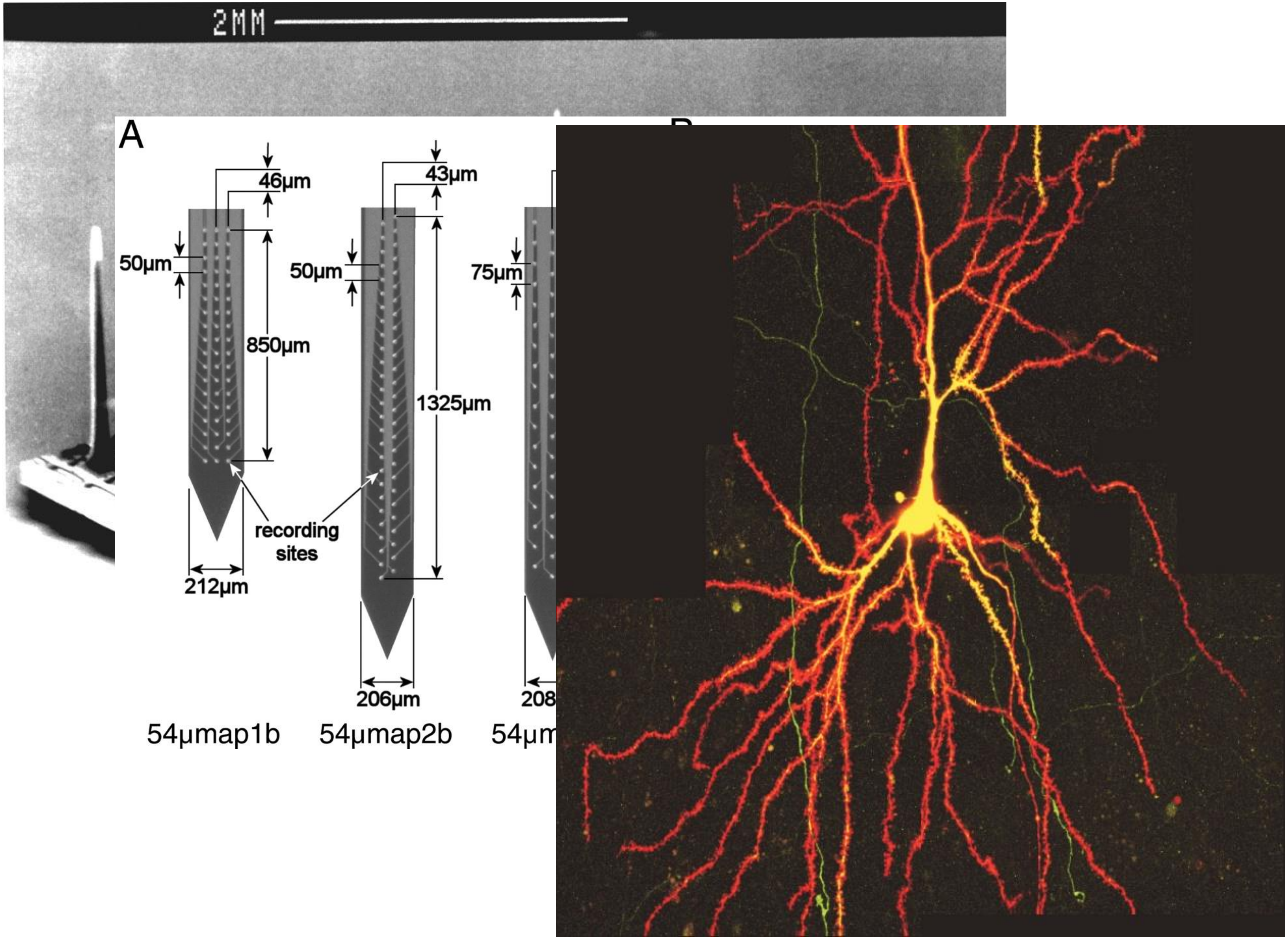
Electrode Interface Board for the
Michigan Silicon Electrode
EIB-MiSi-54
Rev 1.0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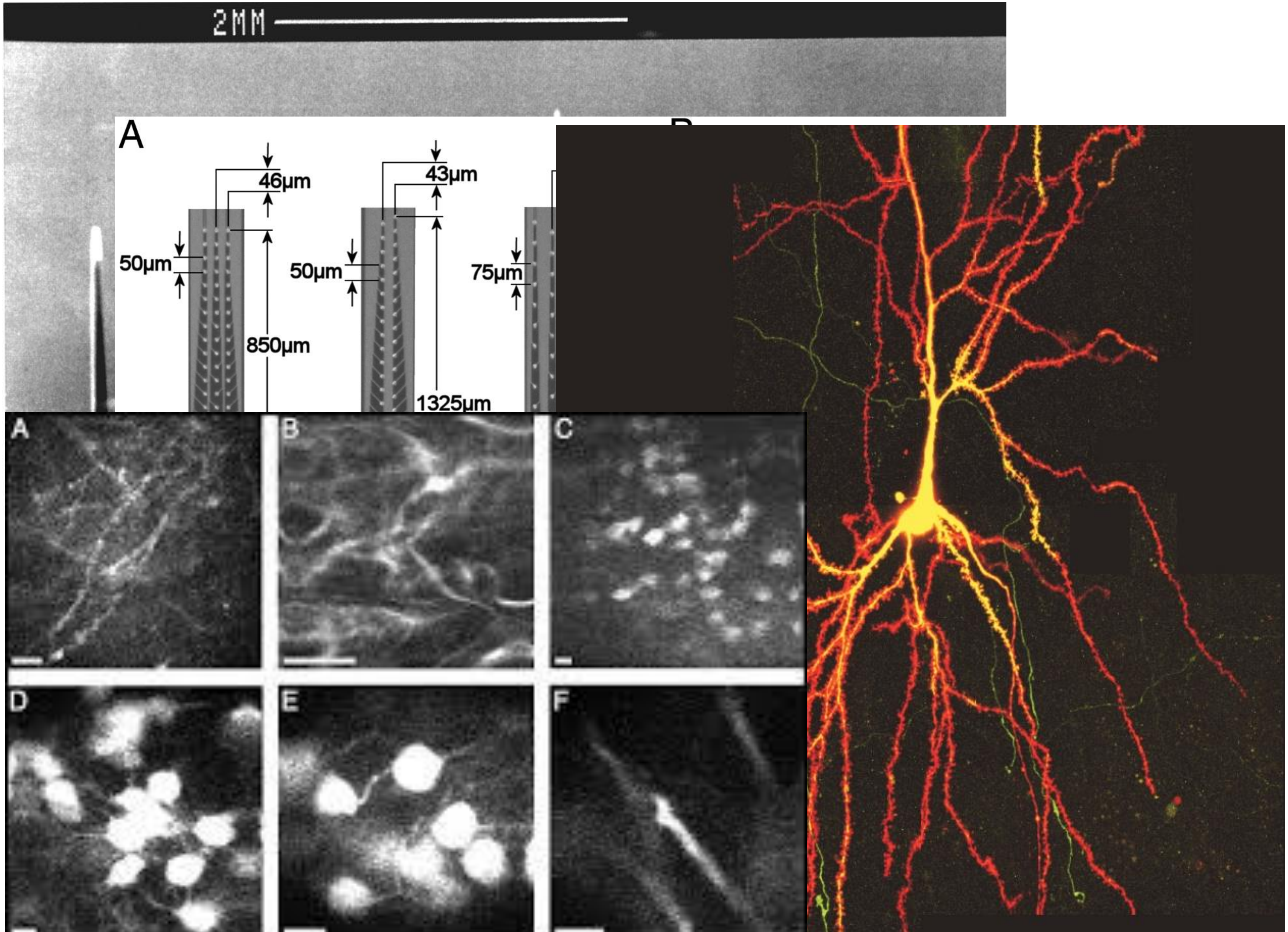
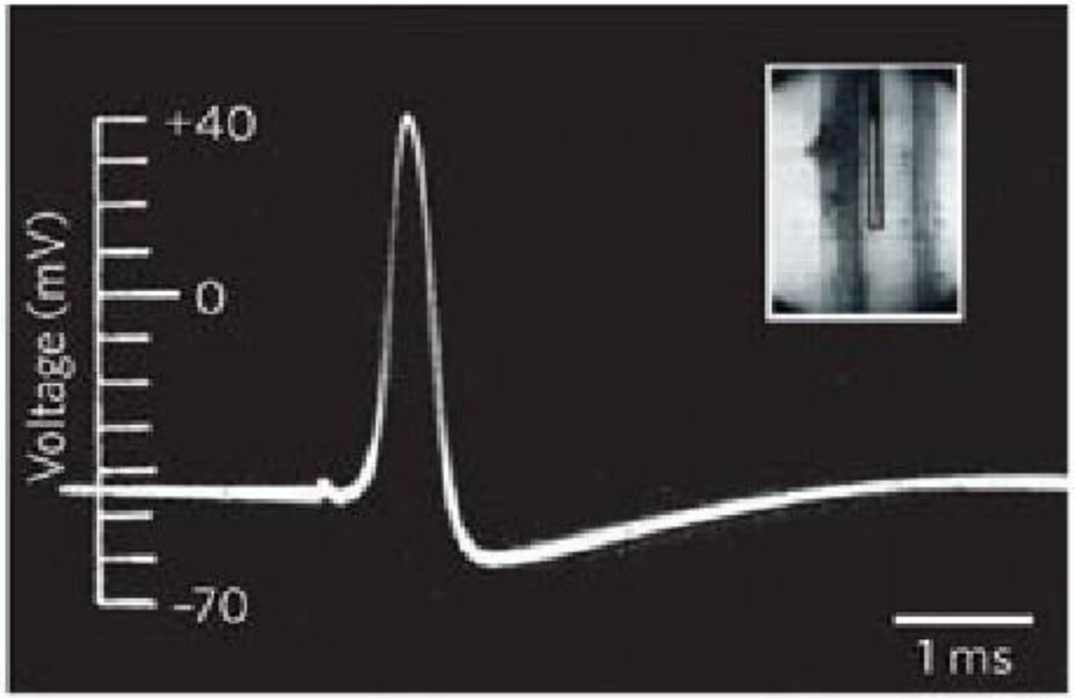
54µmap2b



2MM
A
46µm
43µm
50µm
50µm
75µm
850µm
1325µm
A
B
C
D
E
F


+40
0
-70
Voltage (mV)
1 ms

Intro

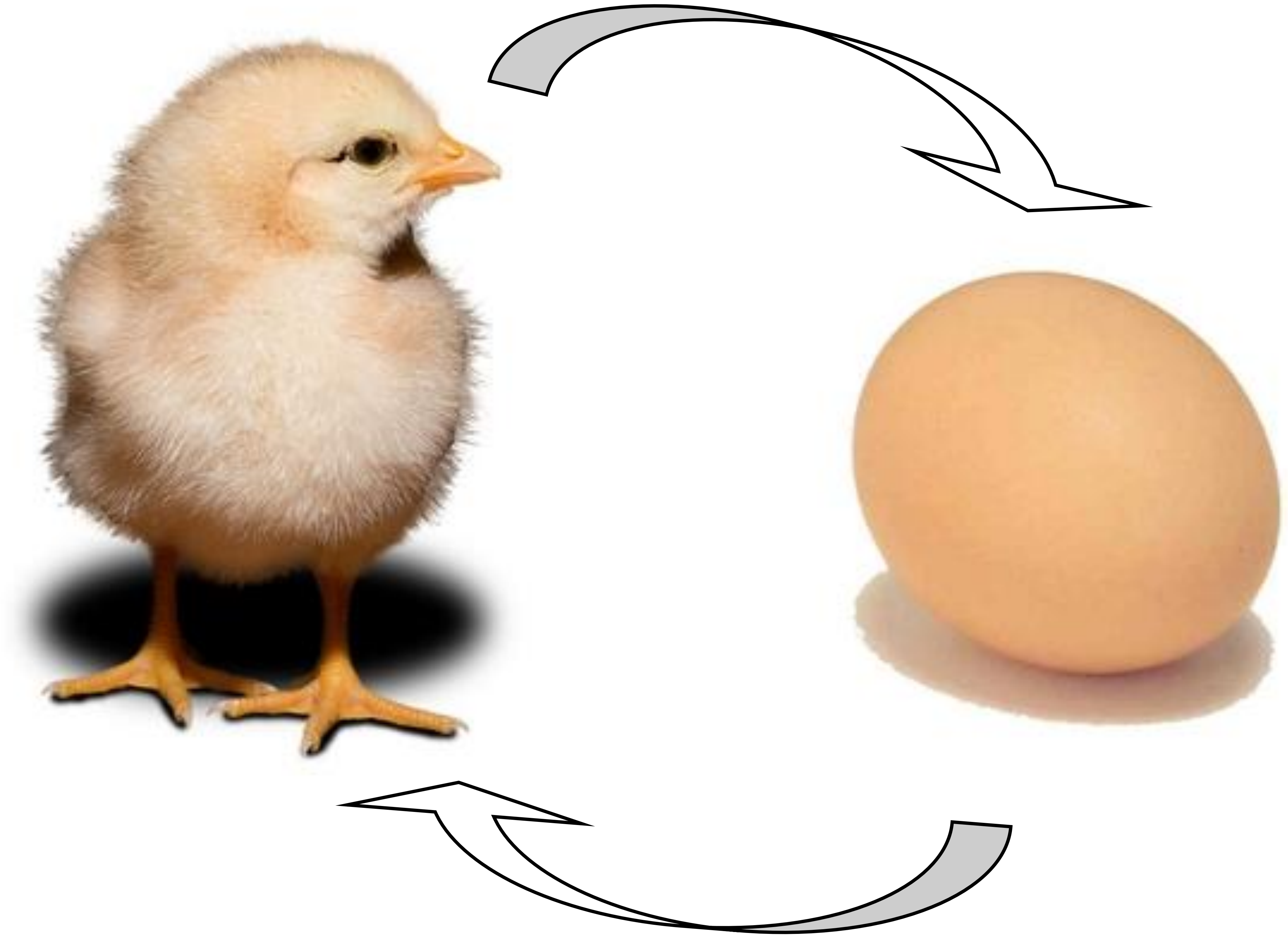
- Stat Phys
- Random walks
- Computational neuroscience
  - Pattern NAP I
  - NAP II
  - Wigner CP
- Computational neuroscience
  - DA
  - ML
  - Math models

joint work with

Zsigmond Benkő, Ádám Zlatniczki, Dániel Fabó,

Zoltán Somogyvári

Which was first?



Modern view of causality

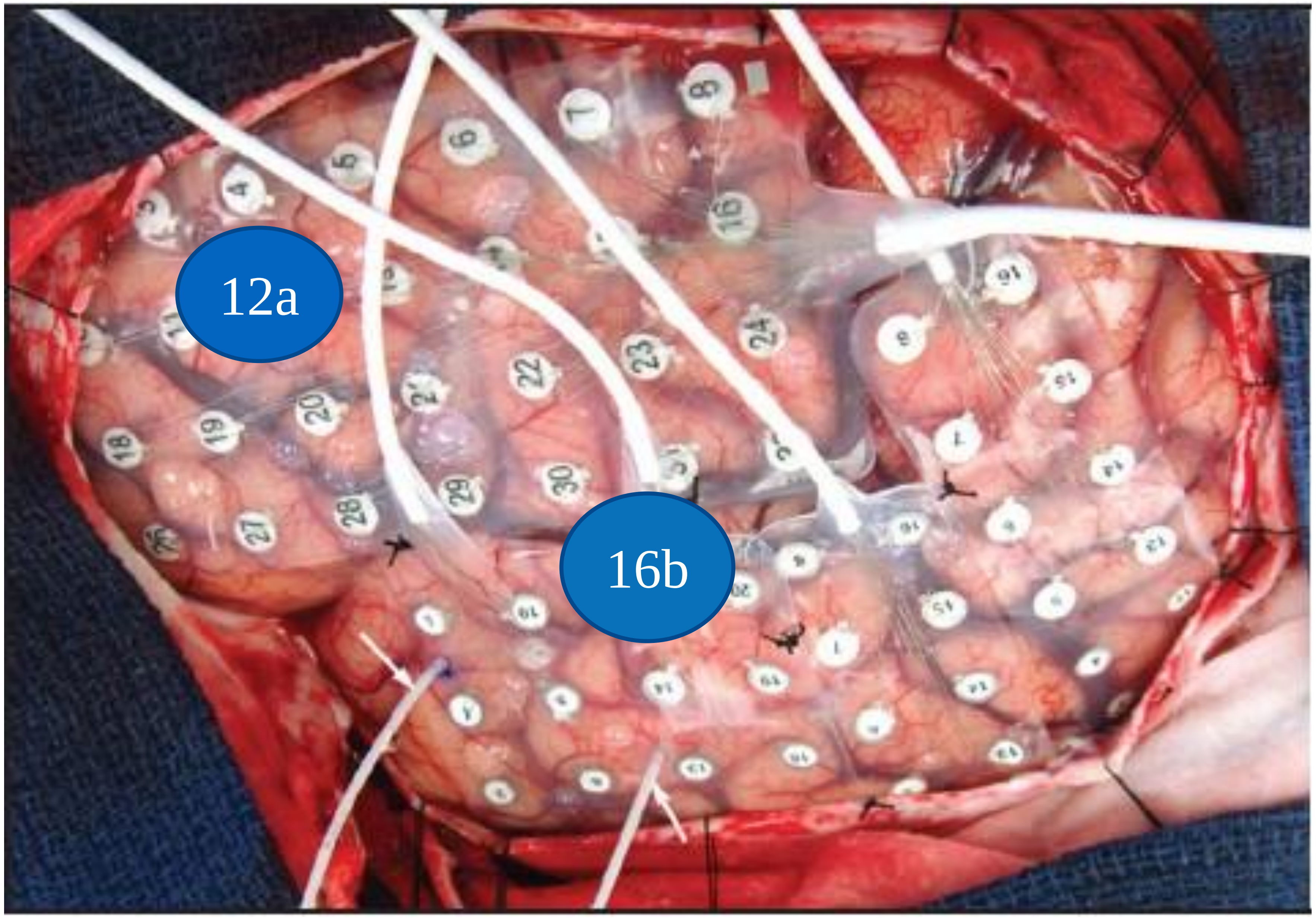
Francis Bacon physics – metaphysics,

only physical causation can be considered causality

Deterministic.. or ?

Application

Which region is the source of the epileptic seizure?

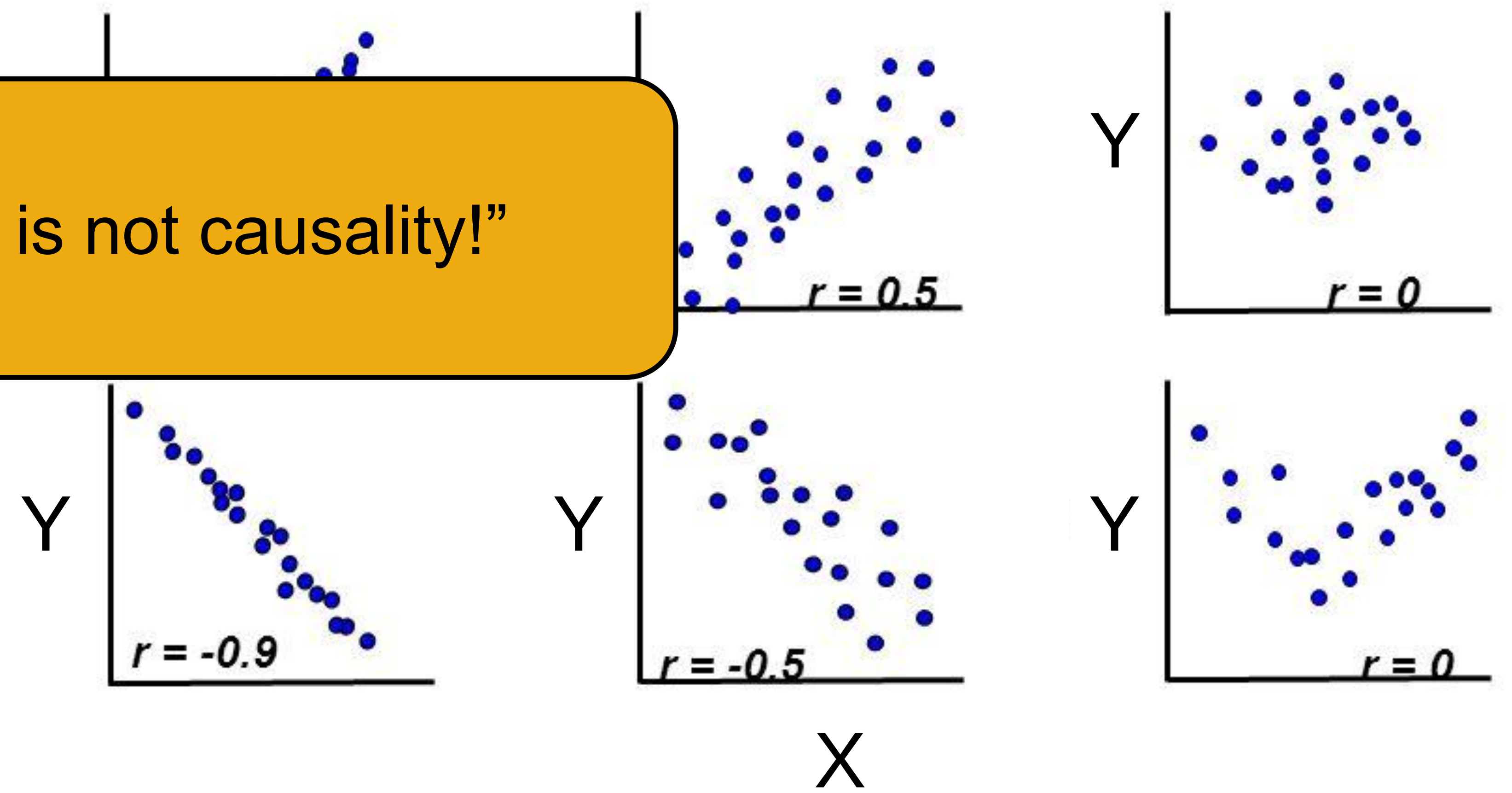


Shah AK, Mittal S. Invasive electroencephalography monitoring: Indications and presurgical planning. Ann Indian Acad Neurol 2014;17, Suppl S1:89-94

Sample: X_i , Y_i i=1,….,n

correlation?

“Correlation is not causality!”



X causes Y ?

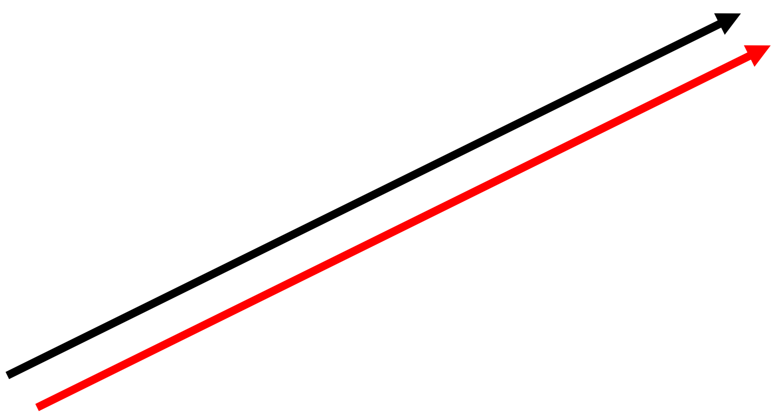
Y causes X ?

From correlation to causality

Can we learn something!

If X moves like that , Y does as well,

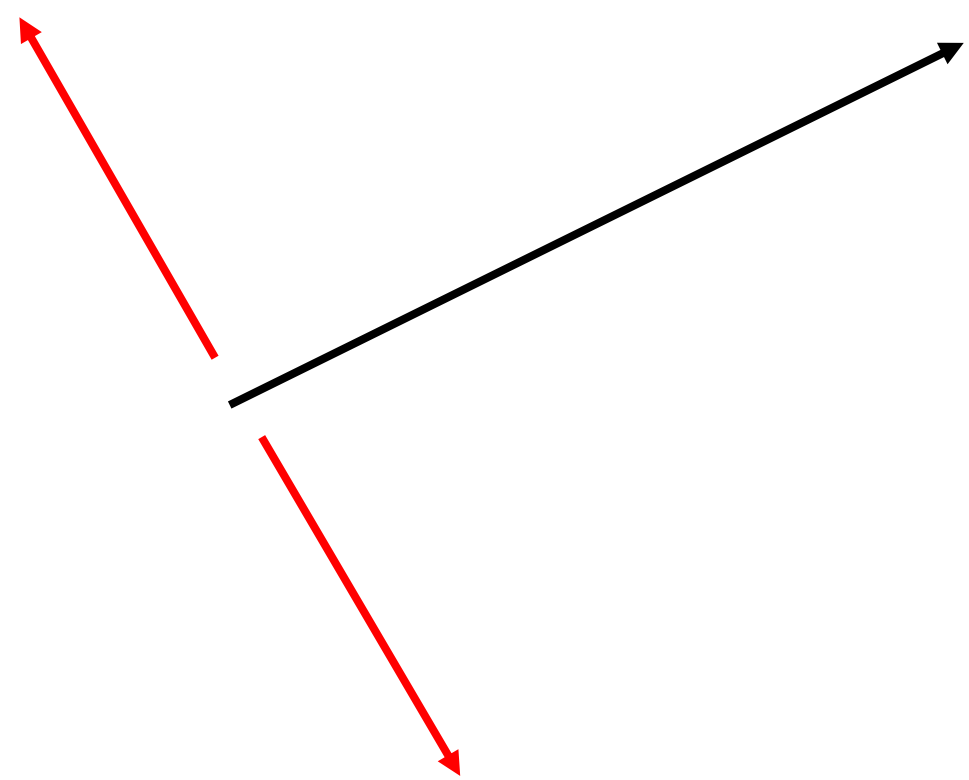
or if X moves like that , Y does not care



X degrees of freedom 1

Y degrees of freedom 1

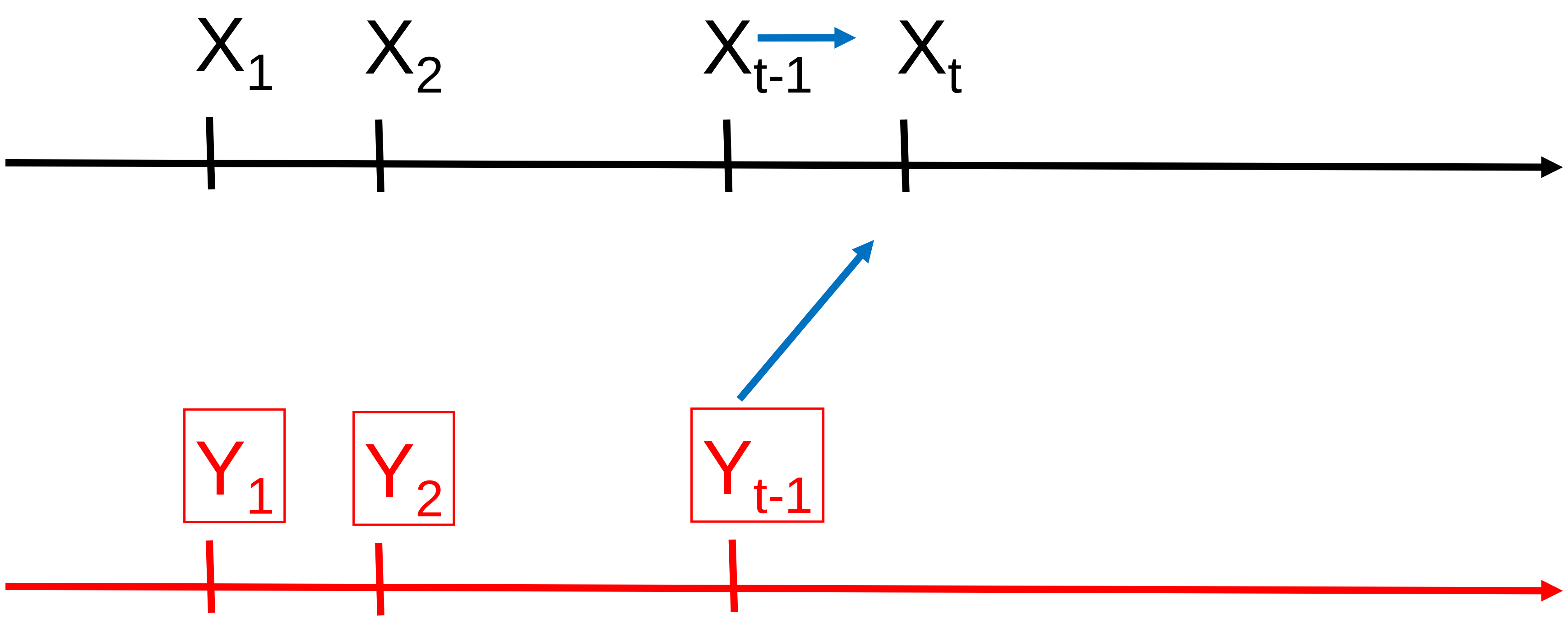
togeather?



Time series

X_t , Y_t two timeseries t=1,….,T

Which causes the other?



Time series

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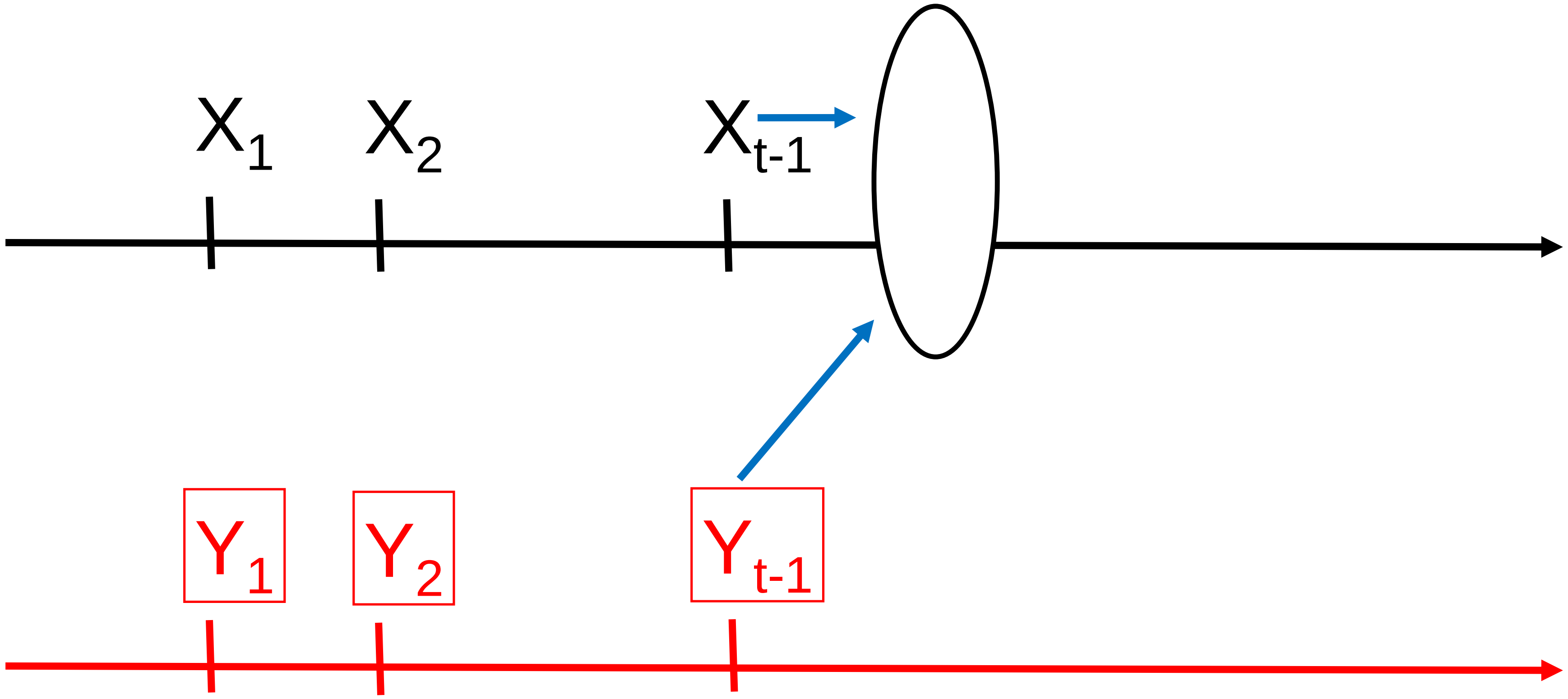


Time series

X_t , Y_t two time series =1,….,T

Let $\mathbf{X_{t+1}}$ the forecast for X_{t+1} and the error

$$e^2=E[(\mathbf{X_{t+1}} - X_{t+1})^2 | \ \dots \ ]$$

The error $e|X_{1,\dots t}$ based on the past of X

$e|X_{1,\dots t},Y_{1,\dots t}$ based on the past X and Y

If the forecast using the past of Y in addition to X decreases the error

then, Y Wiener-Granger – causes X.

Granger causality

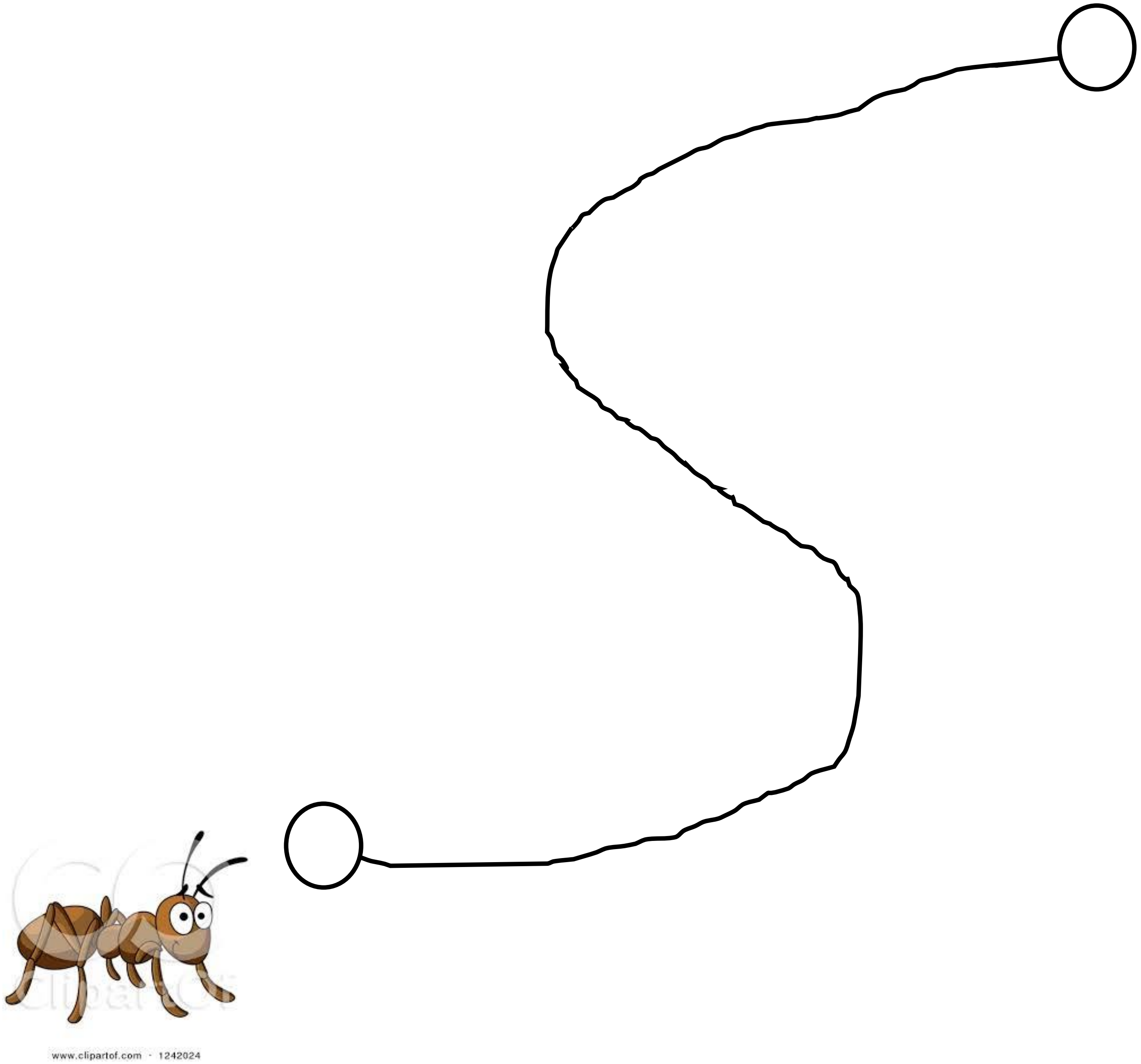
Granger causality

1. Axiom – cause precedes caused

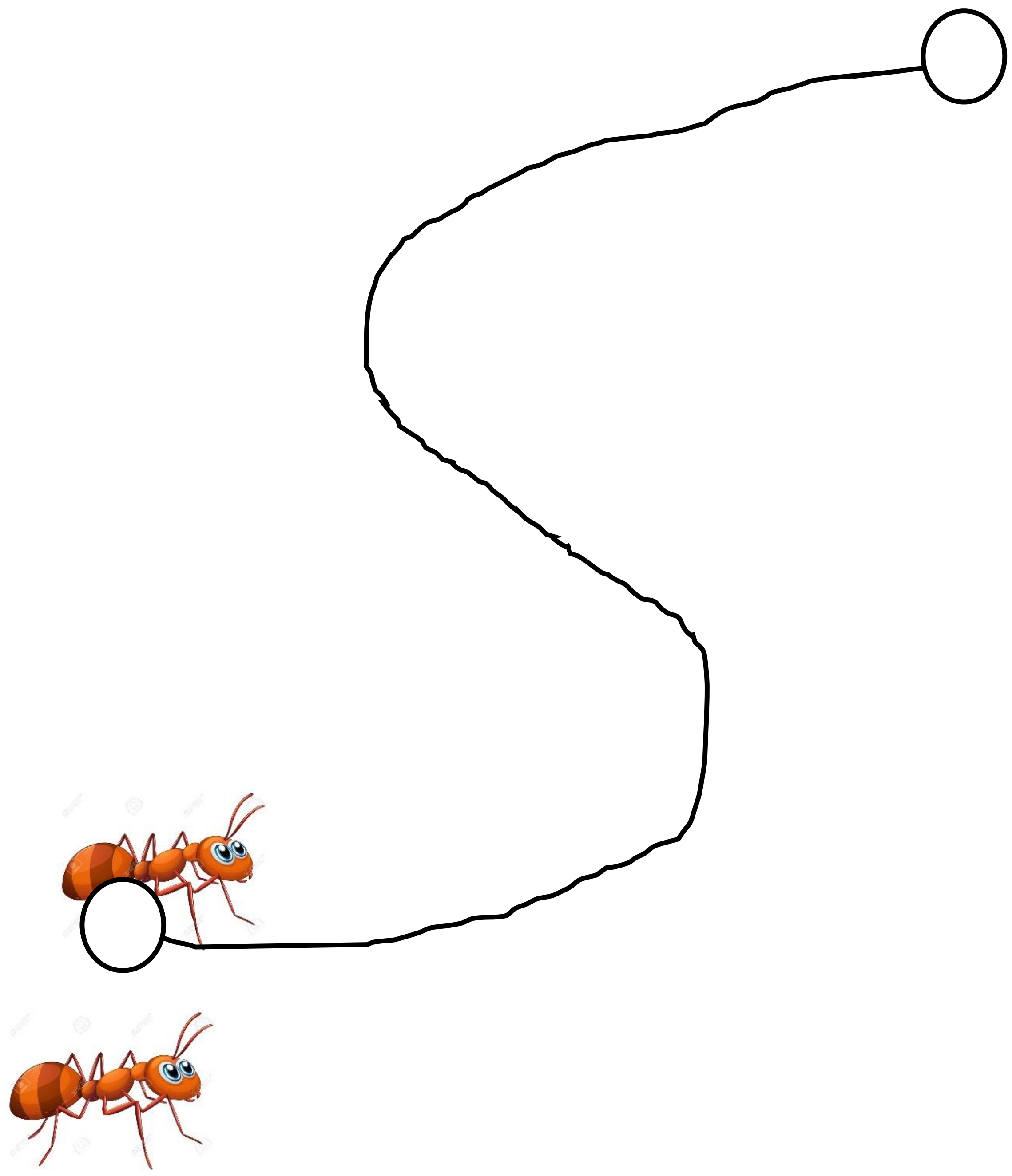
2. Axiom – Using the past of the cause improves the forecast of the caused based solely on its own past..

Granger causality

Life of the ants



Life of the ants



Granger causality

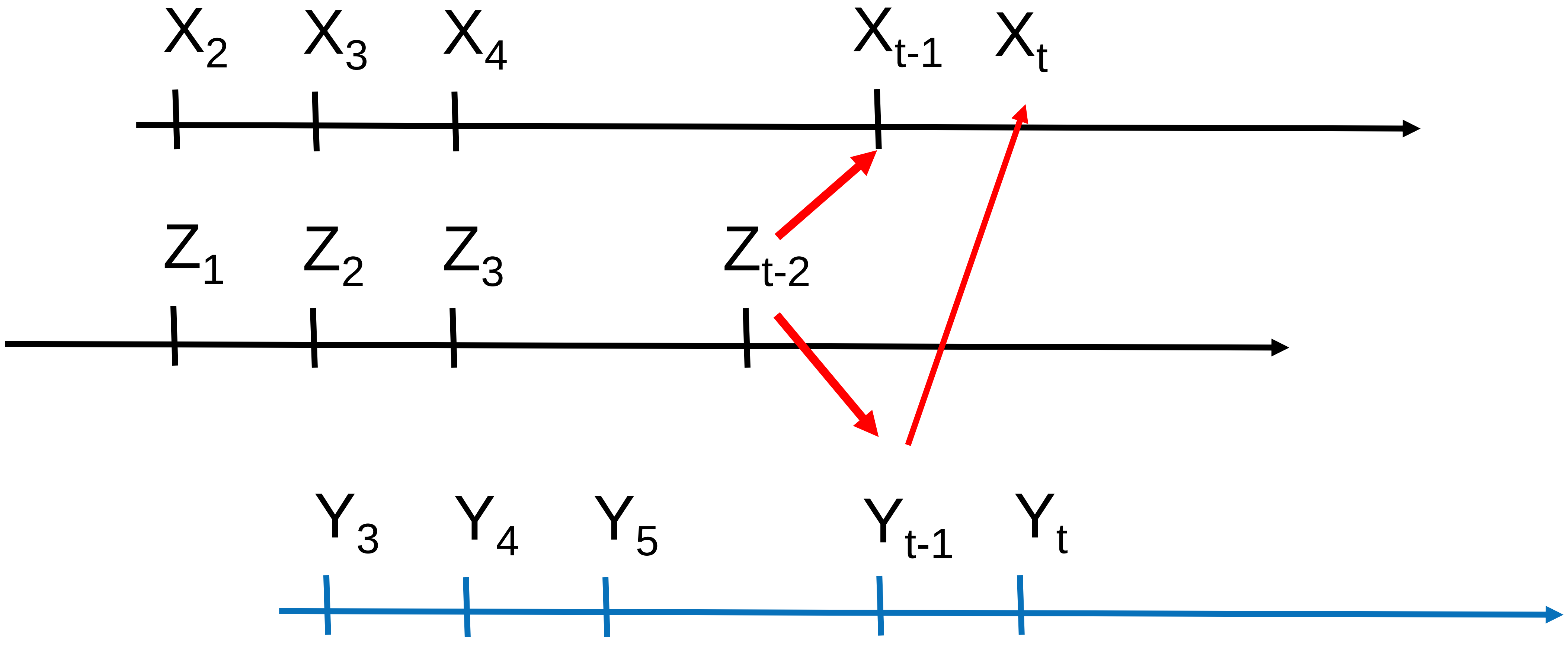
Granger causality confused by hidden common cause

Hidden common cause!

Granger causality

Granger causality confused by hidden common cause

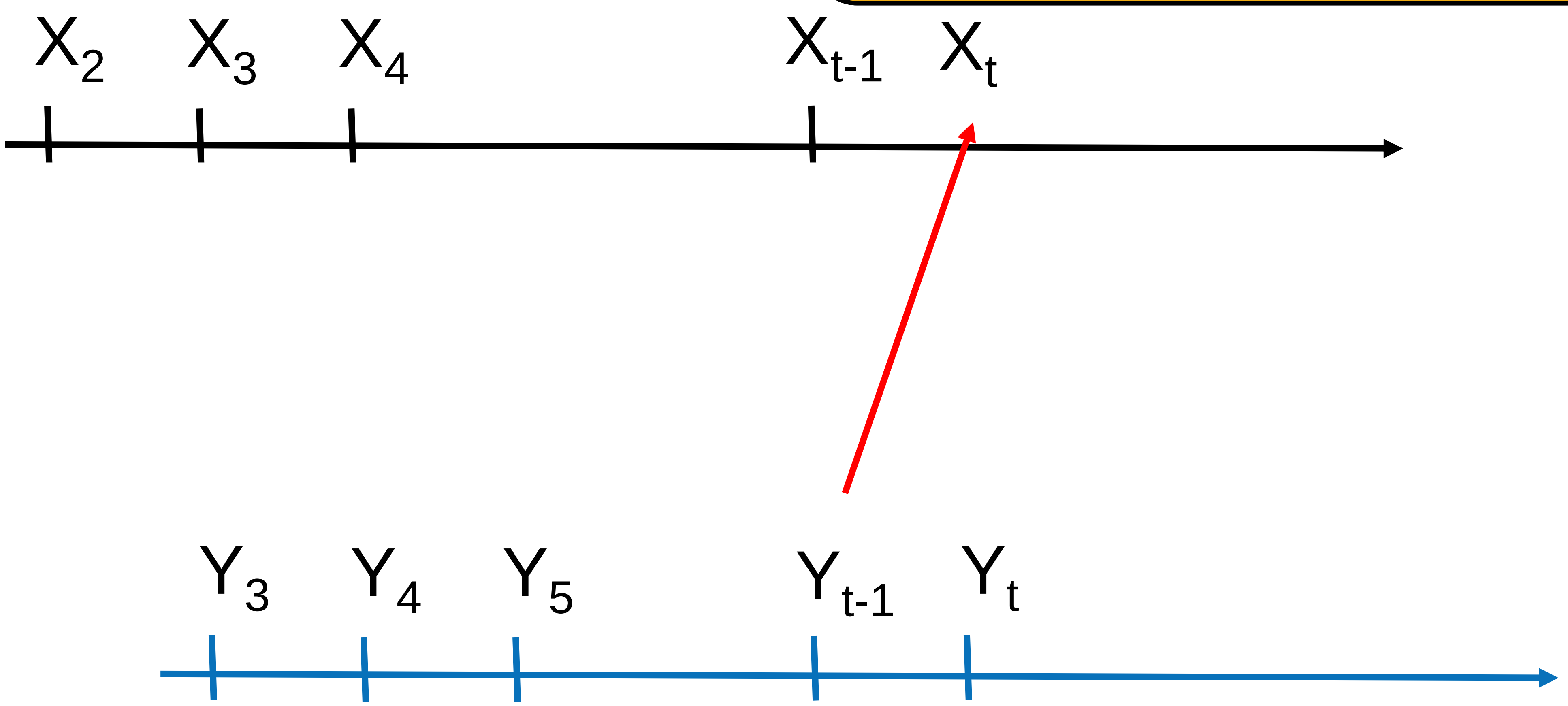
Hidden common cause!



Granger causality

Granger causality confused by hidden common cause

Hidden common cause!



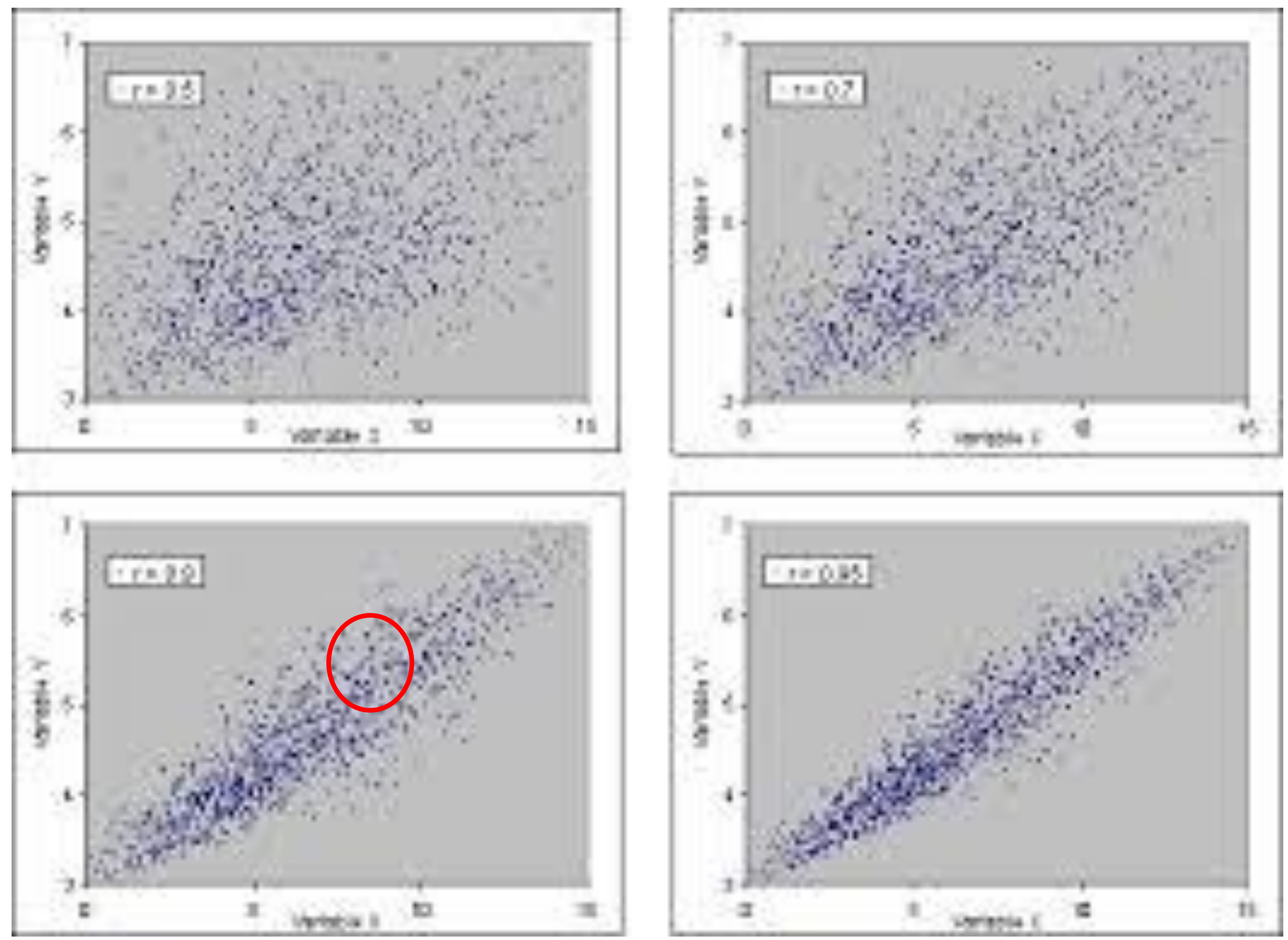
Correlation - a new look

For given i (X_i , Y_i)

How typical is that position?

$N(r) = \# \{ (i,j) \; : \; |(X_i , Y_i)-(X_j , Y_j)|<r \}$

$N(r) \approx r^d$



d the joint
"degrees of freedom"

Correlation - a new look

X_i , Y_i uptil now, IID sample $i=1,\ldots,n$!

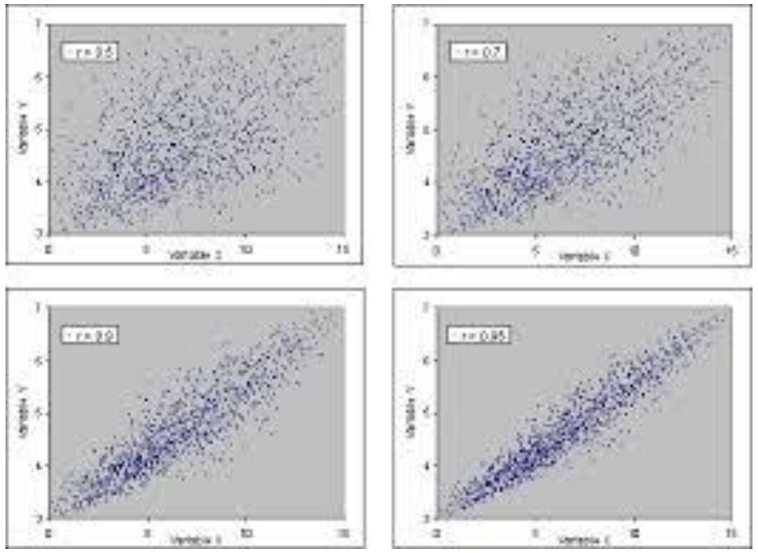
Now let it be time series!

X_t , Y_t reflects only the link between them, but in the series in time $t=1,\ldots,T$, much richer information

Connection? Spot Y if you know X?

$N(r) = \#\{ (s,t) \ : \ |(X_s , Y_s)-(X_t , Y_t)|<r \ \}$

$N(r) \approx r^d$



d is the correlation dimension

Takens' Theorem

Let $f:M \rightarrow M$ the map for a discrete time dynamical system with a strange attractor $\mathcal{A}$ with box counting dimension $d_{\mathcal{A}}$.

$a_{t+1}=f(a_t)$

$x_t=g(a_t)$

α must be twice-differentiable observation function, $m>2d_{\mathcal{A}}$ then, the delay embedding

$F_t(x)=(x_t, x_{t-1}, \ldots x_{t-m+1})$

embeds $\mathcal{A}$ into R^m and left $d_{\mathcal{A}}$ invariant.

On causality

Granger causality (1969)

- Detect uni-directional causality
- Fails to detect bi-directional causality
- *Cheeted by common cause*

Takens' (1981) time delay embedding shows the real dimension

Hirata (2010) all type of causality detected, heuristic

Sugihara - convergence cross embedding (2012)

- Detect uni-directional causality
- Detect bi-directional causality
- In some cases detects common cause (qualitative decision)

Time delay embedding

Embedding of single variables

$$(x_t, x_{t-1}, x_{t-2})$$

$$(y_t, y_{t-1}, y_{t-2})$$

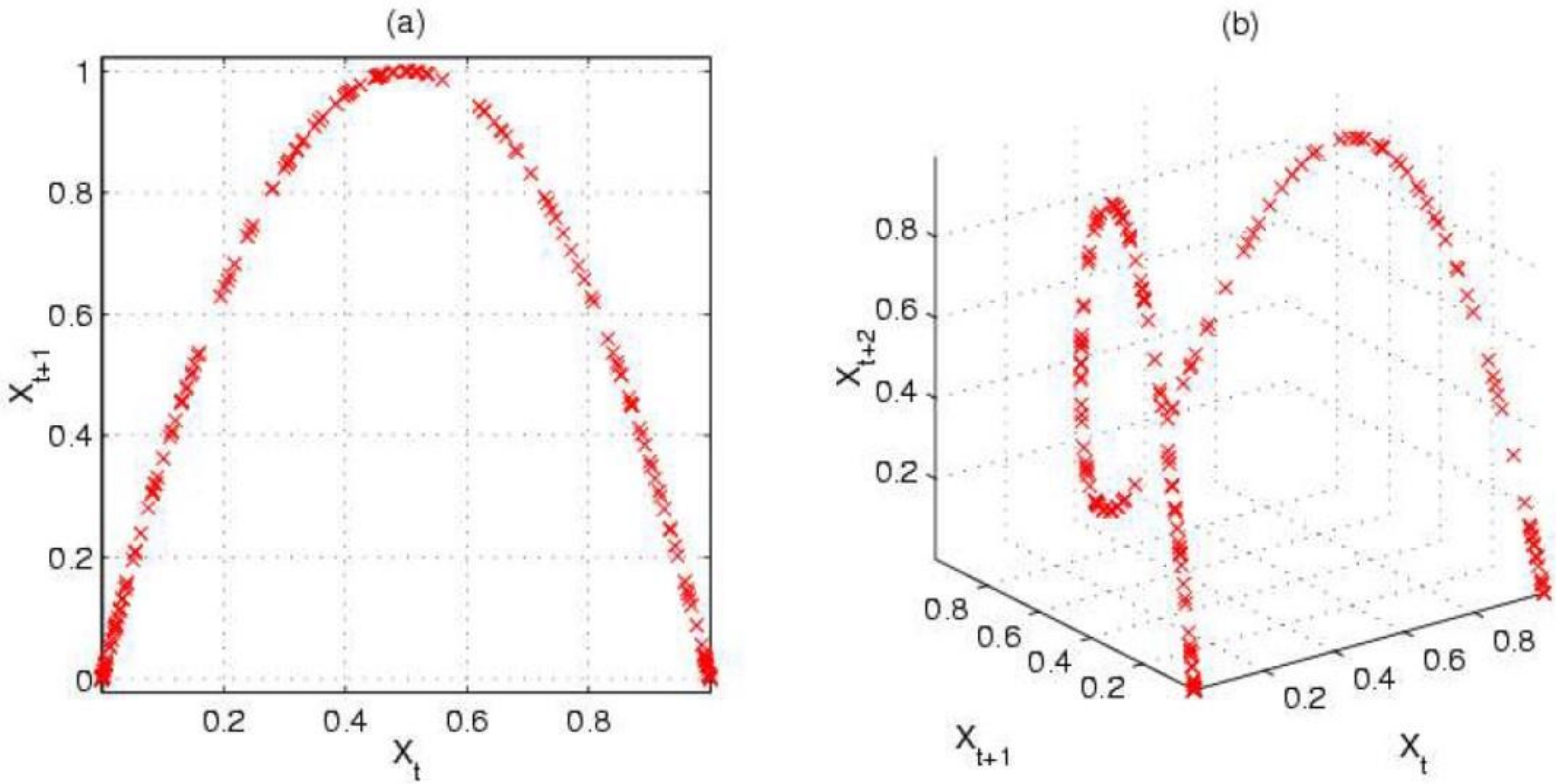
Joint embedding

$$(x_t, x_{t-1}, y_t)$$

Time delay embedding
Takens' Theorem

Example: logistic map

$$x_{n+1}=rx_n(1-x_n)$$

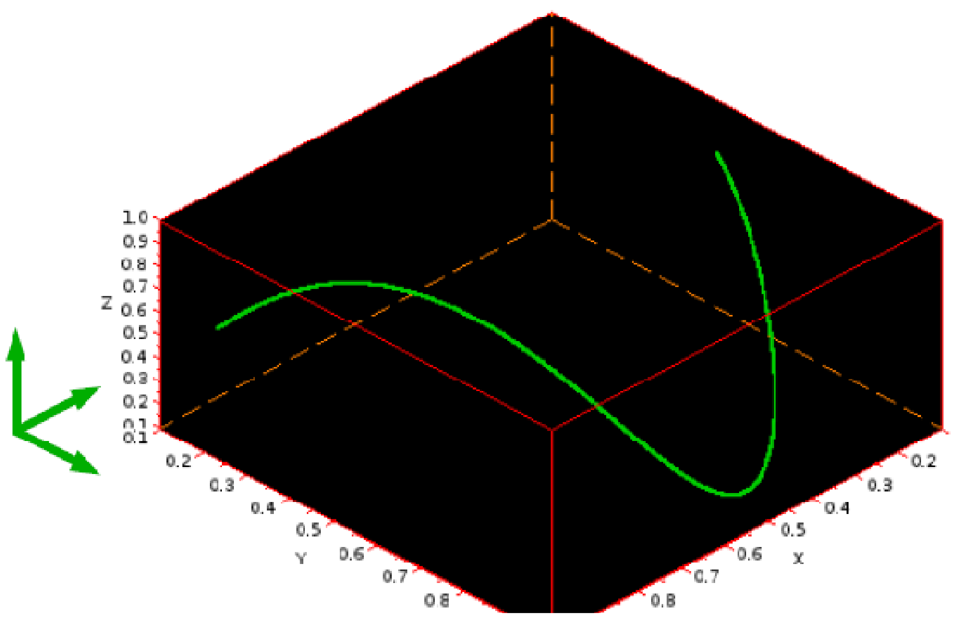


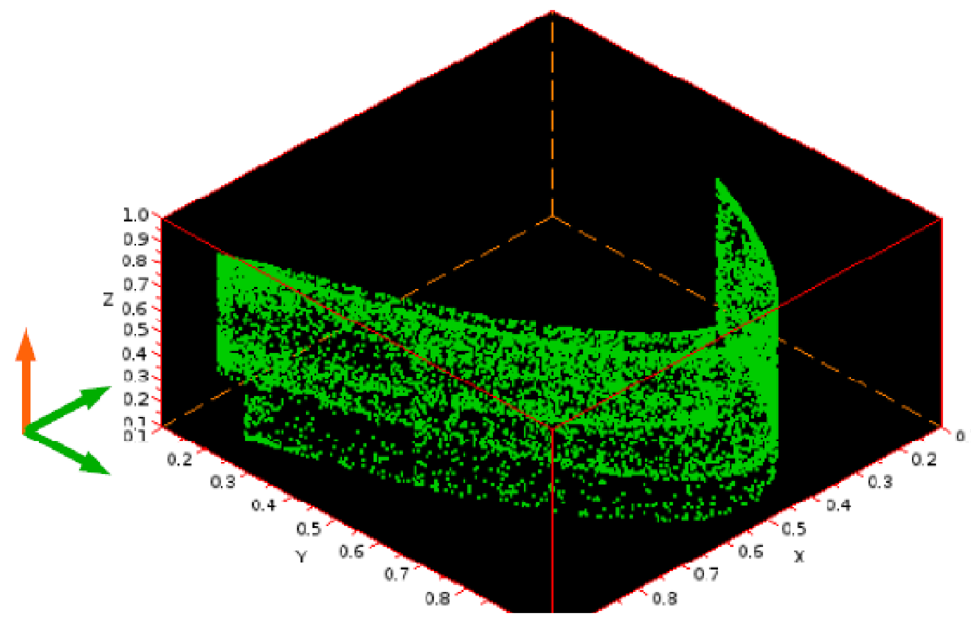
Embedded in D=2,3, the manifold is still one dimensional.

Time delay embedding

$$(y_{t-2}, y_{t-1}, y_t)$$

$$(y_{t-2}, y_{t-1}, x_t)$$



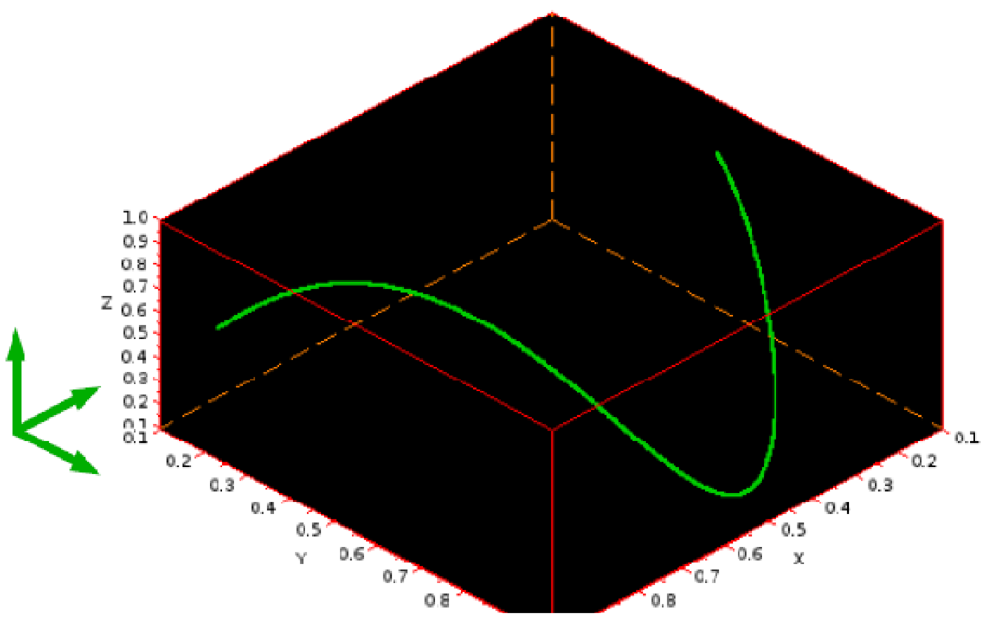


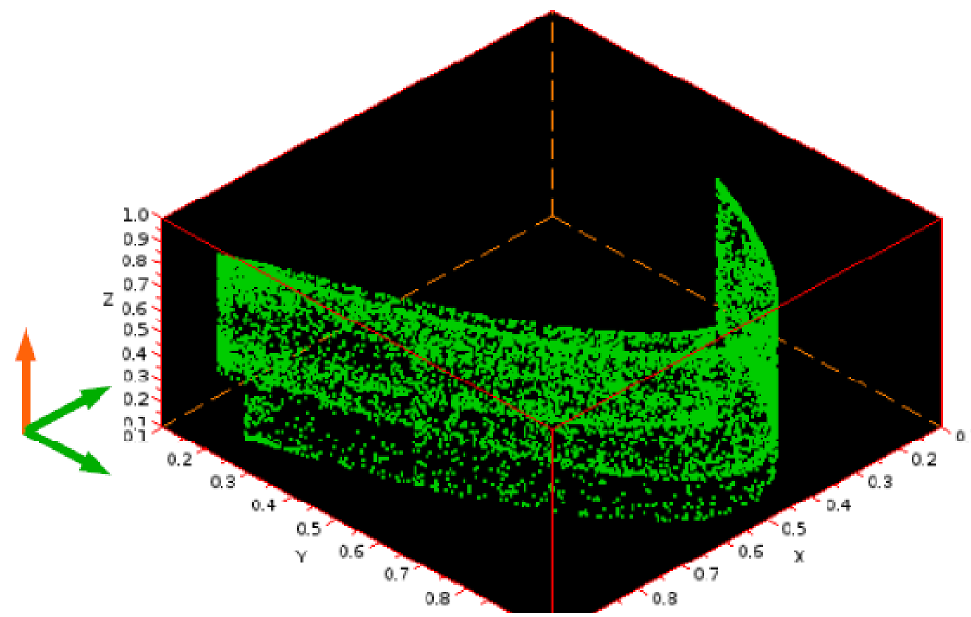
Example 1.

Time delay embedding

$$(y_{t-2}, y_{t-1}, y_t)$$

$$(y_{t-2}, y_{t-1}, x_t)$$





y d=1 (in D=3)

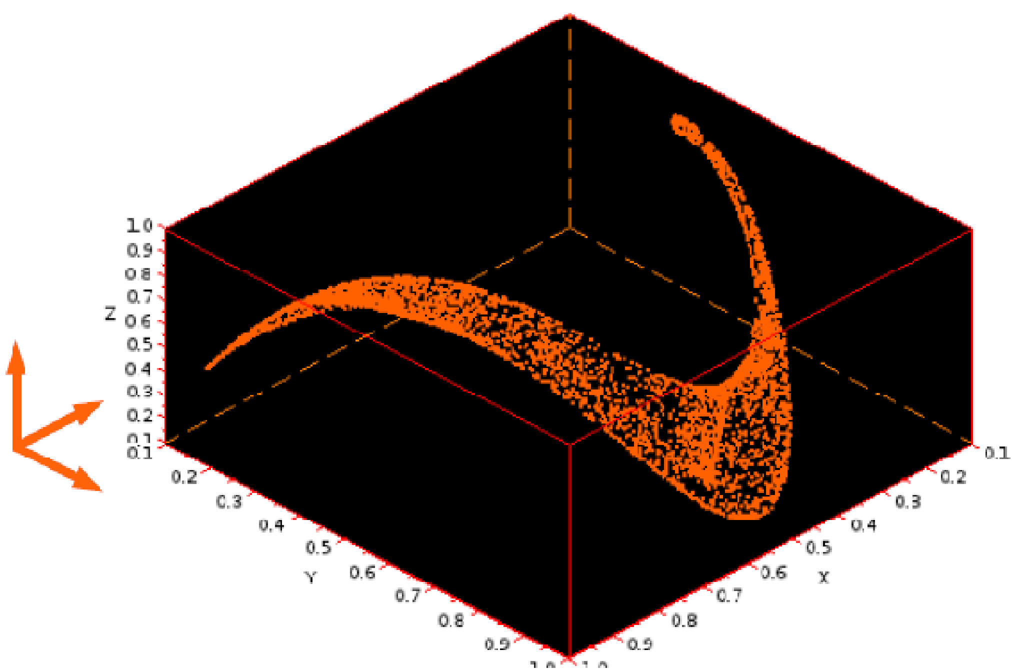
Joint d=2 (in D=3)

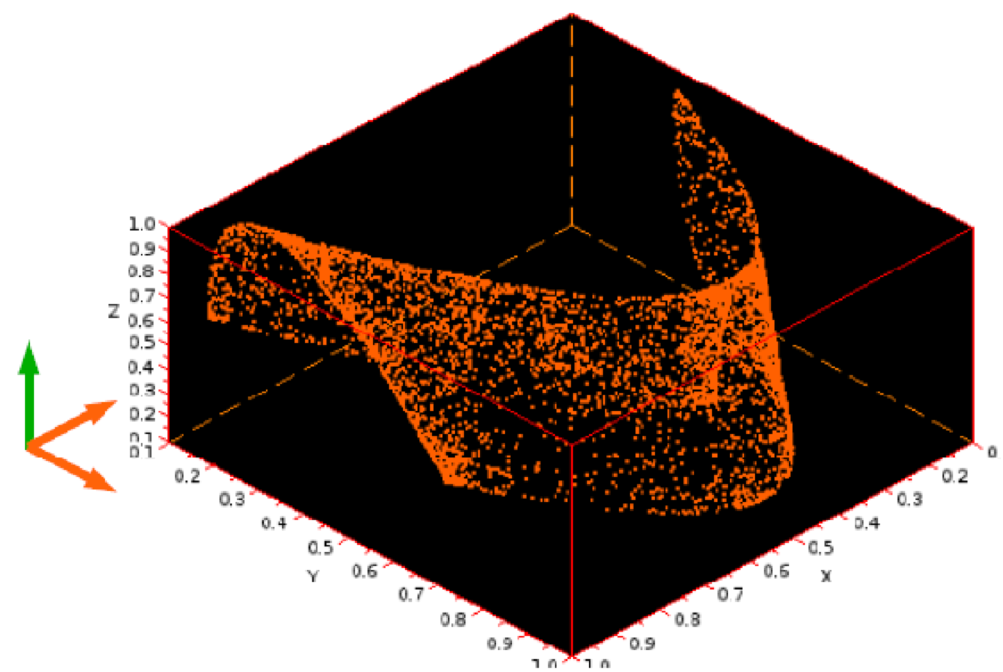
Dimension increase indicates independence

Time delay embedding

(x_{t-2}, x_{t-1}, x_t)

(x_{t-2}, x_{t-1}, y_t)



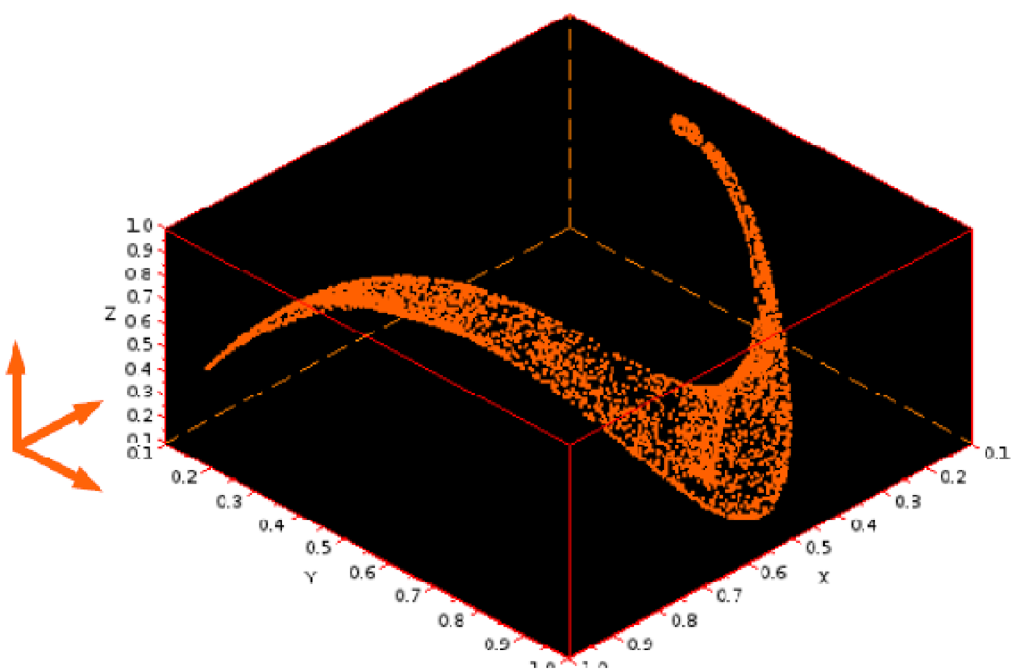


Example 2.

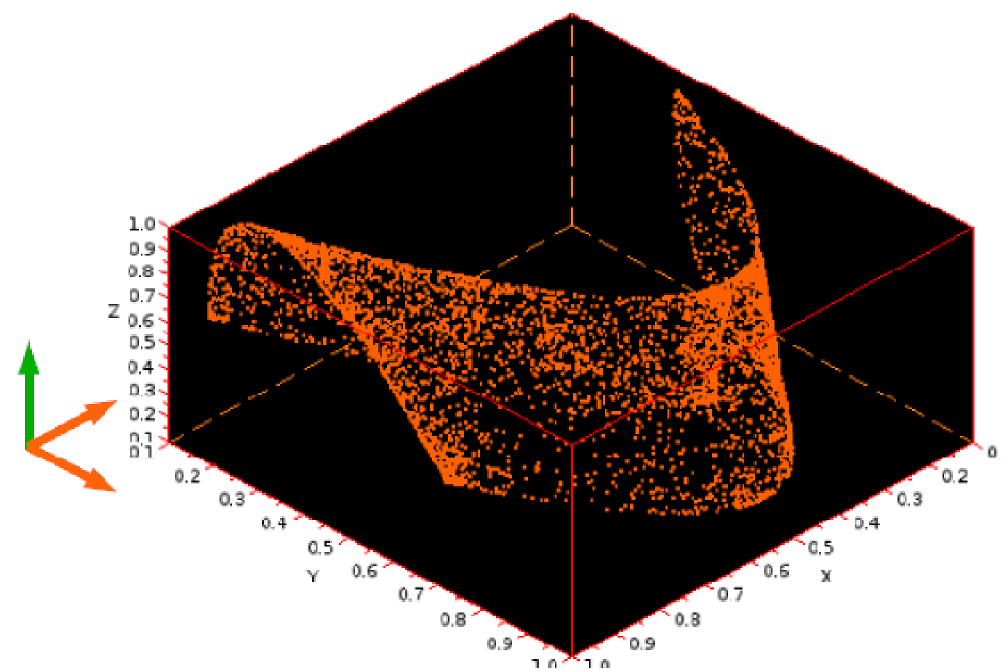
Time delay embedding

(x_{t-2}, x_{t-1}, x_t)

(x_{t-2}, x_{t-1}, y_t)



2d in 3D



joint embedding is still 2d

Lack of dimension increase indicates causality,
y causes x

Dimensions

In general

$$\max\left\{d\left(A\right), d\left(B\right)\right\} \leq d\left(\text{joint}\right) \leq d\left(A\right) + d\left(B\right)$$

Dimensions		Causal relation
d(A) < d(B) = d(A,B)	$\Longleftrightarrow$	A → B
d(B) < d(A) = d(A,B)	$\Longleftrightarrow$	B → A
d(A) = d(B) = d(A,B)	$\Longleftrightarrow$	A ←→B
d(A),d(B) < d(A,B) = d(A)+d(B)	$\Longleftrightarrow$	A and B are independent
d(A),d(B) < d(A,B) < d(A)+d(B)	$\Longleftrightarrow$	A,B have a common cause

Dimensions

- Information dimension
- Intrinsic Dimension (ID)
- ID of the time delayed embedded manifold
- Local ID estimate
- ID as average of local ID-s

Rényi information dimension

$$\langle X \rangle_N = \frac{\lfloor NX \rfloor}{N}$$

$$d = \lim_{N\to\infty} \frac{H\left(\langle X \rangle_N\right)}{\log N}$$

$$H(p) = -\sum_i p_i \log p_i$$

Local intrinsic dimension

Volume of balls for the time delayed manifold

$$V \approx c_d r^{d(x)}$$

or $\mathrm{P}(\mathrm{x},\mathrm{r}) = \mathrm{P}(\mathrm{X} \in \mathrm{B}(\mathrm{x},\mathrm{r})) \approx c_d r^{d(x)}$

$$d(x) = \lim_{r\to 0} \frac{\log(P(x,2r)) - \log(P(x,r))}{\log r}$$

Estimate of the intrinsic dimension

For embedding dimension *m*,

time series $x_0, x_1, \ldots\ldots x_t, x_{t+1}, \ldots$,

the delay vector $X_t = (x_t, x_{t-1}, \ldots x_{t-m+1})$

$$\hat{d} = \frac{1}{n} \sum_t \hat{d}(X_t)$$

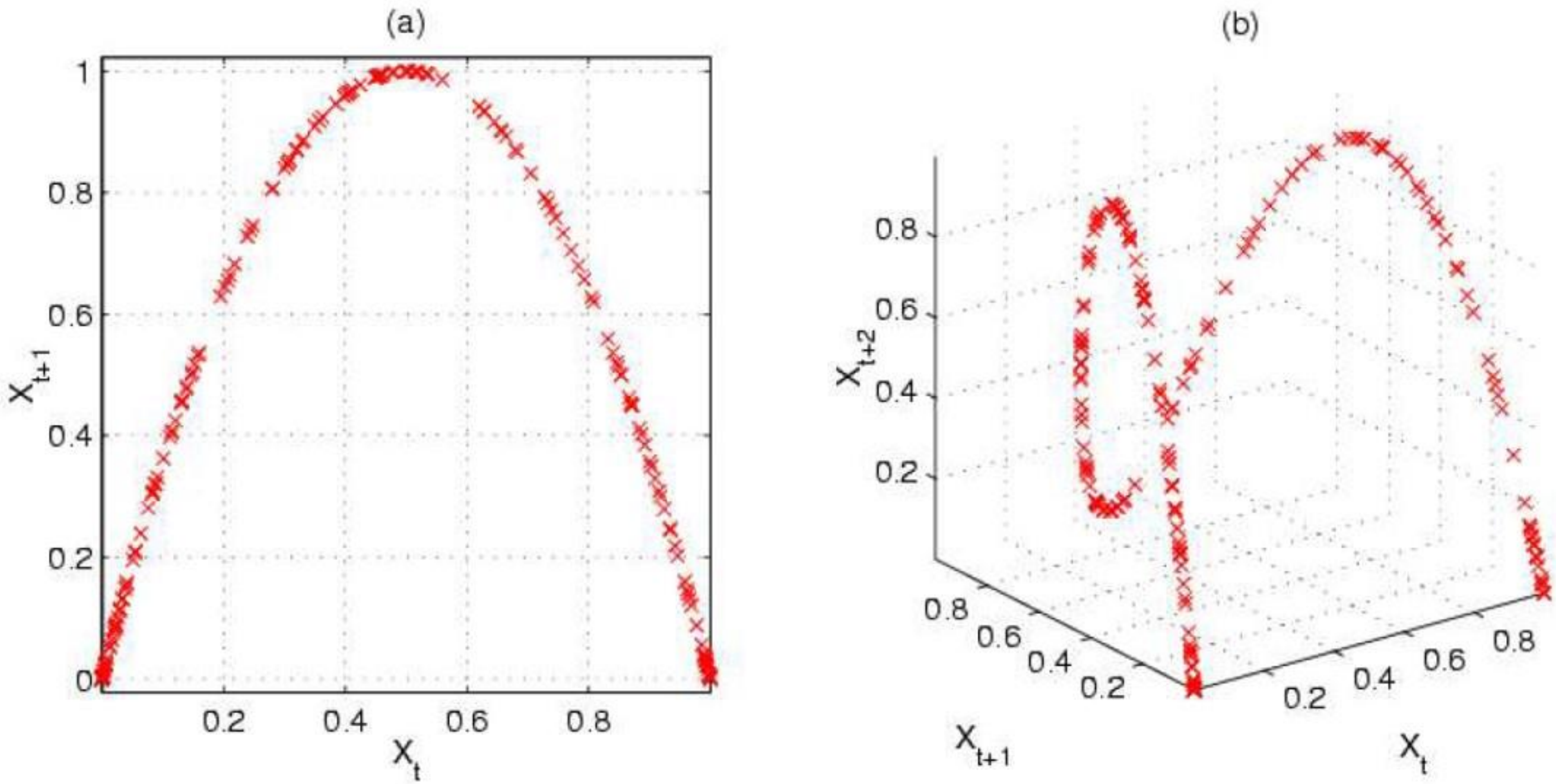
where the LID is estimated for a “good” *r*.

Test and application of the method

- Logistic map
- An old puzzle
- Brain surgery

Logistic map

$$x_{n+1} = r x_n (1 - x_n)$$

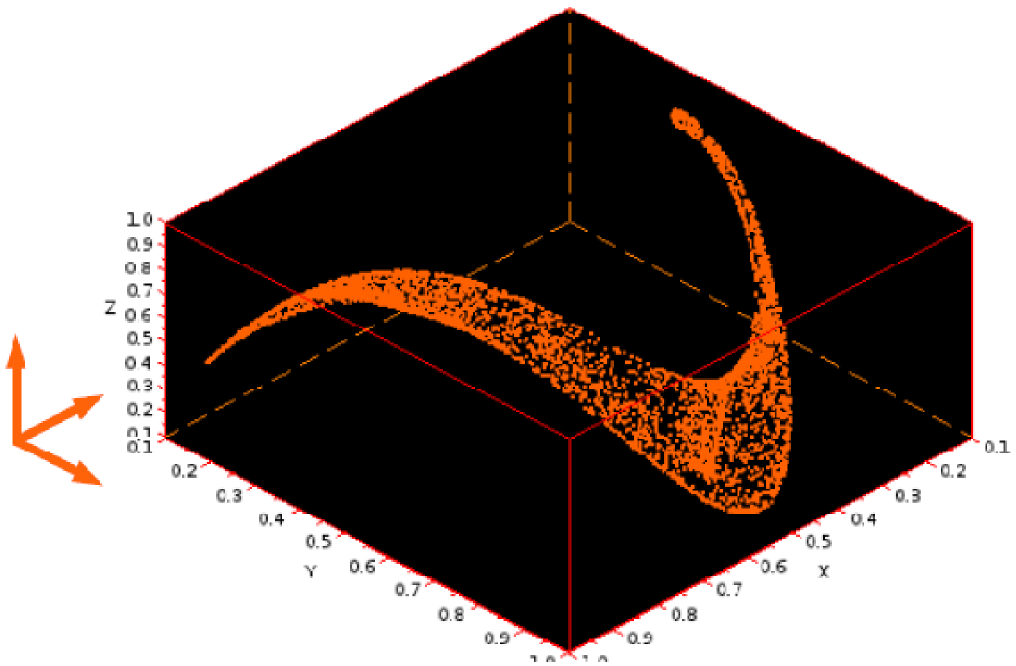


Embedded in D=2,3, the manifold is still one dimensional.

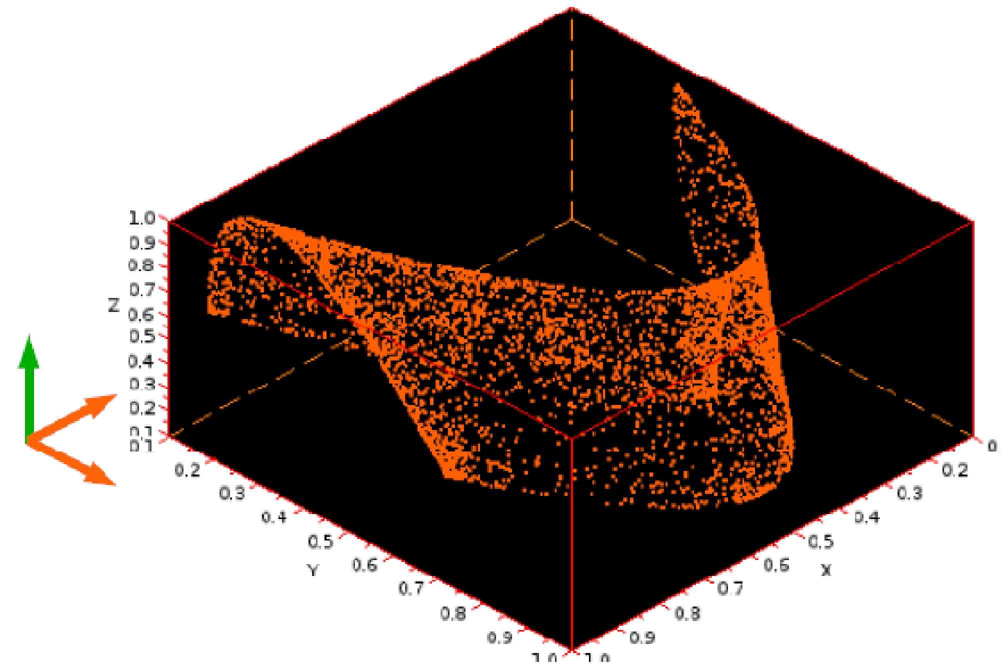
Time delay embedding

(x_{t-2}, x_{t-1}, x_t)

(x_{t-2}, x_{t-1}, y_t)



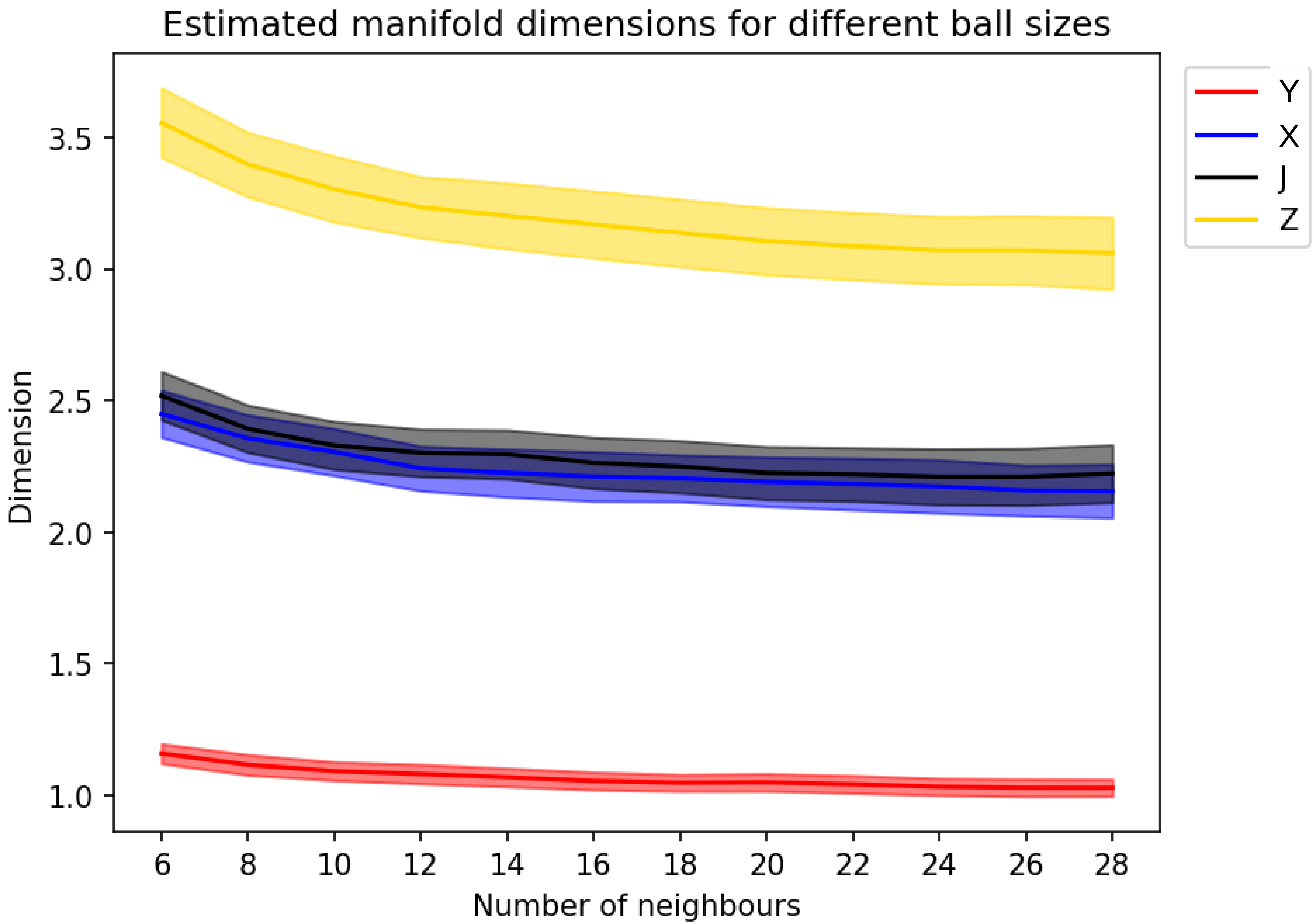
2d in m=3 d



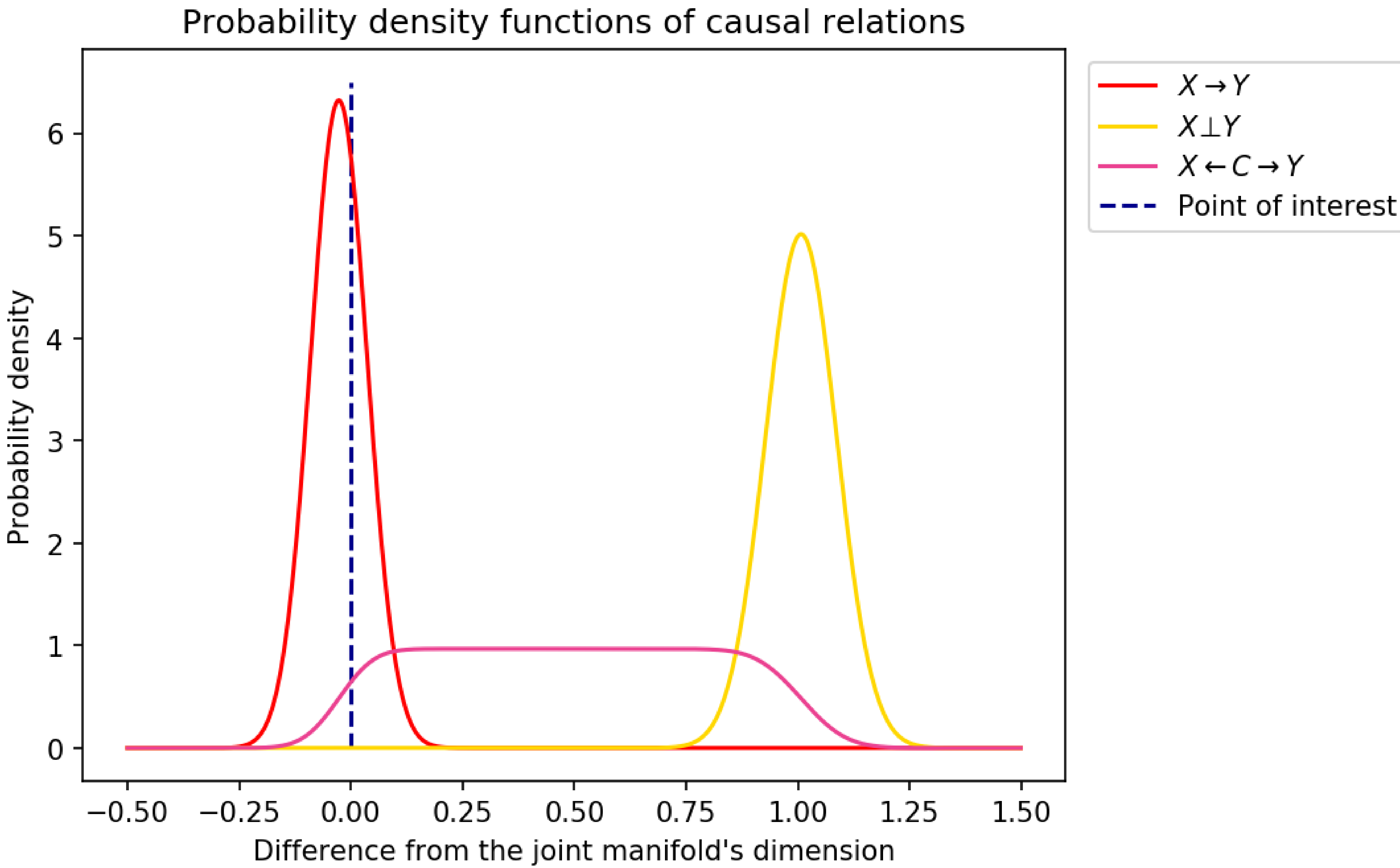
joint embedding is still 2d

Lack of dimension increase indicates causality,y causes x

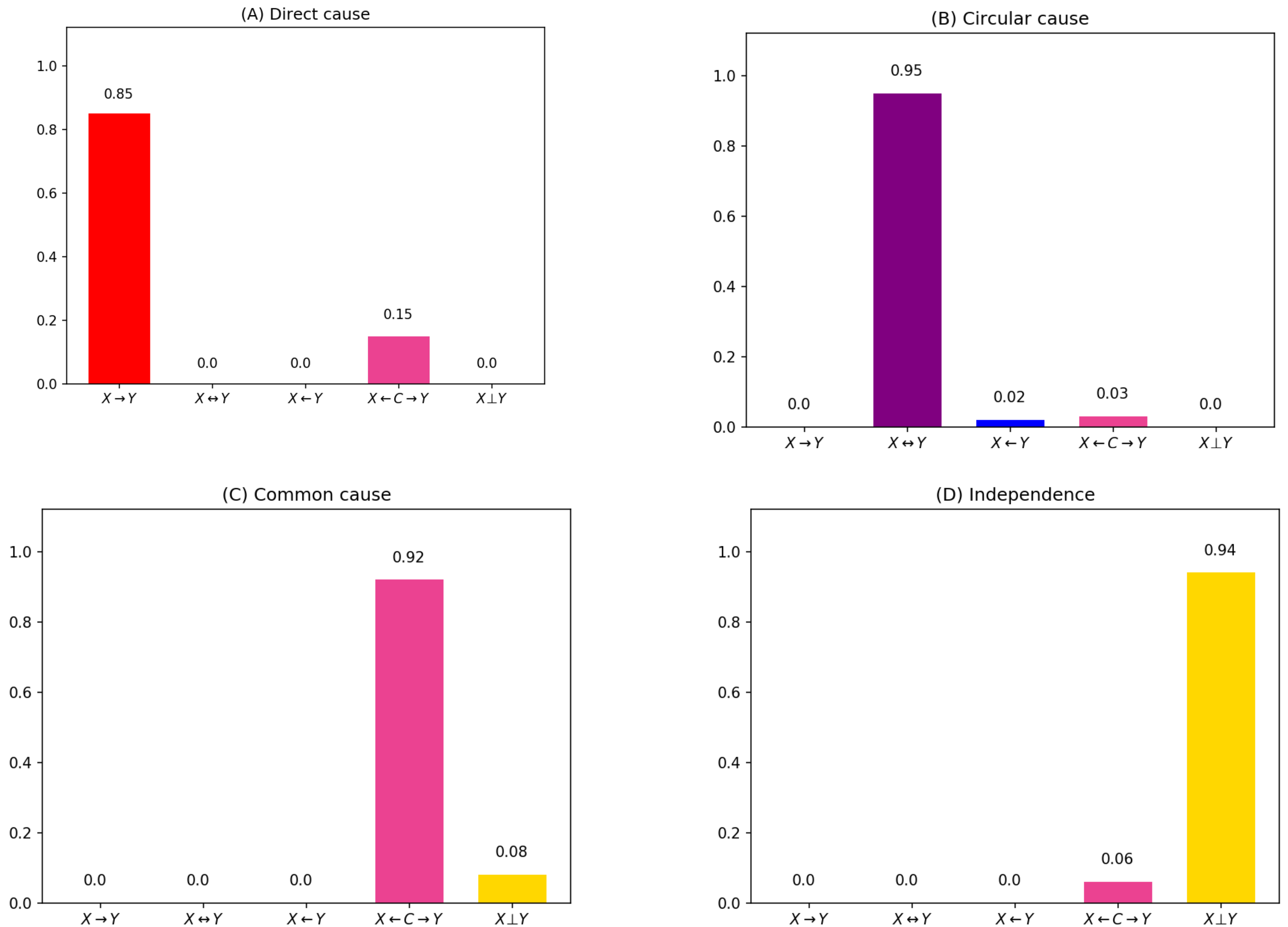
Logistic map



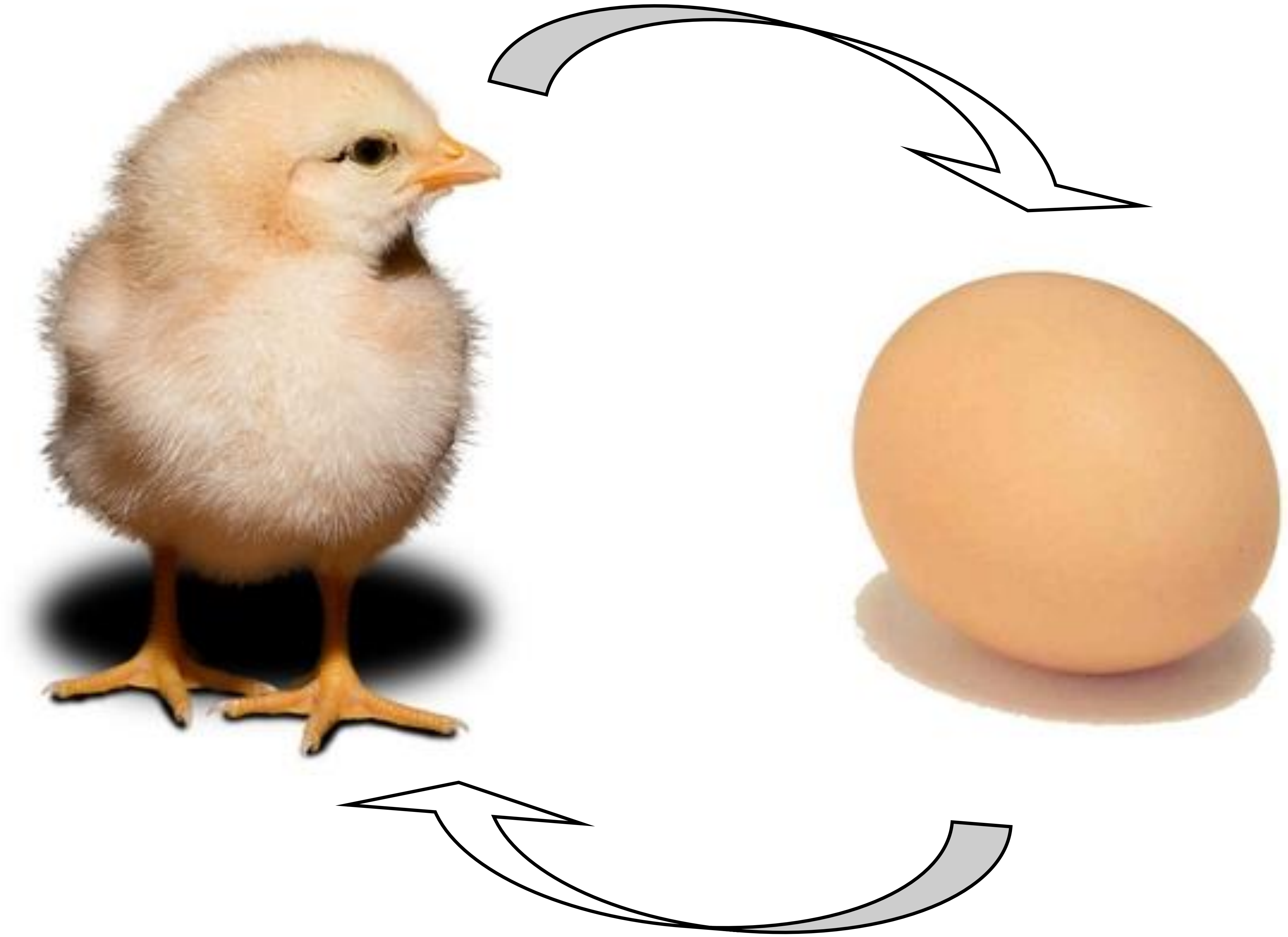
Logistic map



Logistic map



Which one came first?



Chickens, Eggs, and Causality, or Which Came First?

Walter N. Thurman and Mark E. Fisher*

1930-1983 egg production and chicken population

Time-series evidence from the United States indicates unidirectional causality from eggs to chickens.

Key words: causality, chickens, eggs.

Granger's seminal paper entitled "Investigating Causal Relations" has spawned a vast and influential literature. In macroeconomics, for example, the causal relationship between money and income has been investigated time (Sims) and again (Barth and Bennett; Williams, Goodhart, and Gowland; Ciccolo; Feige and Pearce; Hsiao). Some authors have taken exception to Granger's definition of causality *qua* causality (Zellner; Jacobs, Leamer, and Ward; Conway et al.), and even Granger has suggested "a better term might be temporally related" (Granger and Newbold, p. 225). We find ourselves in agreement with the temporal ordering interpretation of Granger causality. In fact, we believe that the most natural application of tests for Granger causality (temporal ordering) has until now been overlooked. We refer, of course, to: "Which came first, the chicken or the egg?" Our purpose in this study is to provide an empirical answer to this venerable question, which theory alone has not resolved.

This measure excludes chickens raised only for meat. Eggs are measured in millions of dozens and include all eggs produced annually in the United States. All are potentially fertilizable.

The notion of Granger causality is simple: If lagged values of X help predict current values of Y in a forecast formed from lagged values of both X and Y, then X is said to Granger cause Y. We implement this notion by regressing eggs on lagged eggs and lagged chickens; if the coefficients on lagged chickens are significant as a group, then chickens cause eggs. A symmetric regression tests the reverse causality.[1] We perform the Granger causality tests using one to four lags. The number of lags in each equation is the same for eggs and chickens.

To conclude that one of the two "came first," we must find unidirectional causality from one to the other. In other words, we must reject the noncausality of the one to the other and at the same time fail to reject the noncausality of the other to the one. If either both cause each other or neither causes the other,

*Mark E Fisher ≠ Ronald Fisher father of modern statistics

Which one came first?

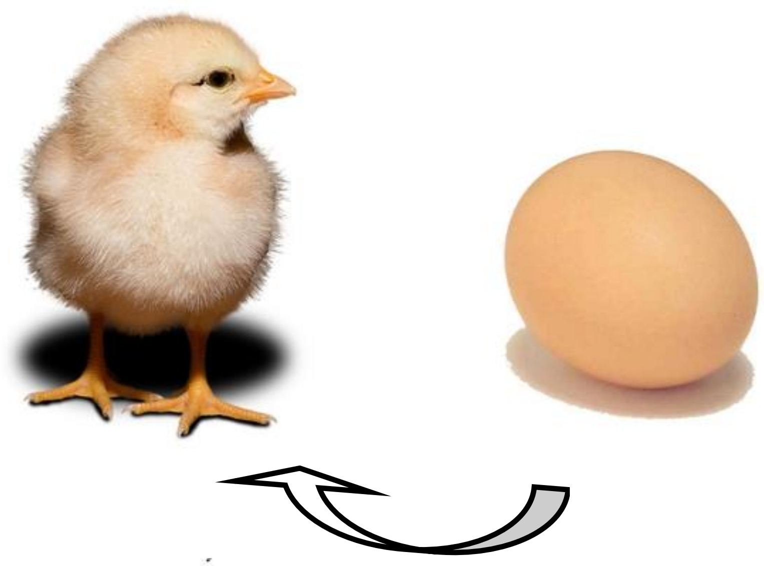


Table 1. Granger Causality Tests

Part 1: Did the Chicken Come First?

The following equation was estimated by OLS:

$$Eggs_t = \mu = \sum_{i=1}^{L} \alpha_i\, Eggs_{t-i} = \sum_{i=1}^{L} \beta_i\, Chickens_{t-i} + \epsilon_t;$$

$H_0 : \beta_1 = \ldots \beta_L = 0$ (chickens do not Granger cause eggs).

L = no. of lags	F-statistic	P-value	R^2 of the regression
1	.04	.85	.96
2	1.71	.19	.97
3	1.10	.36	.97
4	.79	.54	.97

Part 2: Did the Egg Come First?

The following equation was estimated by OLS:

$$Chickens_t = \mu + \sum_{i=1}^{L} \alpha_i\, Chickens_{t-i} + \sum_{i=1}^{L} \beta_i\, Eggs_{t-i};$$

$H_0 : \beta_1 = \ldots = \beta_L = 0$ (eggs do not Granger cause chichens).

L = no. of lags	F-statistic	P-value	R^2 of the regression
1	1.23	.27	.73
2	10.36	.0002	.81
3	5.85	.0019	.81
4	4.71	.0032	.82

Data source: U.S. Department of Agriculture, 1983 and others.
Note: The data are annual, 1930–83.

We perform the Granger causality tests using one to four lags. The number of lags in each equation is the same for eggs and chickens.

To conclude that one of the two "came first," we must find unidirectional causality from one to the other. In other words, we must reject the noncausality of the one to the other and at the same time fail to reject the noncausality of the other to the one. If either both cause each other or neither causes the other, the question will remain unanswered. The test results are presented in table 1. They indicate a clear rejection of the hypothesis that eggs do not Granger cause chickens. They provide no such rejection of the hypothesis that chickens do not Granger cause eggs. Therefore, we conclude that the egg came first.[2]

Which one came first?

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Results

e annual U.S. time series from 1930 gg production and chicken popula- ount as chickens the 1 December of a[illegible] broi[illegible] f the [illegible] all chickens that lay or fertilize ll chickens capable of causing eggs.

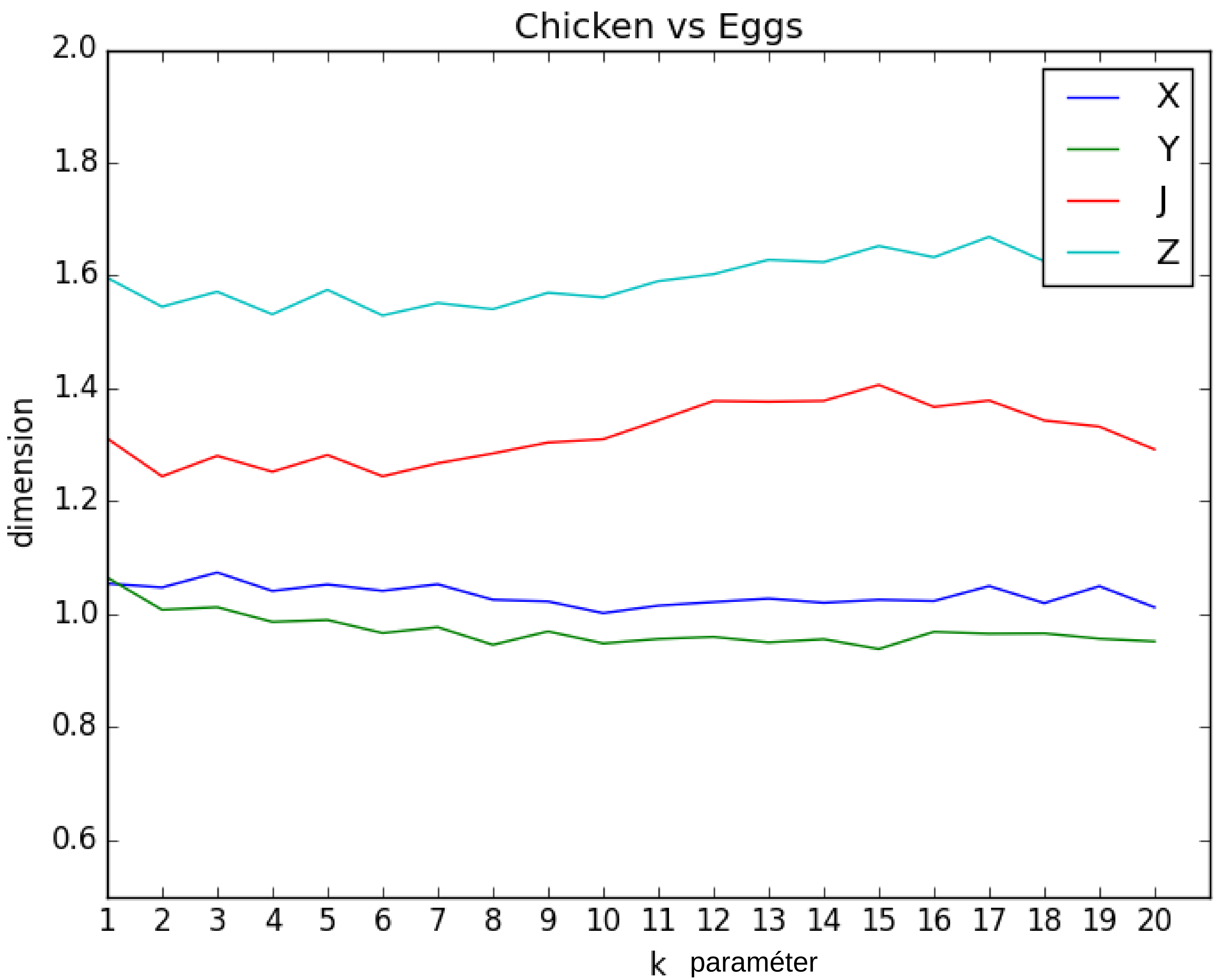
We perform the Granger causality tests using one to four lags. The number of lags in each equation is the same for eggs and chickens.

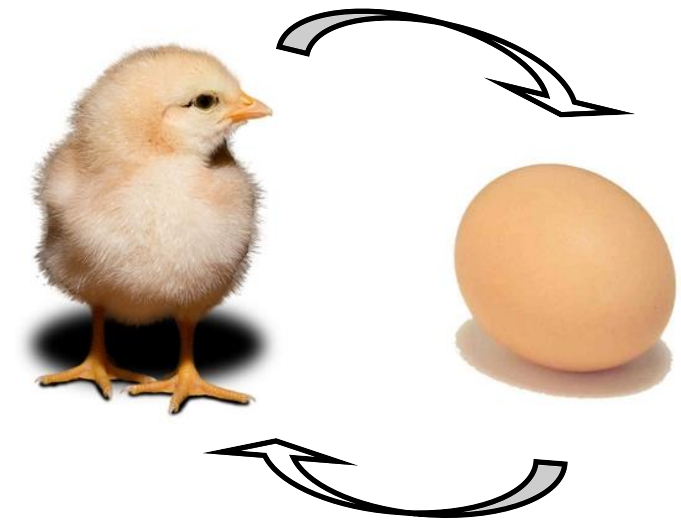
To conclude that one of the two "came first," we must find unidirectional causality from one to the other. In other words, we must reject the noncausality of the one to the other and at the same time fail to reject the noncausality of the other to the one. If either both cause each other or neither causes the other, the question will remain unanswered. The test results are presented in table 1. They indicate a clear rejection of the hypothesis that eggs do not Granger cause chickens. They provide no such rejection of the hypothesis that chickens do not Granger cause eggs. Therefore, we

Therefore, we conclude that chicken came first

[1] Feige and Pearce describe and distinguish among the several Granger causality tests. The validity of our test statistic requires lack of serial correlation, homoskedasticity, and normality of the

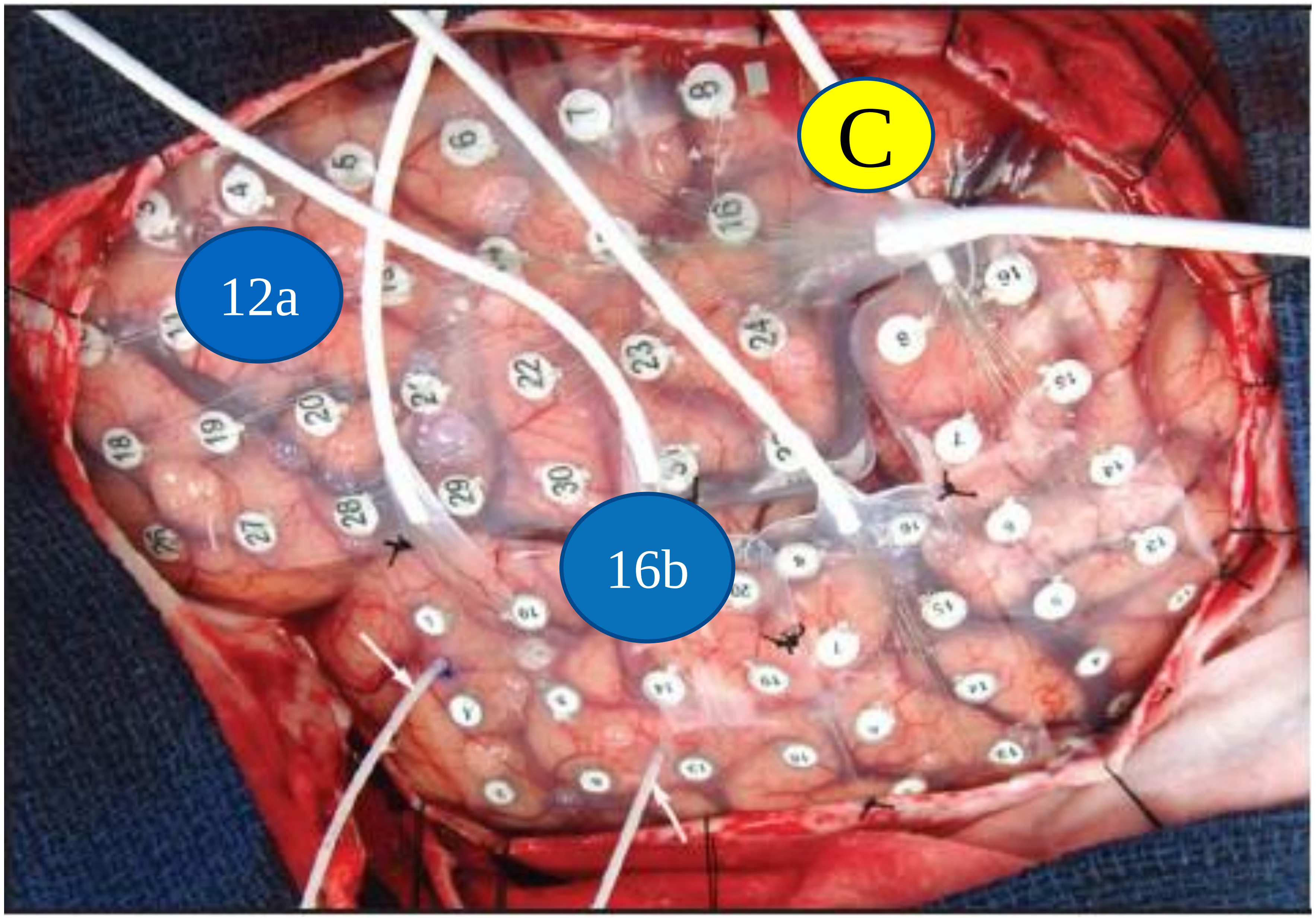
Which one came first?





Application to epilepsy focus detection

Which region is the source of the epileptic seizure?



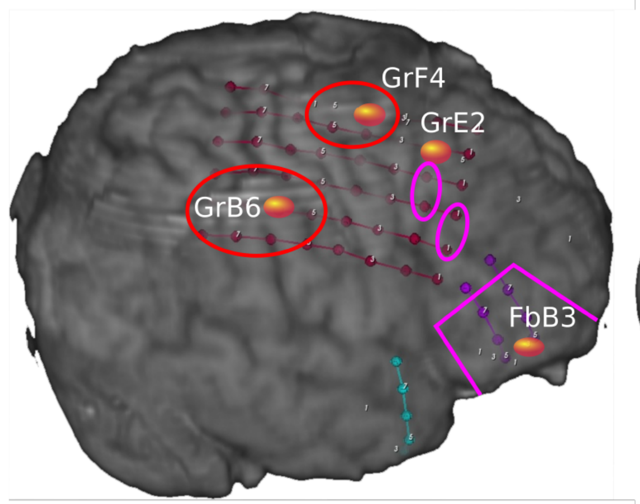
Shah AK, Mittal S. Invasive electroencephalography monitoring: Indications and presurgical planning. Ann Indian Acad Neurol 2014;17, Suppl S1:89-94

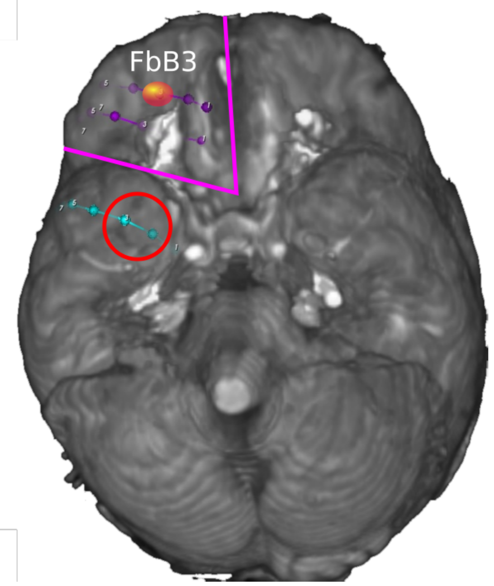
Application to epilepsy focus detection

5.Analysing epileptic patients

The 20-year-old patient suffered from a drug resistant epilepsy with frequent seizures.

The finding of a cortical dysplasia (at GrF4 electrode site) raised the possibility of the surgical treatment





As a part of the pre-surgical examination, a subdural grid and 2 strip electrodes were placed onto the surface of the brain.

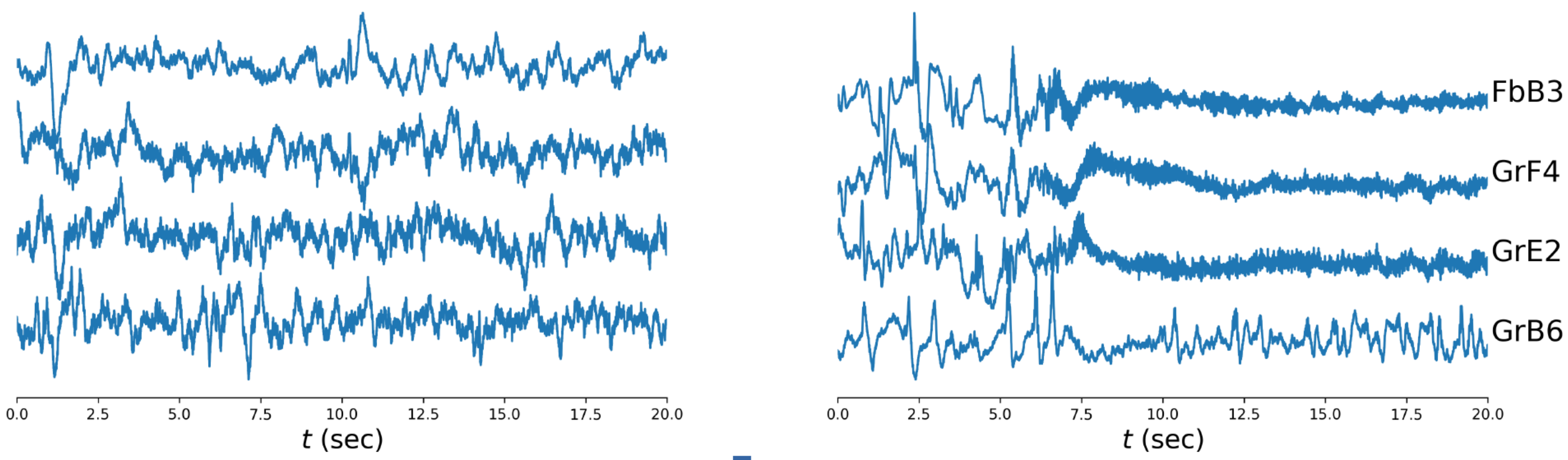
The seizures showed variable and complex picture, where most of the seizure activity were observable on the fronto-basal (FbB3) and the frontal (GrE2) region. The site of the displasia (GrF4) were touched only secondary and the infero-parietal (GrB6) region took part only in the initiation of the seizure but does not exhibit clear high frequency activity.

Based on these observations, and the difficult accessibility of the infero-parietal region (GrB6), the frontal and the fronto-basal region were resected (purple cuts and ellipses), the less active areas were left intact (red ellipses). The patient were seizure free for 1 year, but after that, their seizures returned.

1 – Computing CSD

Application to epilepsy focus detection

Asymptomatic | **Epileptic seizure**

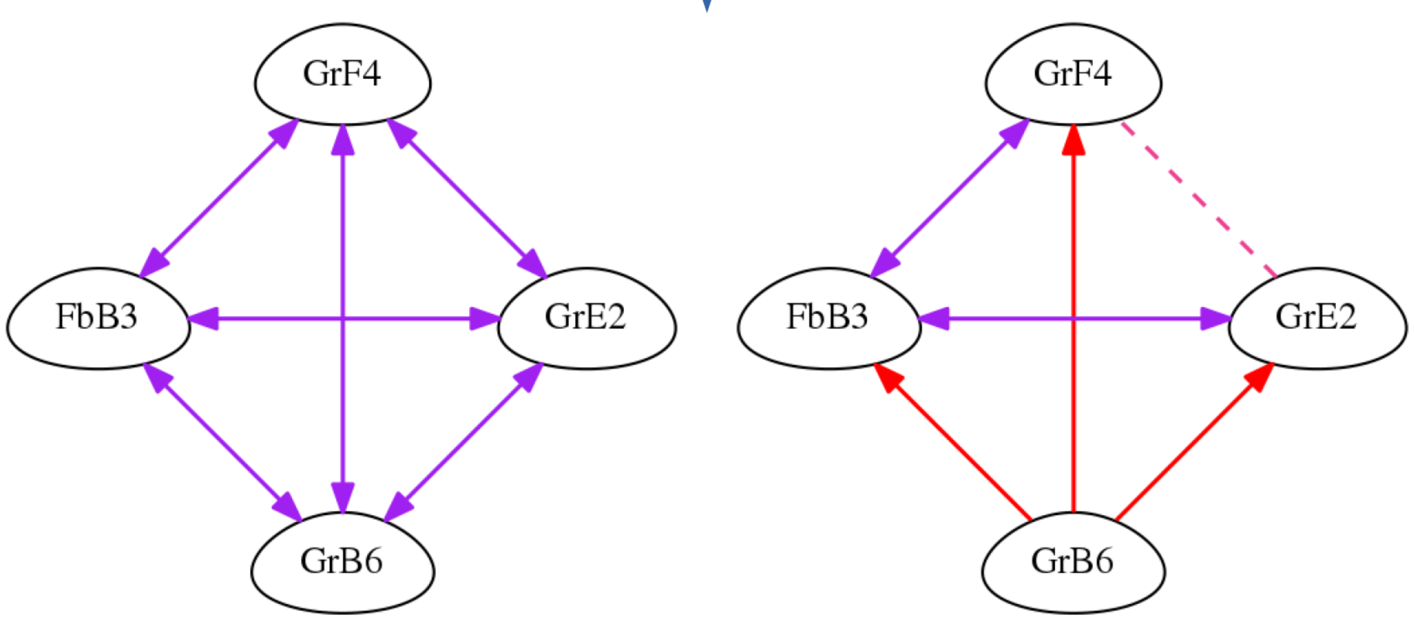


2 – Data preprocessing

Band-pass filtering (1-30 Hz)
Normalization

3 – Dimension-causality analysis

embedding dimension: 5
embedding delay: 11 step

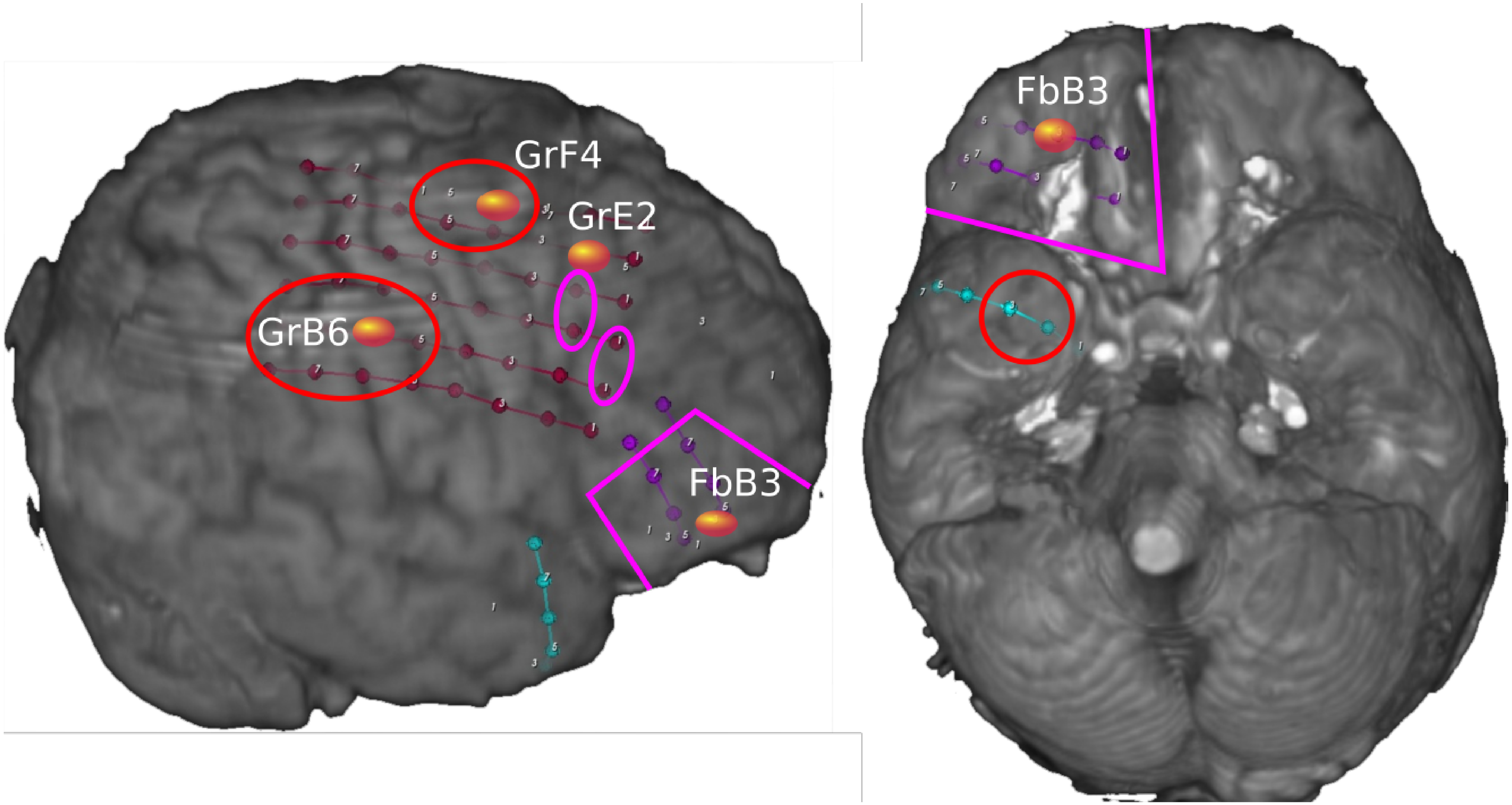


4 – Result

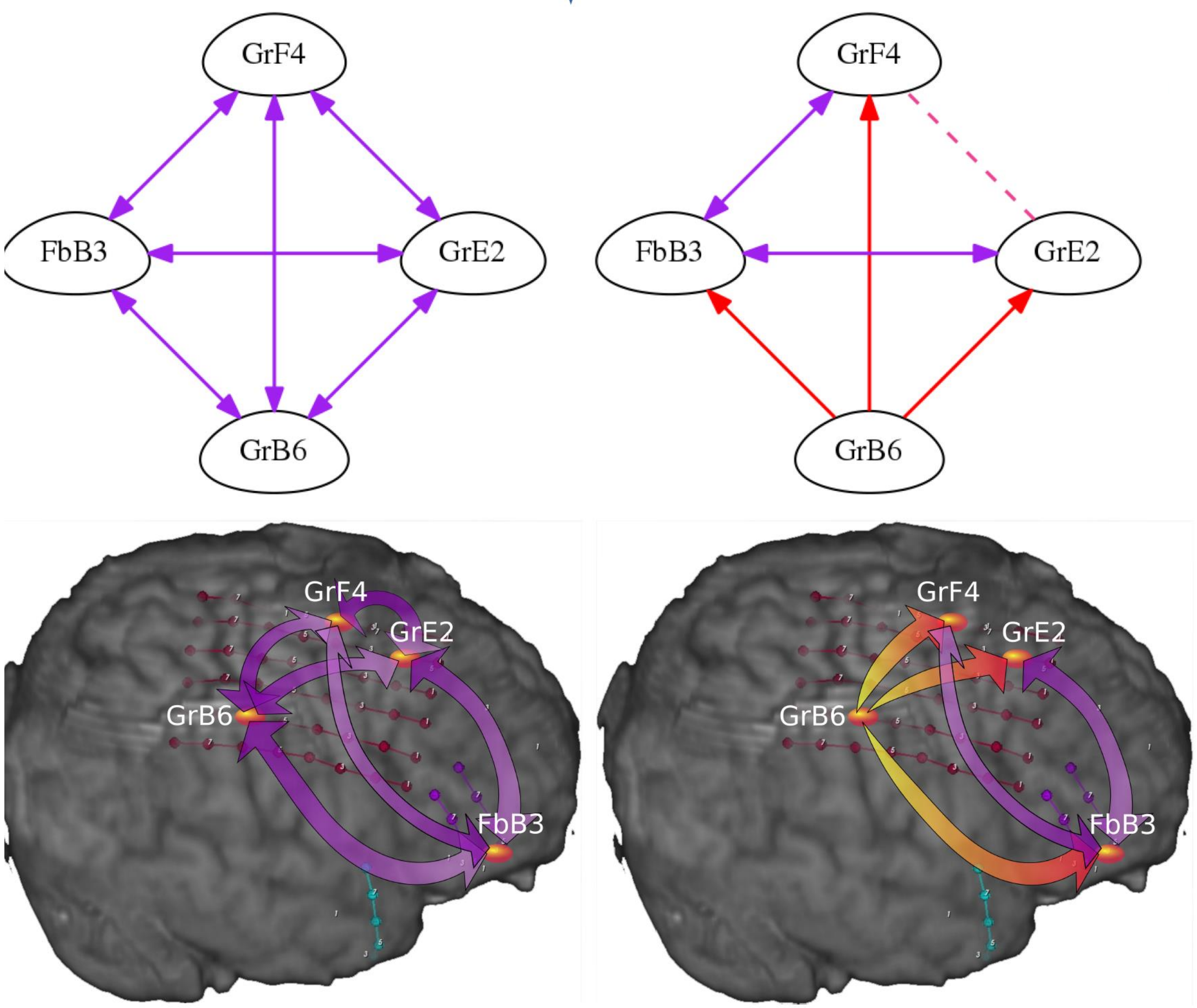
Our causality analysis showed that all the 4 area in question were mutually interconnected during normal, interictal activity, but the infero-temporal (GrB6) area became the dominant cause during seizure.

Application to epilepsy focus detection

Magenta areas have been removed



Application to epilepsy focus detection



Discussion

These results can be interpreted that, although, the resection of the large part of a highly interconnected epileptic network significantly reduced the seizure activity for a while, the un-touched primary cause transformed the remained tissue towards epilepsy and the seizures were restored.

Summary

Granger causality (1969)
Takens (1981) time delay embedding
Hirata (2010) recurrence maps - heuristic

Sugihara - convergence cross mapping (2012)

- – qualitative on causality
- – common cause detection in some cases

Our method

- Detects and distinguish all causality relations (expect cc in the shadow of bi-directional.)
- Provides probability to all causality relations

Thanks for the attention