Why do we live in hierarchies?

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Original motivation: Pigeons (see later)

A few words about hierarchy

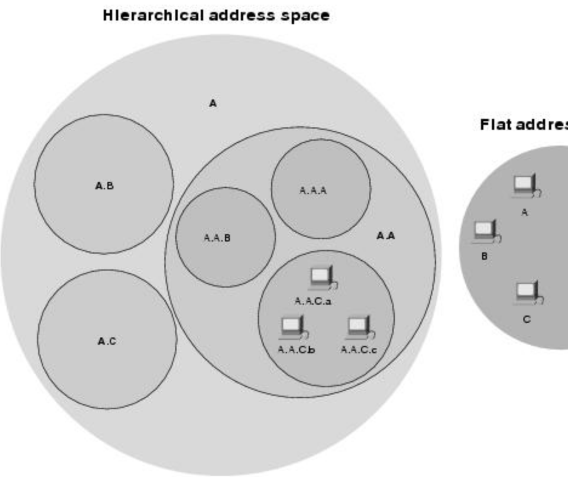
It is a kind of collective behaviour

HIERARCHY	**No:**	**bacteria, locusts, small fish..**
	Simple (2-3 levels):	**ants, bees, wasps ...**
	Complex(multi-level):	**some birds and mammals i.e., dolphins, people**

Types of hierarchy

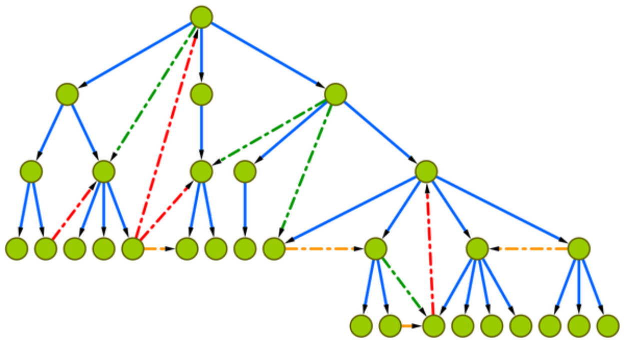
→ as classified based on relations among the members

Order a →b →c → d →.....



Embedded (those who belong to the smaller group AB also belong to the larger group A, with possible overlaps, etc)

Flow (relations are directed)

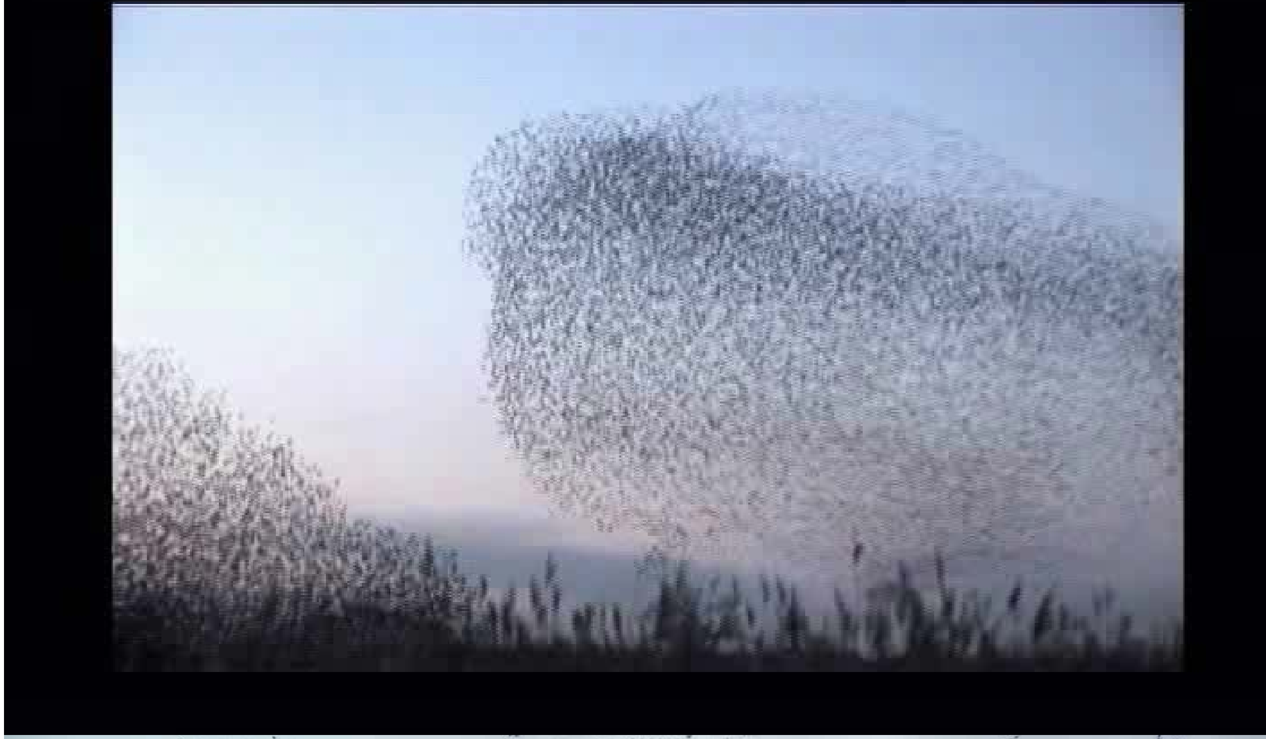


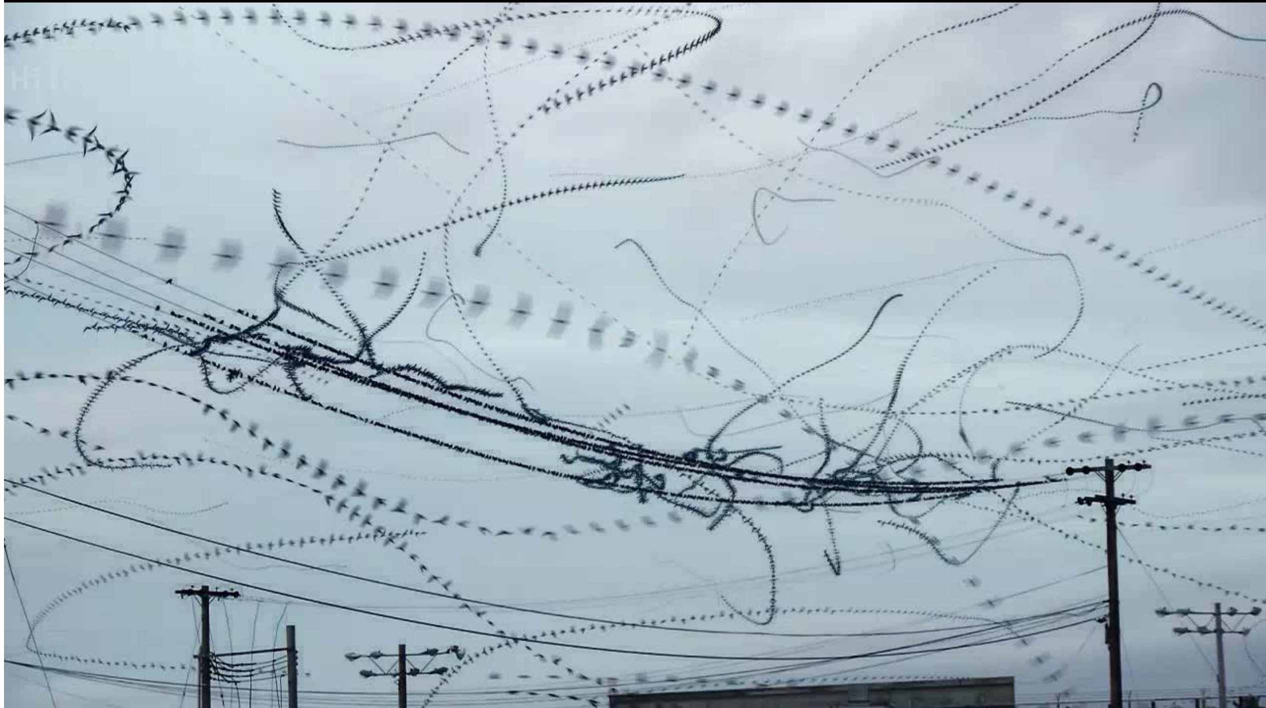
→ as classified based on its function

Dominance (goal: acquire, distribute food, mating partner)

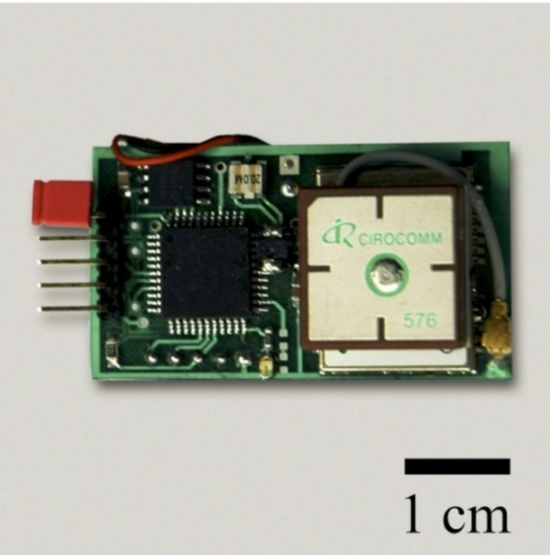
Knowledge or competence (goal: solve problems)

Collective motion of starlings





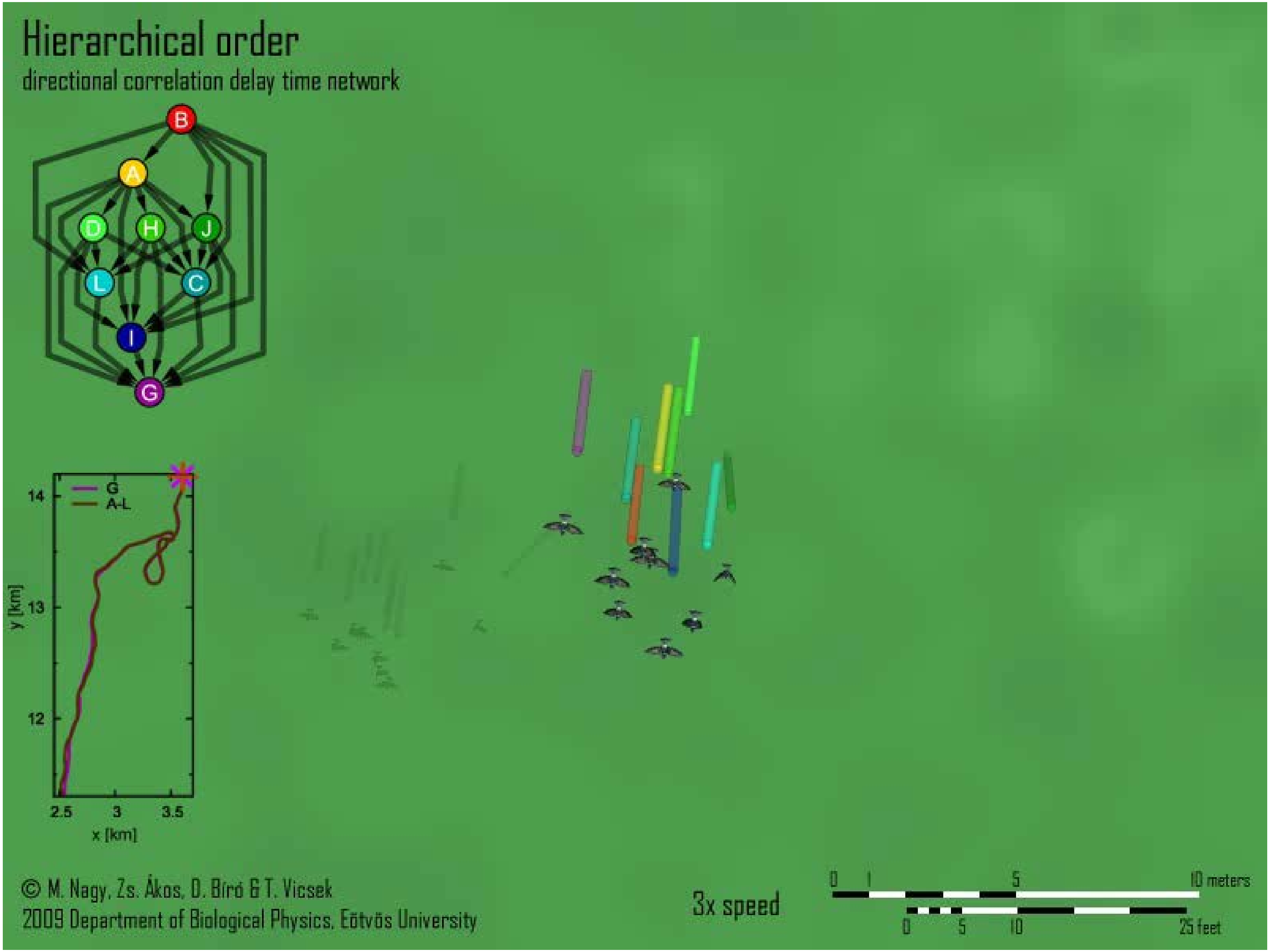
By Dennis Hlynsky
RISD



Group decision making in pigeon flocks

GPS : Switzerland, U-blox, (17 X 22 mm, 2,1g), 10Hz
antenna, Ireland, Taoglas
battery : lipoly 2,9g (100mAh)-3,5 g (145mAh)
Weight: 12,5g

+ accelerometer, pressure, temperature, goniometer


Hierarchical order
directional correlation delay time network
B
A
D
H
J
L
C
I
G
G
A-L
14
13
12
y [km]
2.5
3
3.5
x [km]
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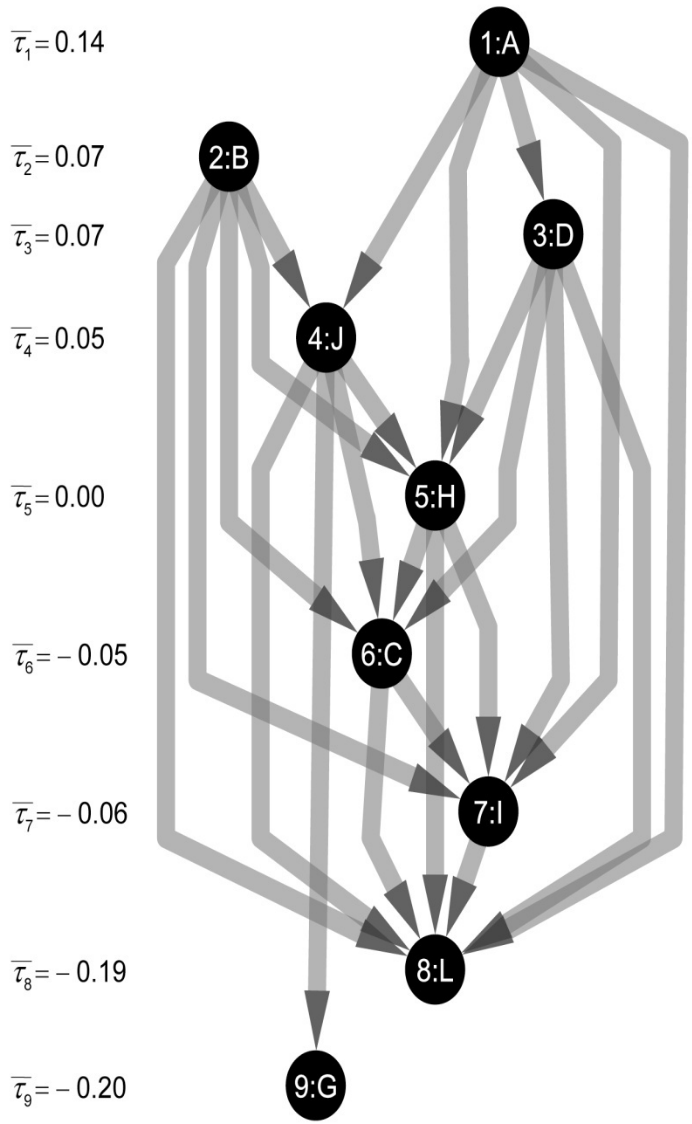
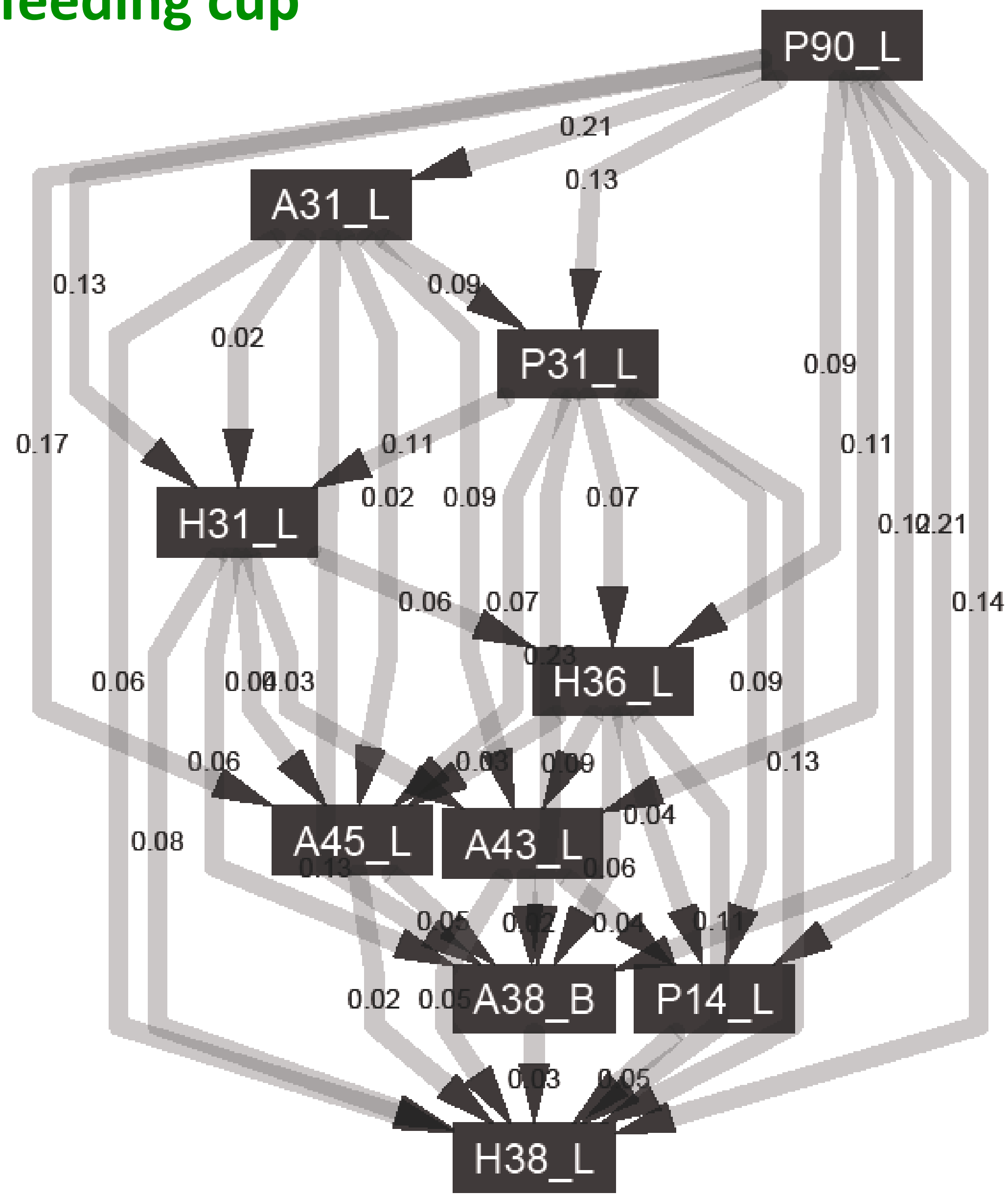
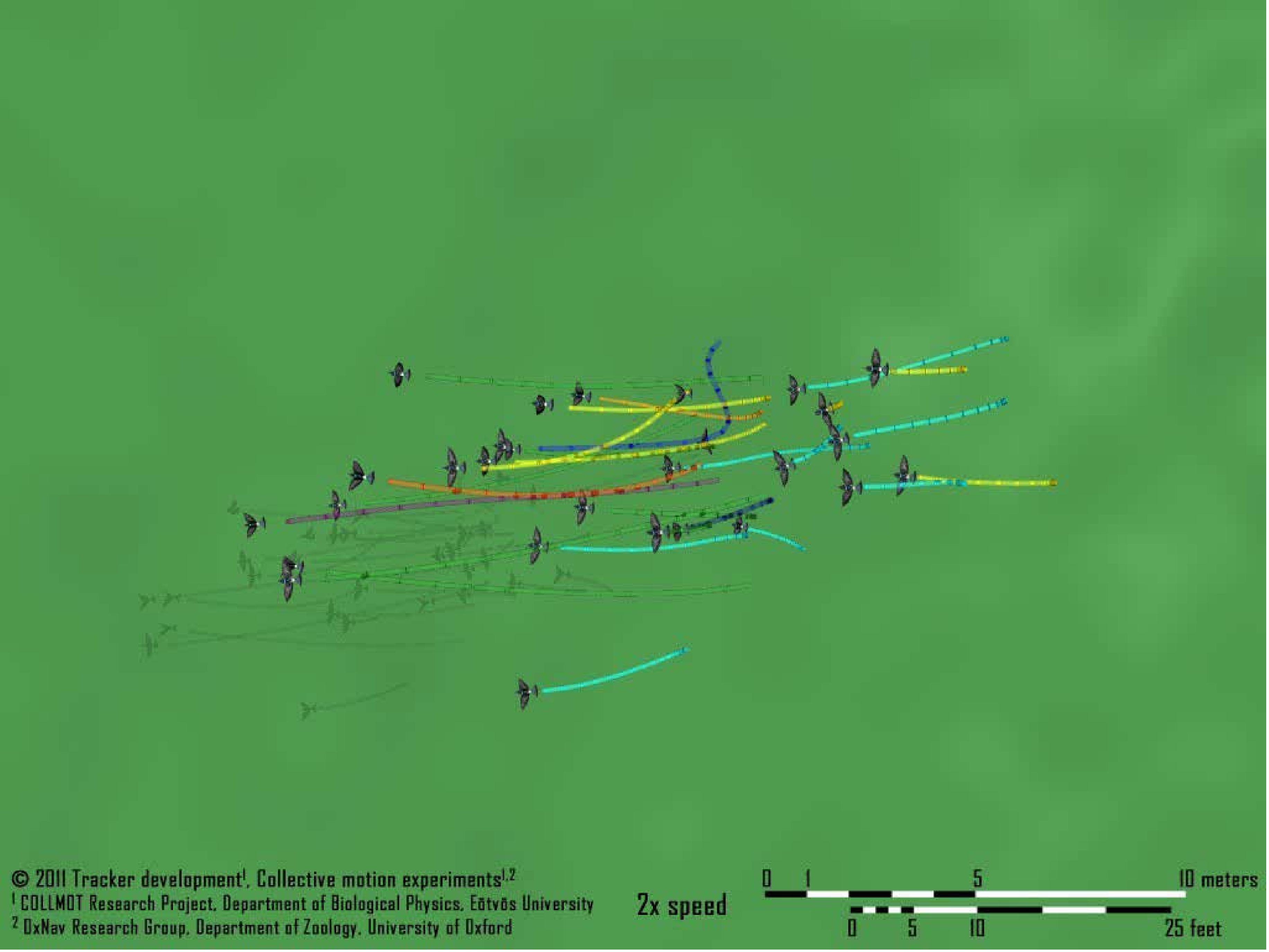
2009 Department of Biological Physics, Eötvös University
3x speed
0
1
5
10 meters
0
5
10
25 feet


$\overline{\tau}_1 = 0.14$
$\overline{\tau}_2 = 0.07$
$\overline{\tau}_3 = 0.07$
$\overline{\tau}_4 = 0.05$
$\overline{\tau}_5 = 0.00$
$\overline{\tau}_6 = -0.05$
$\overline{\tau}_7 = -0.06$
$\overline{\tau}_8 = -0.19$
$\overline{\tau}_9 = -0.20$
1:A
2:B
3:D
4:J
5:H
6:C
7:I
8:L
9:G

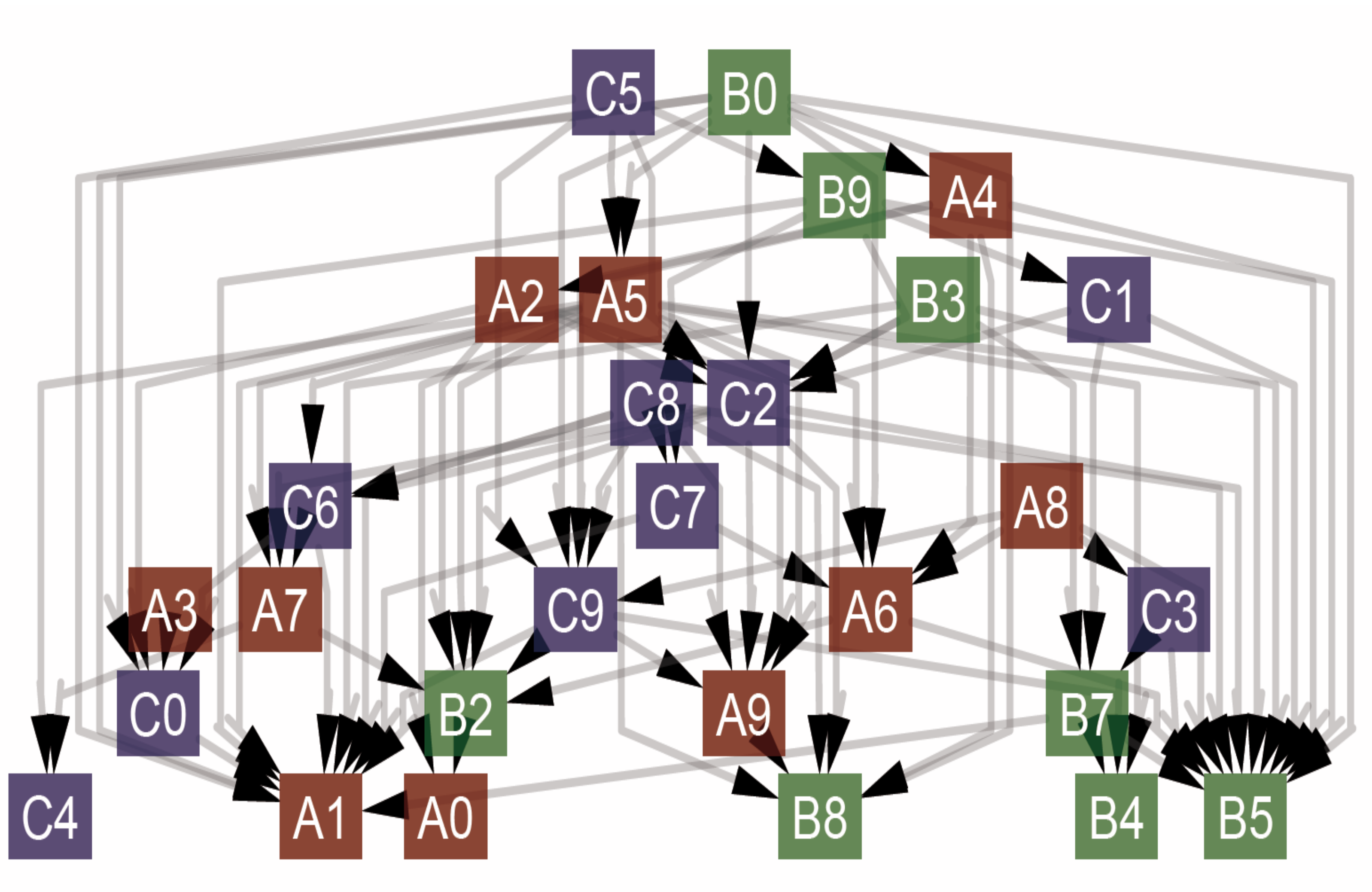
Digital video analysis of the moving pigeons around the feeding cup

Pair-wise dominance graph as determined from „who is closer to the feeding cup"



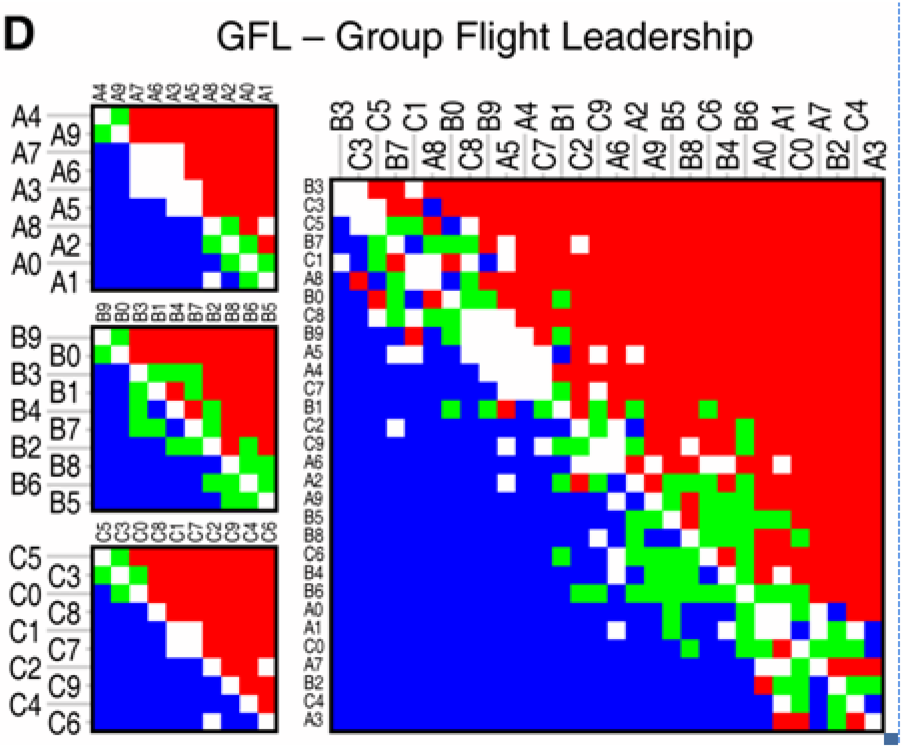


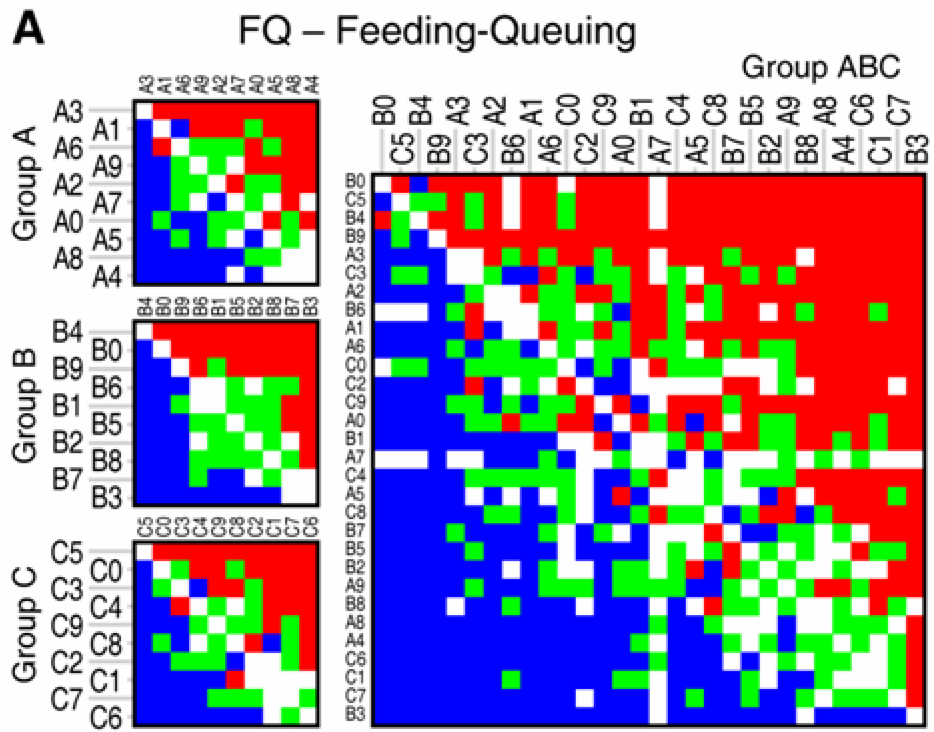
© 2011 Tracker development[1], Collective motion experiments[1,2]
[1] COLLMOT Research Project, Department of Biological Physics, Eötvös University
[2] OxNav Research Group, Department of Zoology, University of Oxford
2x speed
0 1 5 10 meters
0 5 10 25 feet

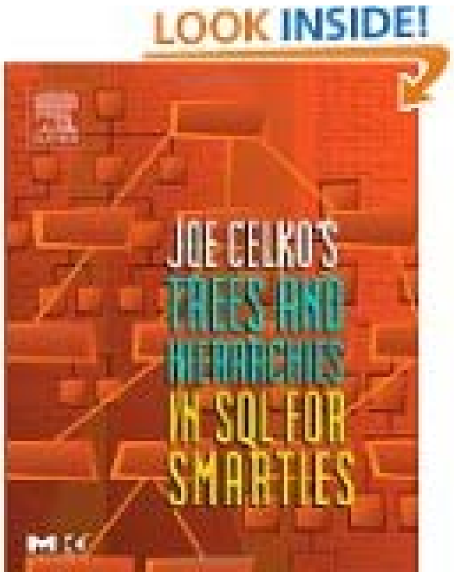

C5
B0
B9
A4
A2
A5
B3
C1
C8
C2
C6
C7
A8
A3
A7
C9
A6
C3
C0
B2
A9
B7
C4
A1
A0
B8
B4
B5

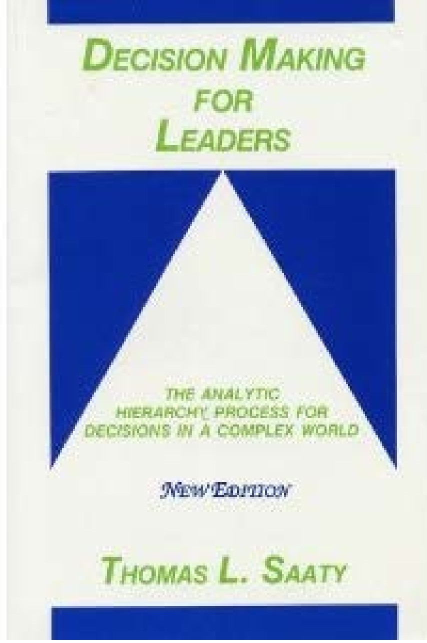
Correlation of interaction matrices is nearly zero:

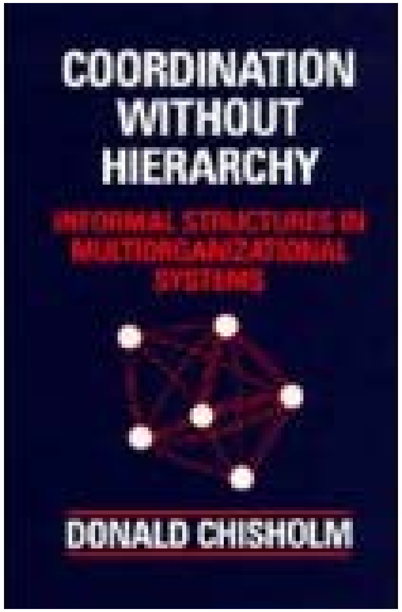
For pigeons the knowledge-based and the dominance hierarchies are independent



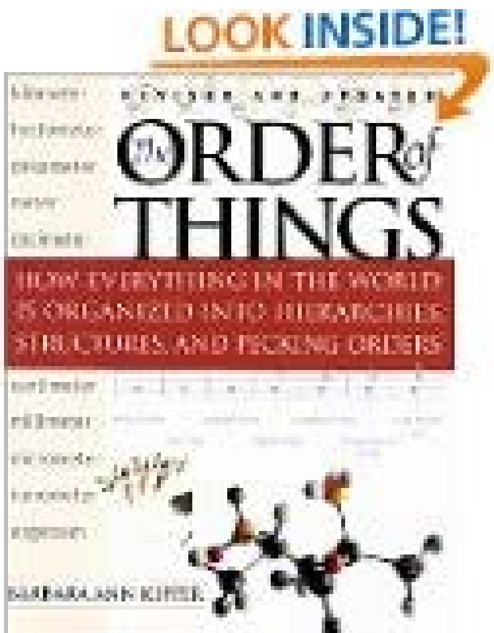


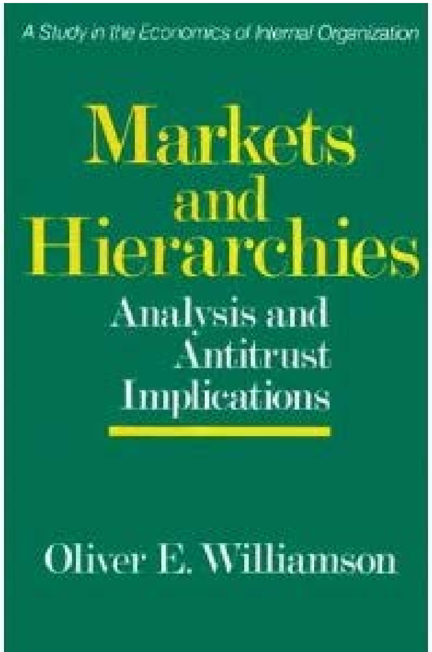


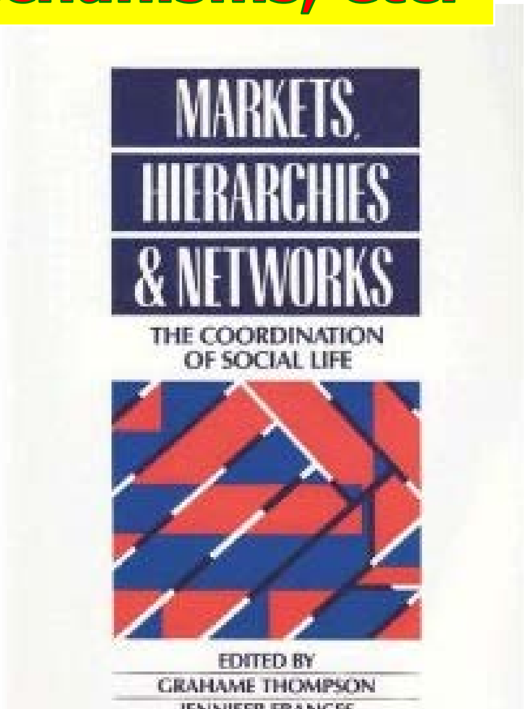




But we want numbers, universal mechanisms, etc.







Provocative statements

- Every real network is hierarchical

- Why? Because such a structure is more efficient (performs better)

- A flock of autonomous flying robots (drones)
  with two levels of hierarchy can be built

In particular:

- Group performance is maximized by a hierarchical competence distribution

- Hierarchical networks are more easily controllable using switchboard dynamics (controlling the edges

- Copying decisions from more competent „palyers” results in the spontaneous build-up of hierarchical leader-follower relationships

The case of optimal order hierarchy

Egalitarian? 2 levels (bimodal)? Many levels?

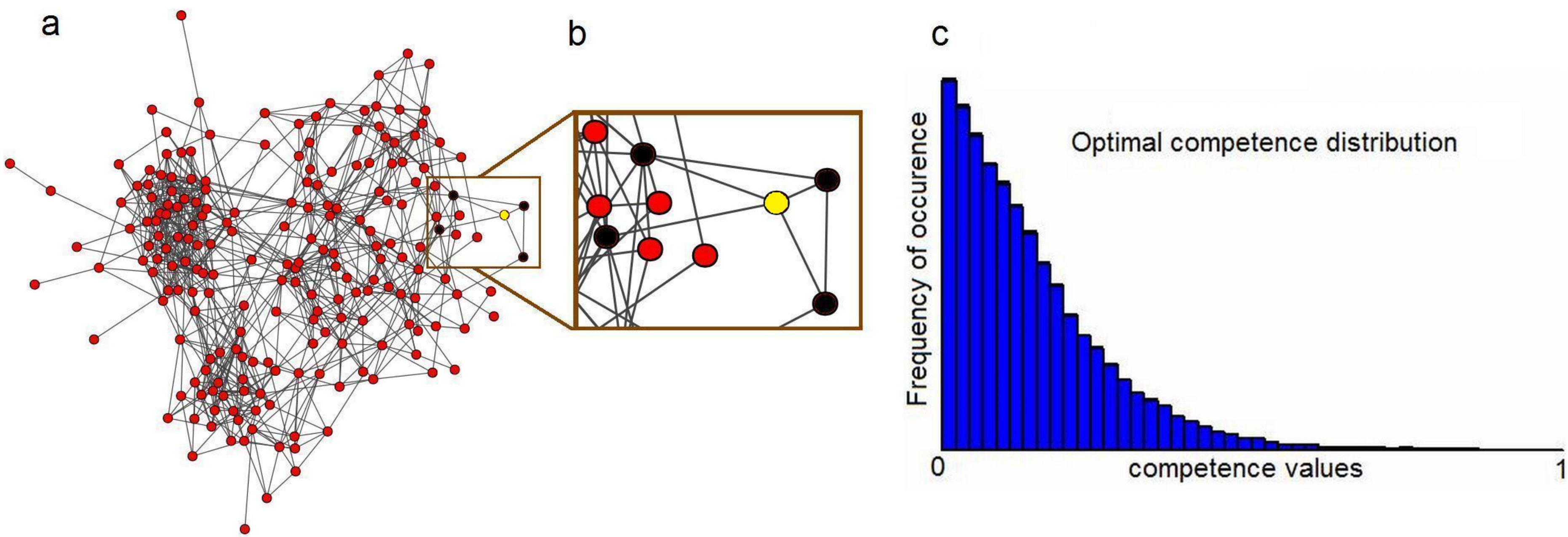
Group performance is maximized by a multi-level hierarchical competence distribution

with A. Zafeiris

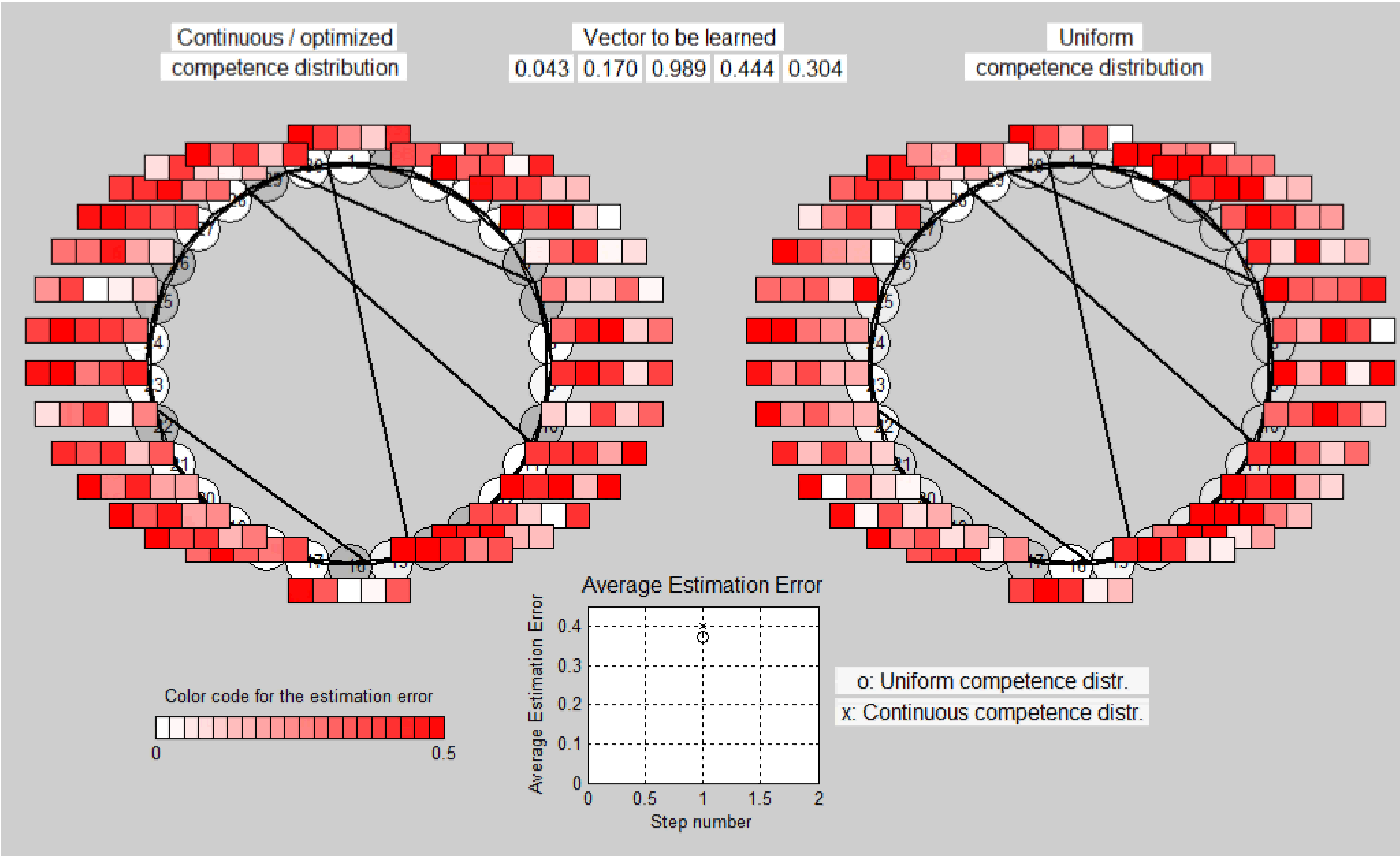
You have X dollars to spend on composing a group of n advisers delivering a collective decision on, e.g., where to invest

Voting model

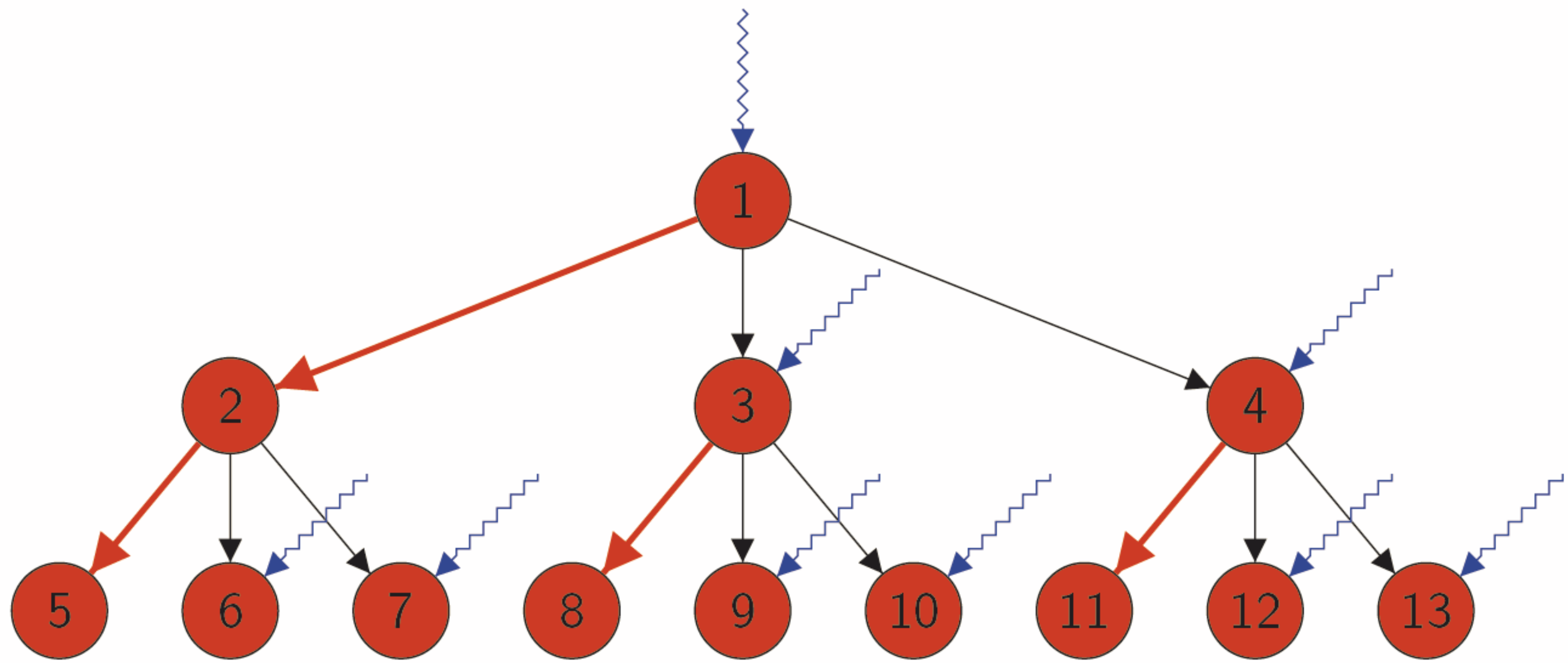
Simplest:
- **guess whether up or down is the true state**
- **ask nearest neighbours**
- **take majority vote**
- **make one more round**



Number sequence guessing game on a small world graph



Hierarchical networks require a very large number of driver nodes (counter-intuitive)



$$n_D \to 1 - \frac{1}{d}, \qquad n \to \infty$$

Linear edge dynamics in complex networks

With Tamás Nepusz

We want to control several edges leading out from a single node using a „switchboard"

$$\dot{\mathbf{y}}_i^+(t) = \mathbf{M}_i \mathbf{y}_i^-(t) - \boldsymbol{\tau}_i \otimes \mathbf{y}_i^+(t) + \sigma_i \mathbf{u}_i(t)$$

$\mathbf{y}_i^+(t)$	States of edges outgoing from node i
$\mathbf{M}_i$	Mixing or switching matrix
$\boldsymbol{\tau}_i$	Damping
$\mathbf{u}_i(t)$	State of the „driver node"

Edge control is more efficient for hierarchical networks

Proportion of nodes that have to be controlled

Table 1: Controllability properties of the real networks analysed in this pap

Type	#	Name	Nodes	Edges	n_D^{SBD}	n_D^{Liu}	n_D^{ER}
Regulatory	1.	Ownership-USCorp	7,253	6,726	0.160	0.820	0.339
	2.	TRN-EC-2	418	519	0.222	0.751	0.366
	3.	TRN-Yeast-1	4,441	12,873	0.034	0.965	0.415
	4.	TRN-Yeast-2	688	1,079	0.177	0.821	0.381
Trust	5.	College*	32	96	0.344	0.188	0.418
	6.	Epinions*	75,888	508,837	0.336	0.549	0.445
	7.	Prison*	67	182	0.403	0.134	0.411
	8.	Slashdot*	82,168	948,464	0.323	0.045	0.458
	9.	WikiVote*	7,115	103,689	0.281	0.666	0.463
Food web	10.	Grassland	88	137	0.318	0.523	0.381
	11.	Little Rock	183	2,494	0.639	0.541	0.463
	12.	Seagrass	49	226	0.449	0.265	0.436
	13.	Ythan	135	601	0.304	0.511	0.432

Emergence of hierarchical cooperation among non-cooperating individuals

With T. Nepusz

Task: guess the actual state of the environment

Essential model parameters:

n – the number of agents
p – the probability of a state flip in the environment
a_i – the ability of agent *i* (probability that it makes the right guess)

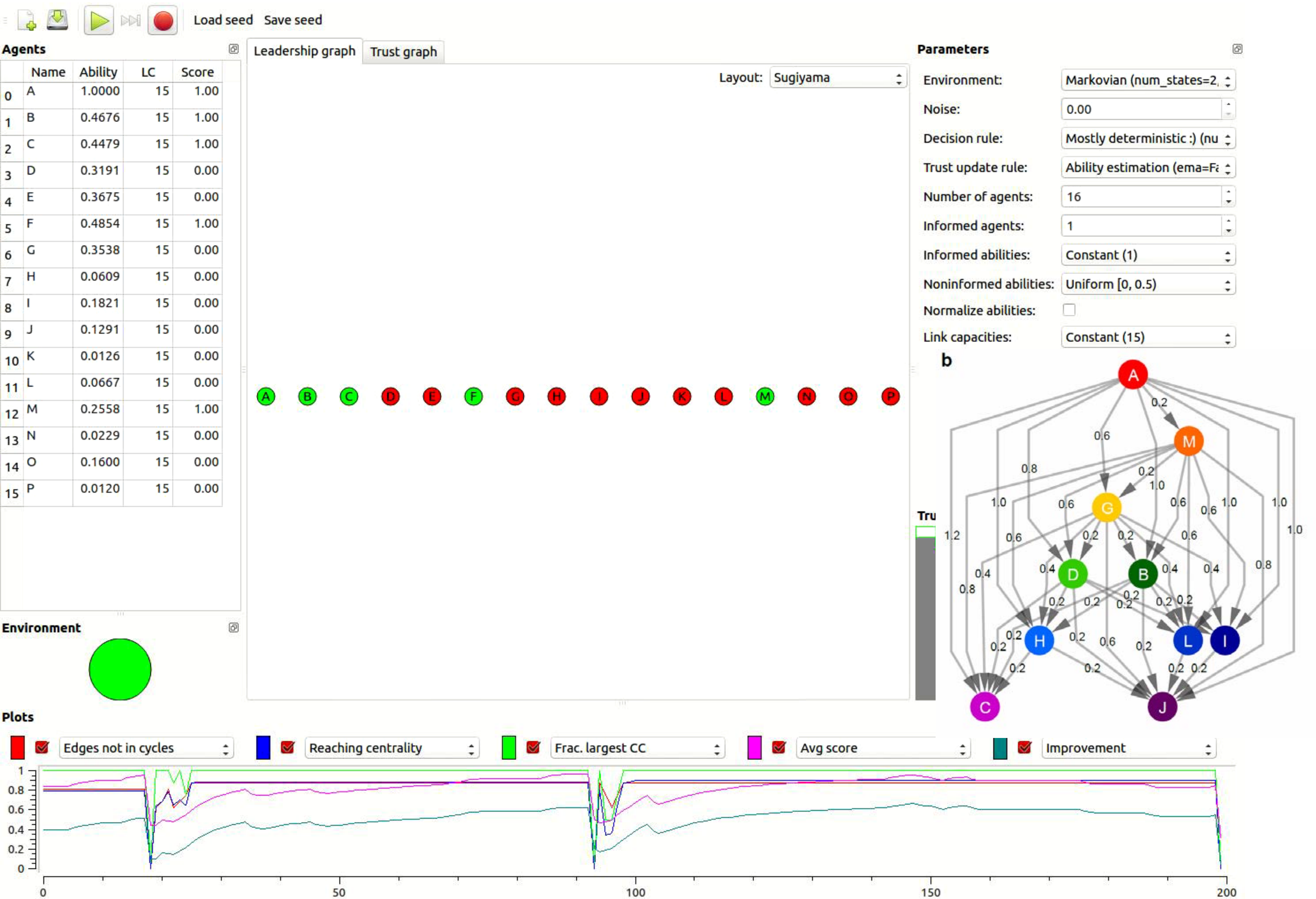
T – the „trust"matrix of abiliy estimates

individuals are trying to follow (interested in copying) the decisions of their more successful group mates (learn from them)

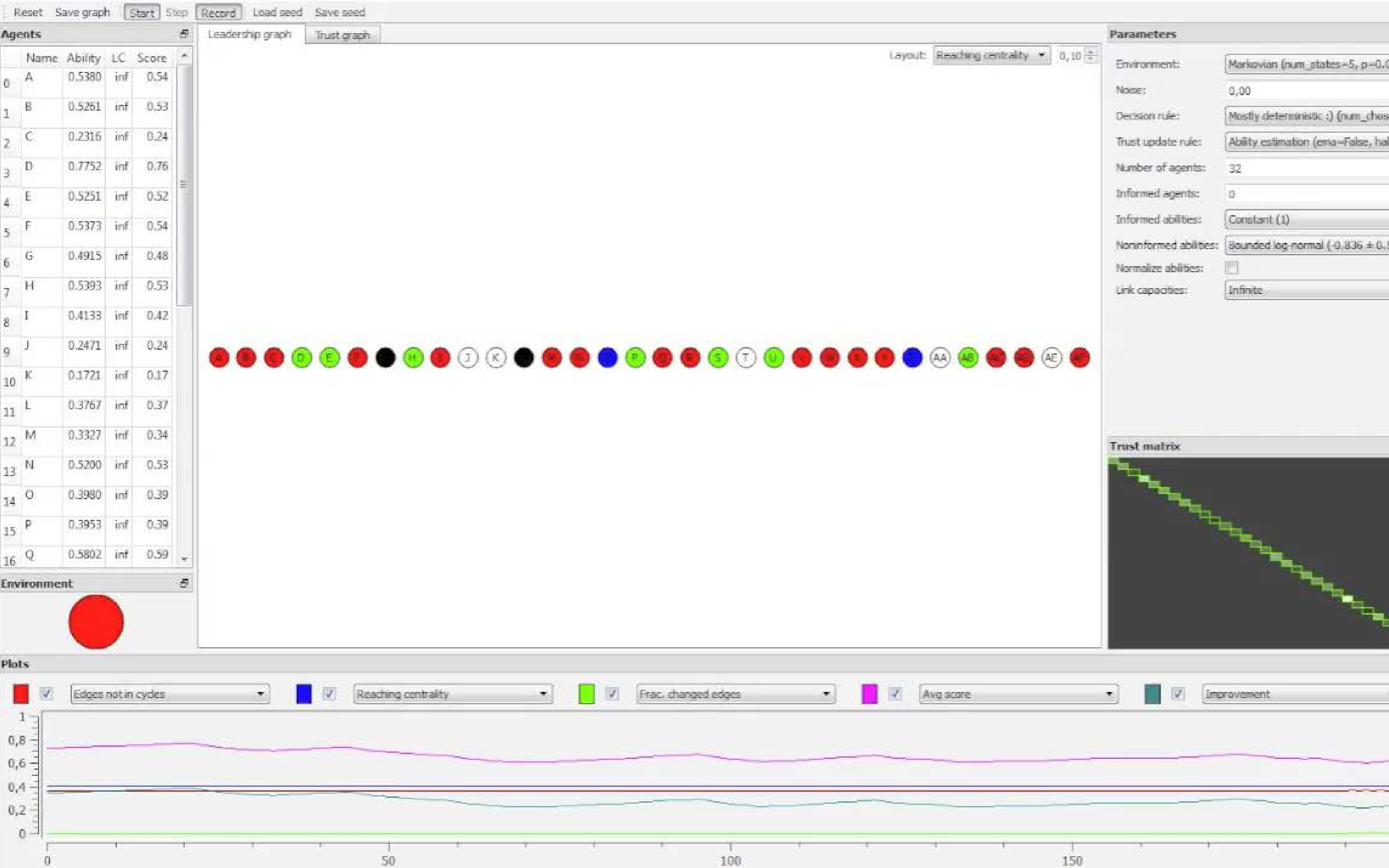
in proportion with the degree they trust the level of judgment of the other actors as compared to their own level of competence

The corresponding model results in the emergence of a collaboration structure in which the

leadership-followership relations manifest themselves in the form of a multi-level, directed hierarchical network.



Load seed
Save seed
Agents
Name
Ability
LC
Score
0 A 1.0000 15 1.00
1 B 0.4676 15 1.00
2 C 0.4479 15 1.00
3 D 0.3191 15 0.00
4 E 0.3675 15 0.00
5 F 0.4854 15 1.00
6 G 0.3538 15 0.00
7 H 0.0609 15 0.00
8 I 0.1821 15 0.00
9 J 0.1291 15 0.00
10 K 0.0126 15 0.00
11 L 0.0667 15 0.00
12 M 0.2558 15 1.00
13 N 0.0229 15 0.00
14 O 0.1600 15 0.00
15 P 0.0120 15 0.00
Environment
Leadership graph
Trust graph
Layout:
Sugiyama
Parameters
Environment:
Markovian (num_states=2,
Noise:
0.00
Decision rule:
Mostly deterministic :) (nu
Trust update rule:
Ability estimation (ema=Fa
Number of agents:
16
Informed agents:
1
Informed abilities:
Constant (1)
Noninformed abilities:
Uniform [0, 0.5)
Normalize abilities:
Link capacities:
Constant (15)
b
Plots
Edges not in cycles
Reaching centrality
Frac. largest CC
Avg score
Improvement
0
50
100
150
200



Reset
Save graph
Start
Step
Record
Load seed
Save seed
Agents
Name Ability LC Score
0 A 0.5380 inf 0.54
1 B 0.5261 inf 0.53
2 C 0.2316 inf 0.24
3 D 0.7752 inf 0.76
4 E 0.5251 inf 0.52
5 F 0.5373 inf 0.54
6 G 0.4915 inf 0.48
7 H 0.5393 inf 0.53
8 I 0.4133 inf 0.42
9 J 0.2471 inf 0.24
10 K 0.1721 inf 0.17
11 L 0.3767 inf 0.37
12 M 0.3327 inf 0.34
13 N 0.5200 inf 0.53
14 O 0.3980 inf 0.39
15 P 0.3953 inf 0.39
16 Q 0.5802 inf 0.59
Environment
Leadership graph
Trust graph
Layout:
Reaching centrality
0,10
Parameters
Environment:
Noise:
0,00
Decision rule:
Trust update rule:
Number of agents:
32
Informed agents:
0
Informed abilities:
Constant (1)
Noninformed abilities:
Normalize abilities:
Link capacities:
Infinite
Trust matrix
Plots
Edges not in cycles
Reaching centrality
Frac. changed edges
Avg score
Improvement
1
0,8
0,6
0,4
0,2
0
0
50
100
150

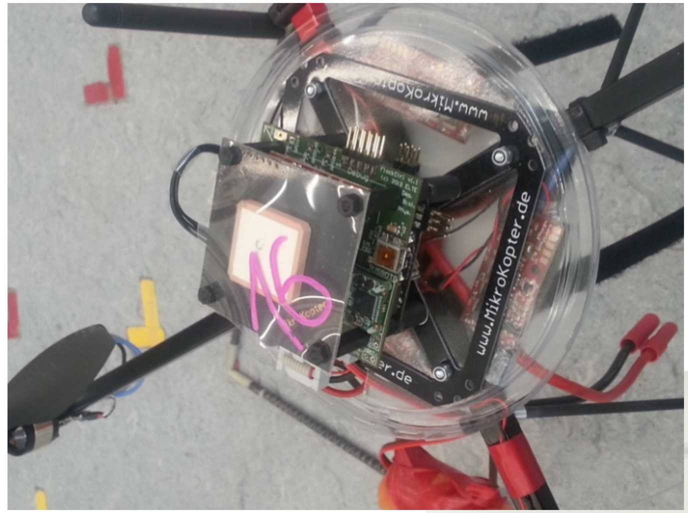
Hierarchy is gradually built up, with those with higher abilities „climbing to the top”

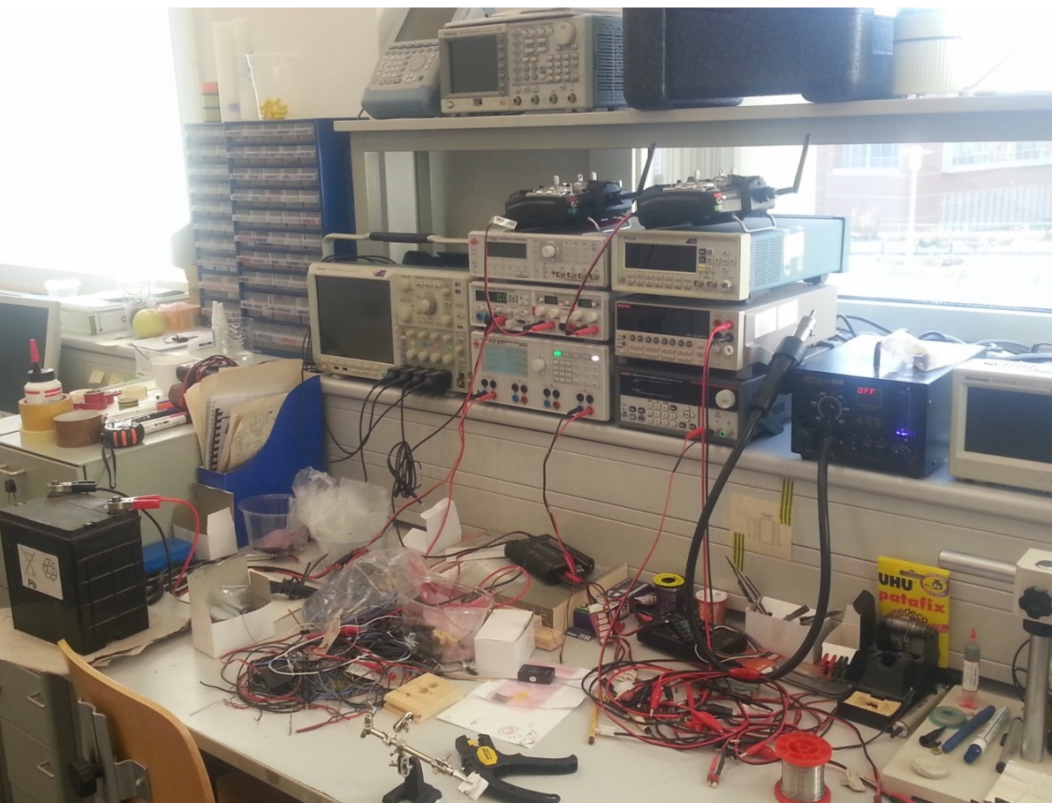
It is „metastable”, both adaptability and robustness are present

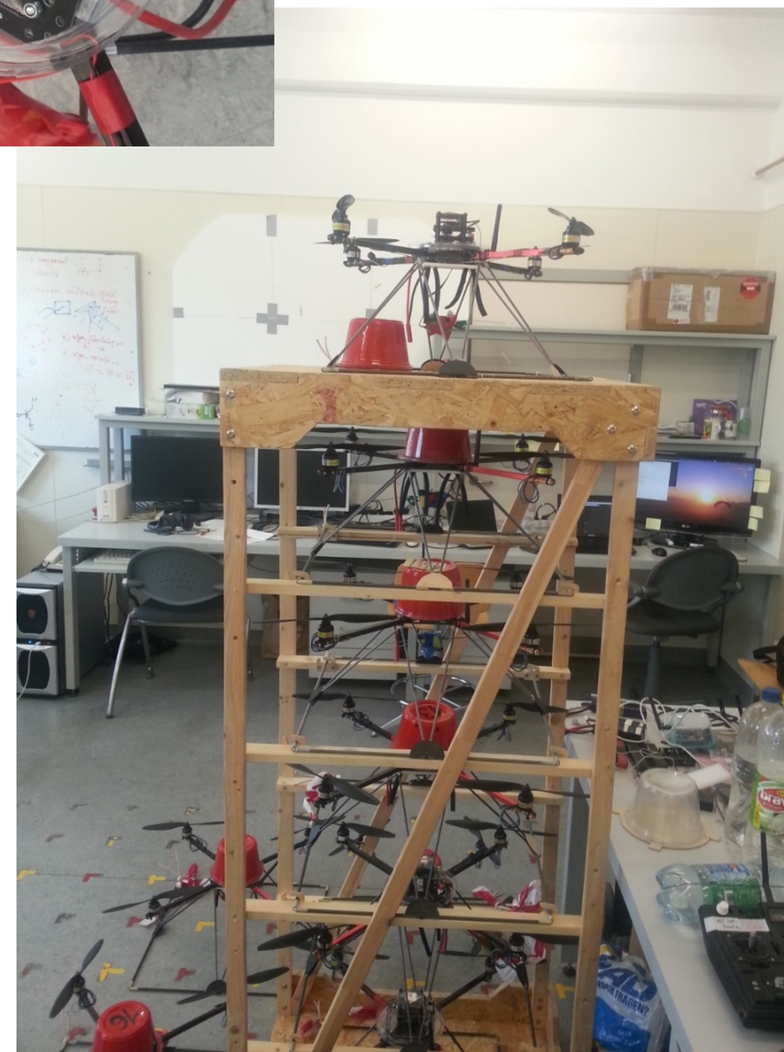
Overall performance is above the average of the individuals (information „flows down” efficiently”)

Next, we plan to apply hierarchical control to a flock of autonomous drones (quadcopters)

Our copters and lab

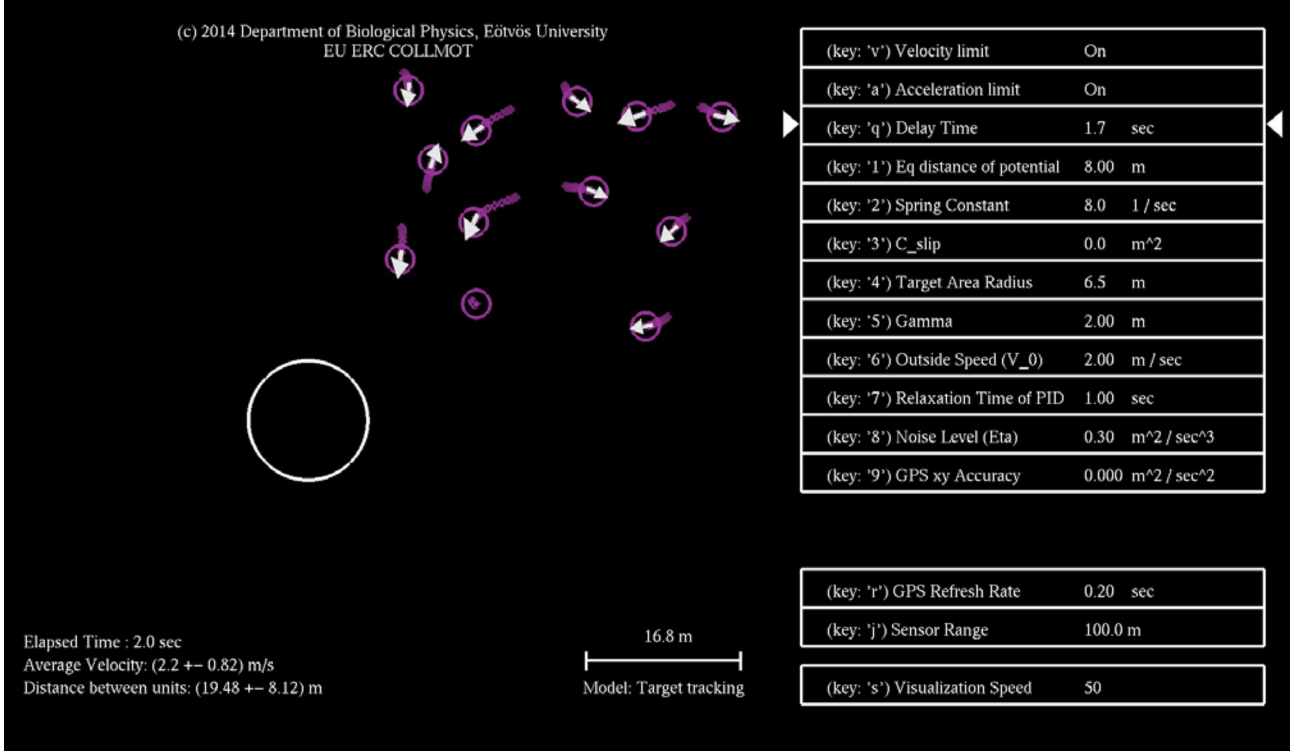


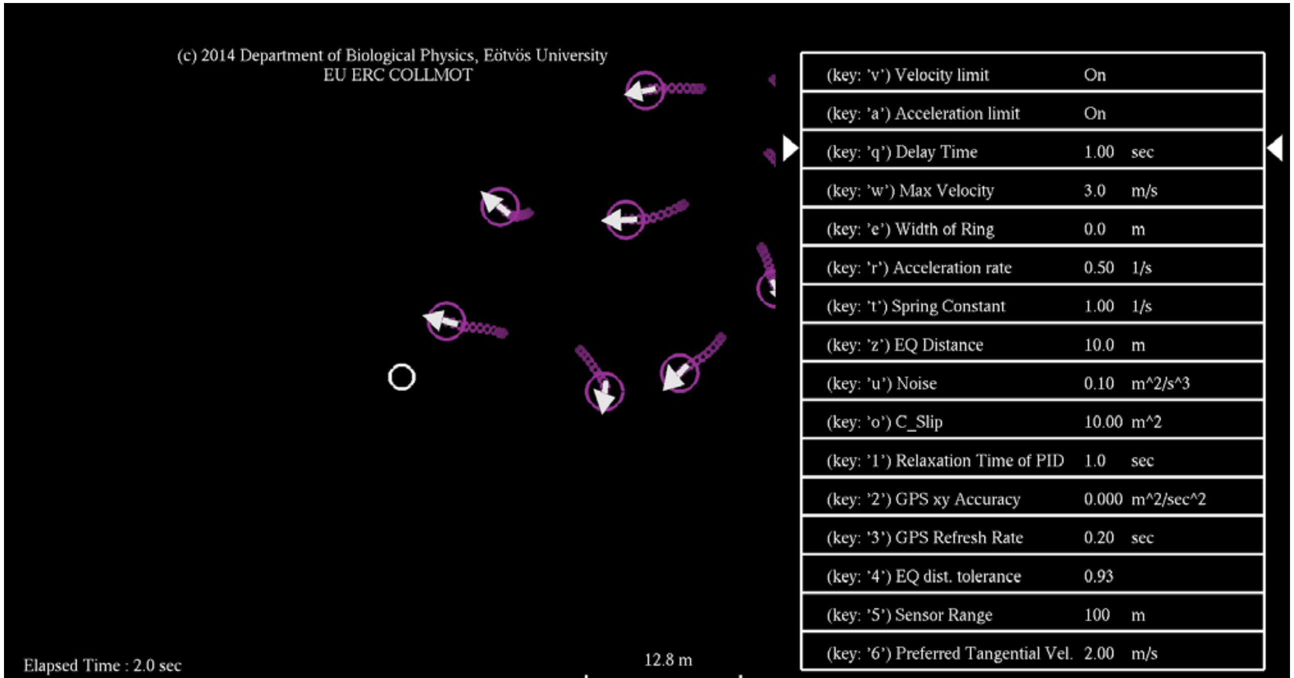




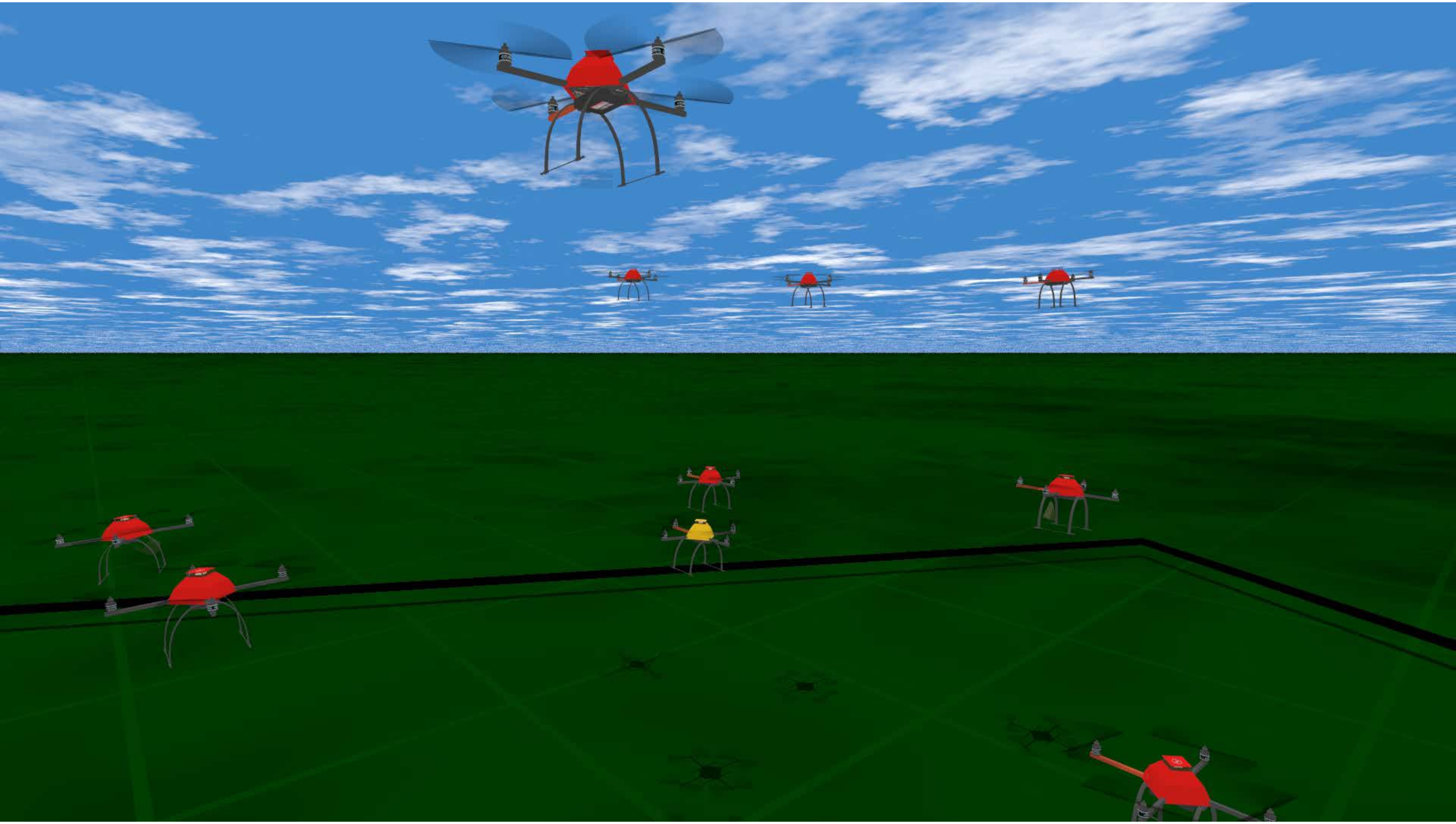
Copters:

Realistic simulations

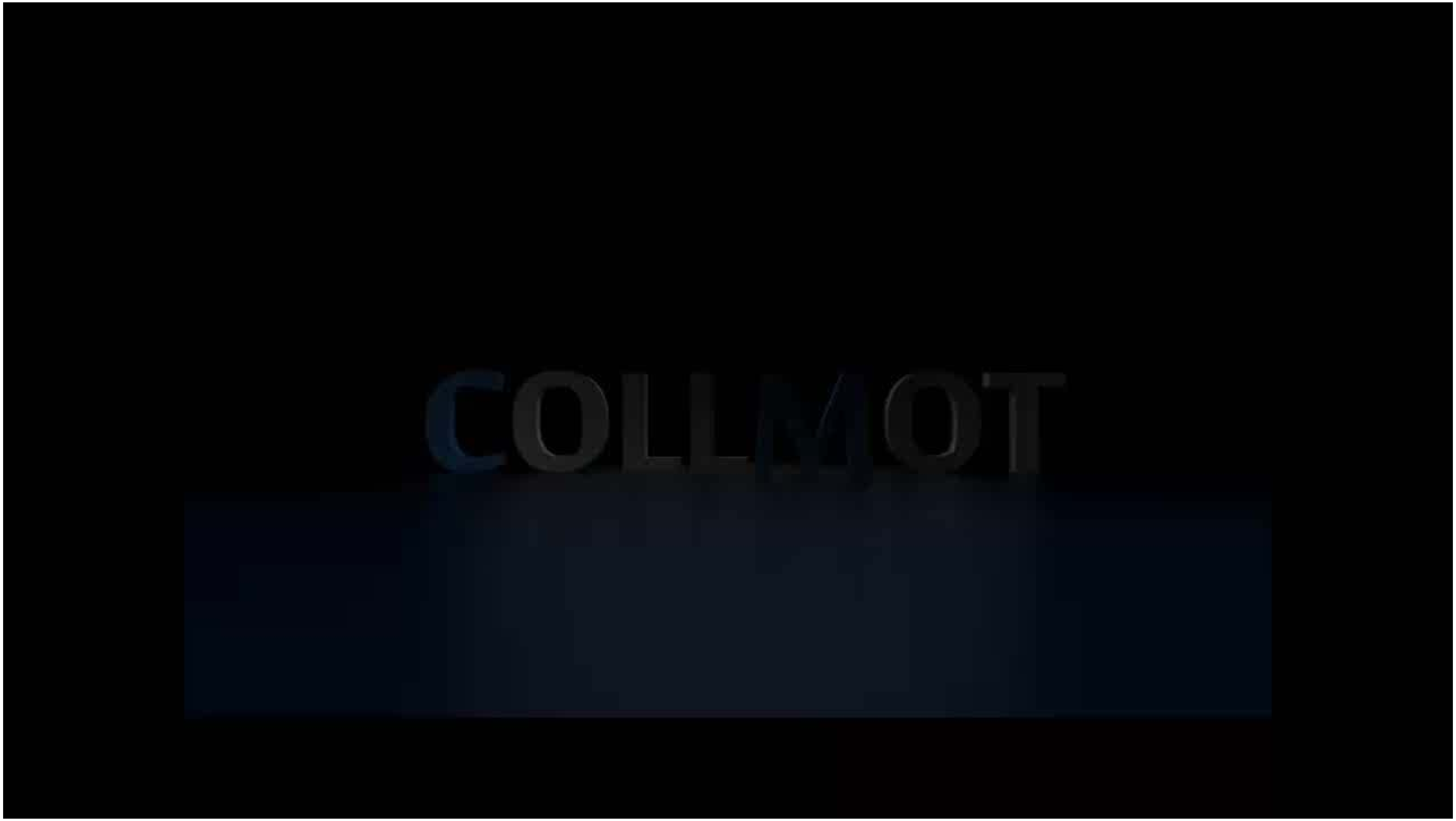




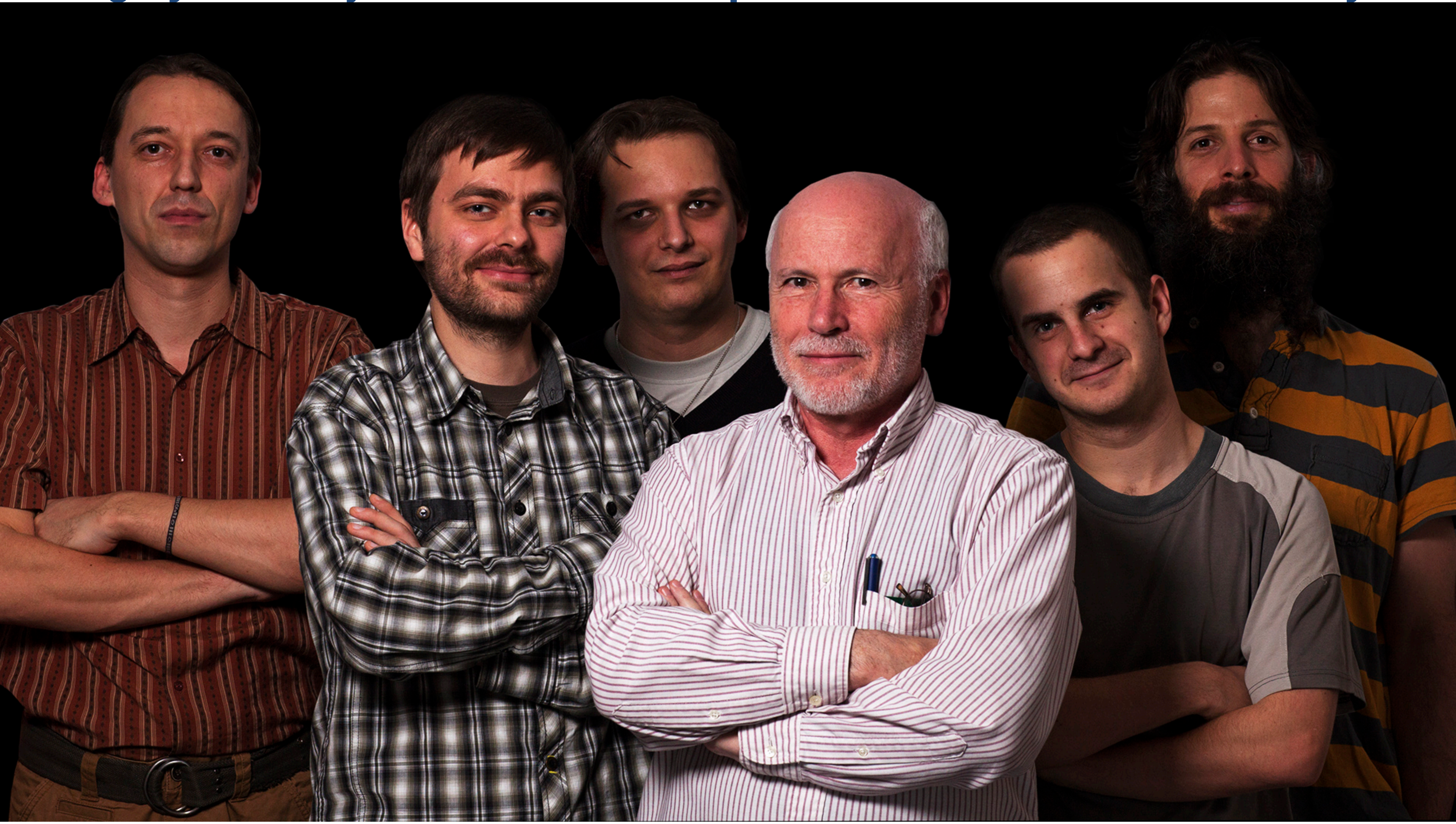
Copters: Animated actual GPS data



A few seconds from the video we produced

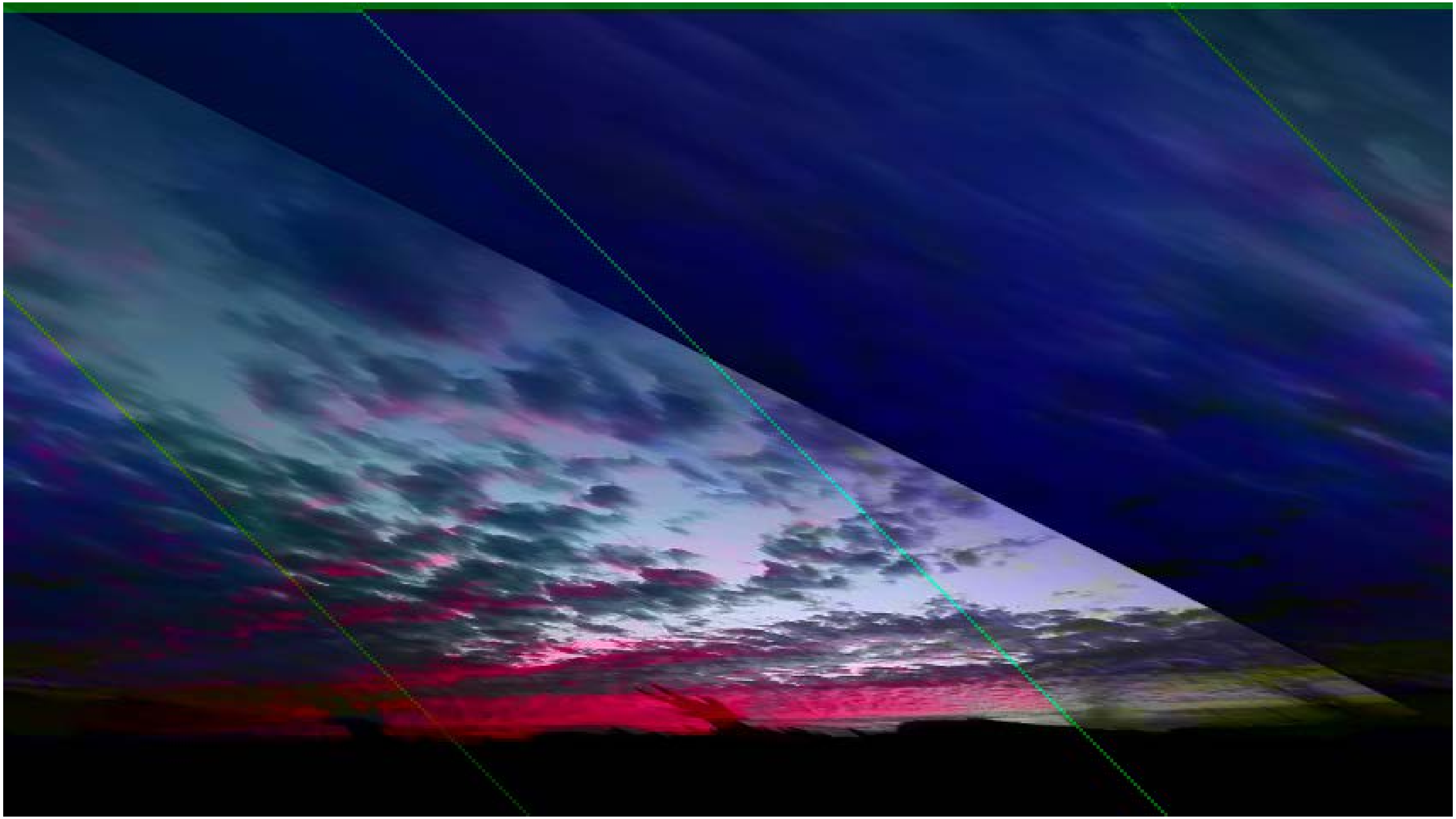


Gergely Somorjai Tamás Nepusz Gábor Vásárhelyi



Norbert Tarcai Tamás Vicsek Csaba Virágh

According to Monty Python Flying Circus….



Thank you for your attention

The rules can be formulated as follows:

1. First we define a changing environment (the state of which the individuals have to guess to gain benefit) as simple as possible, but still varying in an unpredictable way. The state of the environment is chosen to have a value of *1* or *0* with a probability *p.* Such a definition corresponds to a random walk with a characteristic time of changing its direction proportional to *1/p*

2. The individuals have a pre-defined ability (according to a given distribution with values between 0 and 1) to make a proper guess of the state of the environment. Their guess in each turn is based on their interactions with the agents they trust the most by making a weighted average of the decisions of the most trusted *k=1,2 ..m* friends/colleagues/players and his/her own estimate.

3. The trust matrix is thus used to update the guess of a player in the next round (the elements of this matrix correspond to the degree agent *i* trusts agent *j*). Individuals are trusted on the basis of their prior performance. More trusted agents are „listened to" more frequently. Naturally, the trust matrix is updated as the collective decision making process progresses.

4. A network is constructed based on the frequency of how many times agent *i* takes into account the guess of agent *j*.

(the actual realization/algorithm has a few more less relevant rules)