Why do we live in hierarchies?

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EU FP7  Hung.Acad. Sci.  Eötvös Univ.
Original motivation: Pigeons (see later)

A few words about hierarchy

It is a kind of collective behaviour

HIERARCHY

No: bacteria, locusts, small fish...

Simple (2-3 levels): ants, bees, wasps ...

Complex (multi-level): some birds and mammals i.e., dolphins, people
Types of hierarchy
→ as classified based on relations among the members

Order  a → b → c → d → .....  

Embedded (those who belong to the smaller group AB also belong to the larger group A, with possible overlaps, etc)

Flow (relations are directed)
→ as classified based on its function

Dominance (goal: acquire, distribute food, mating partner)

Knowledge or competence (goal: solve problems)
Collective motion of starlings

By Dennis Hlynsky
RISD
Group decision making in pigeon flocks

GPS: Switzerland, U-blox, (17 X 22 mm, 2.1g), 10Hz antenna, Ireland, Taoglas
Battery: lipoly 2.9g (100mAh)-3.5 g (145mAh)
Weight: 12.5g

+ accelerometer, pressure, temperature, goniometer
Hierarchical order

directional correlation delay time network

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2009 Department of Biological Physics, Eötvös University

3x speed

25 feet
Digital video analysis of the moving pigeons around the feeding cup
Pair-wise dominance graph as determined from „who is closer to the feeding cup”
Correlation of interaction matrices is nearly zero:

For pigeons the knowledge-based and the dominance hierarchies are independent.
But we want numbers, universal mechanisms, etc.
Provocative statements

- Every real network is hierarchical

- Why? Because such a structure is more efficient (performs better)

- A flock of autonomous flying robots (drones) with two levels of hierarchy can be built
In particular:

- Group performance is maximized by a hierarchical competence distribution

- Hierarchical networks are more easily controllable using switchboard dynamics (controlling the edges)

- Copying decisions from more competent “palyers” results in the spontaneous build-up of hierarchical leader-follower relationships
The case of optimal order hierarchy

Egalitarian? 2 levels (bimodal)? Many levels?

Group performance is maximized by a multi-level hierarchical competence distribution

You have $X$ dollars to spend on composing a group of $n$ advisers delivering a collective decision on, e.g., where to invest

with A. Zafeiris
Voting model

Simplest:
- guess whether up or down is the true state
- ask nearest neighbours
- take majority vote
- make one more round
Number sequence guessing game on a small world graph
Hierarchical networks require a very large number of driver nodes (counter-intuitive)

\[ n_D \to 1 - \frac{1}{d}, \quad n \to \infty \]
Linear edge dynamics in complex networks

With Tamás Nepusz

We want to control several edges leading out from a single node using a „switchboard”

\[ \dot{y}_i^+(t) = M_i y_i^-(t) - \tau_i \otimes y_i^+(t) + \sigma_i u_i(t) \]

\( y_i^+(t) \) States of edges outgoing from node i

\( M_i \) Mixing or switching matrix

\( \tau_i \) Damping

\( u_i(t) \) State of the „driver node”
Edge control is more efficient for hierarchical networks

Proportion of nodes that have to be controlled

Table 1: Controllability properties of the real networks analysed in this paper

<table>
<thead>
<tr>
<th>Type</th>
<th>#</th>
<th>Name</th>
<th>Nodes</th>
<th>Edges</th>
<th>$n_D^{SBD}$</th>
<th>$n_D^{Liu}$</th>
<th>$n_D^{ER}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regulatory</td>
<td>1.</td>
<td>Ownership-USCorp</td>
<td>7,253</td>
<td>6,726</td>
<td>0.160</td>
<td>0.820</td>
<td>0.339</td>
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<td></td>
<td>2.</td>
<td>TRN-EC-2</td>
<td>418</td>
<td>519</td>
<td>0.222</td>
<td>0.751</td>
<td>0.366</td>
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<tr>
<td></td>
<td>3.</td>
<td>TRN-Yeast-1</td>
<td>4,441</td>
<td>12,873</td>
<td>0.034</td>
<td>0.965</td>
<td>0.415</td>
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<tr>
<td></td>
<td>4.</td>
<td>TRN-Yeast-2</td>
<td>688</td>
<td>1,079</td>
<td>0.177</td>
<td>0.821</td>
<td>0.381</td>
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<tr>
<td>Trust</td>
<td>5.</td>
<td>College*</td>
<td>32</td>
<td>96</td>
<td>0.344</td>
<td>0.188</td>
<td>0.418</td>
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<tr>
<td></td>
<td>6.</td>
<td>Epinions*</td>
<td>75,888</td>
<td>508,837</td>
<td>0.336</td>
<td>0.549</td>
<td>0.445</td>
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<tr>
<td></td>
<td>7.</td>
<td>Prison*</td>
<td>67</td>
<td>182</td>
<td>0.403</td>
<td>0.134</td>
<td>0.411</td>
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<tr>
<td></td>
<td>8.</td>
<td>Slashdot*</td>
<td>82,168</td>
<td>948,464</td>
<td>0.323</td>
<td>0.045</td>
<td>0.458</td>
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<tr>
<td></td>
<td>9.</td>
<td>WikiVote*</td>
<td>7,115</td>
<td>103,689</td>
<td>0.281</td>
<td>0.666</td>
<td>0.463</td>
</tr>
<tr>
<td>Food web</td>
<td>10.</td>
<td>Grassland</td>
<td>88</td>
<td>137</td>
<td>0.318</td>
<td>0.523</td>
<td>0.381</td>
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<td></td>
<td>11.</td>
<td>Little Rock</td>
<td>183</td>
<td>2,494</td>
<td>0.639</td>
<td>0.541</td>
<td>0.463</td>
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<td></td>
<td>12.</td>
<td>Seagrass</td>
<td>49</td>
<td>226</td>
<td>0.449</td>
<td>0.265</td>
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<tr>
<td></td>
<td>13.</td>
<td>Ythan</td>
<td>135</td>
<td>601</td>
<td>0.304</td>
<td>0.511</td>
<td>0.432</td>
</tr>
</tbody>
</table>
Emergence of hierarchical cooperation among non-cooperating individuals

*With T. Nepusz*

**Task:** guess the actual state of the environment

**Essential model parameters:**

- $n$ – the number of agents
- $p$ – the probability of a state flip in the environment
- $a_i$ – the ability of agent $i$ (probability that it makes the right guess)

$T$ – the „trust” matrix of ability estimates
individuals are trying to follow (interested in copying) the decisions of their more successful group mates (learn from them) in proportion with the degree they trust the level of judgment of the other actors as compared to their own level of competence.

The corresponding model results in the emergence of a collaboration structure in which the leadership-followership relations manifest themselves in the form of a multi-level, directed hierarchical network.
Hierarchy is gradually built up, with those with higher abilities „climbing to the top”

It is „metastable”, both adaptability and robustness are present

Overall performance is above the average of the individuals (information „flows down” efficiently”)

Next, we plan to apply hierarchical control to a flock of autonomous drones (quadcopters)
Our copters and lab
Copters:
Realistic simulations

Elapsed Time: 2.0 sec
Average Velocity: (2.3 ± 0.82) m/s
Distance between units: (19.48 ± 9.82) m
Copters: Animated actual GPS data
A few seconds from the video we produced
According to Monty Python Flying Circus....
Thank you for your attention
The rules can be formulated as follows:

1. First we define a changing environment (the state of which the individuals have to guess to gain benefit) as simple as possible, but still varying in an unpredictable way. The state of the environment is chosen to have a value of 1 or 0 with a probability \( p \). Such a definition corresponds to a random walk with a characteristic time of changing its direction proportional to \( 1/p \).

2. The individuals have a pre-defined ability (according to a given distribution with values between 0 and 1) to make a proper guess of the state of the environment. Their guess in each turn is based on their interactions with the agents they trust the most by making a weighted average of the decisions of the most trusted \( k=1,2 \ldots m \) friends/colleagues/players and his/her own estimate.
3. The trust matrix is thus used to update the guess of a player in the next round (the elements of this matrix correspond to the degree agent $i$ trusts agent $j$). Individuals are trusted on the basis of their prior performance. More trusted agents are „listened to” more frequently. Naturally, the trust matrix is updated as the collective decision making process progresses.

4. A network is constructed based on the frequency of how many times agent $i$ takes into account the guess of agent $j$.

*(the actual realization/algorithm has a few more less relevant rules)*