DATA-BASED MODELING:
System Identification in practice
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• Introduction
• System identification cycle
• Applications in the automotive industry
• Example
SYSTEM IDENTIFICATION
Basically, how to represent the reality in the virtual space as mathematical model based on measurements:

- Which can be used to predict/simulate the behavior of the reality in a much compact and cheaper way.

**SYSTEM IDENTIFICATION**
SYSTEM IDENTIFICATION

Modeling — Validation

Validation — Optimization

Optimization — Experiment

Experiment — Experiment Design

Experiment Design — Modeling
SYSTEM IDENTIFICATION

Modeling

Validation

Optimization

Experiment Design

Experiment
SYSTEM IDENTIFICATION

Modeling → Validation → Optimization → Experiment → Experiment Design
SYSTEM IDENTIFICATION

Modeling — Validation — Optimization

Experiment Design — Experiment
SYSTEM IDENTIFICATION

Modeling → Validation → Optimization → Experiment → Experiment Design → Modeling
- **Black-box**
  - Input/output behavior

- **White-box**
  - Model is created based on the field specific knowledge: differential equations, reaction equations, etc...

- **Frequency-domain representation**
  - Integral transformed linear differential equation.
  - Discrete-time models are applied usually for computational purposes.

- **Identifiability**
  - Which makes modeling the most crucial step in identification.
How to excite the system in order to get the most information possible?

- Can contain also optimization.
- Select the best input signal (magnitude, shape and frequency)
Perform the activity that is needed to gather the data for identification and validation.

- Highly process dependent task.
  - Physical measurements.
  - Big-data analysis.
  - ...

EXPERIMENT
The frontline application of numerical mathematics

- Classical parameter estimation using least squares for dynamic systems in time- and frequency domain.

\[ J(\theta) = \frac{1}{N} \sum_{k=0}^{N-1} \| y(k) - \hat{y}(k, \theta) \|^2 \]

- \( \hat{y}(k, \theta) \) can be generated using a large set of different models.

- Modern one-step method using subspace-based techniques.

\[
\begin{align*}
x_{k+1} &= Ax_k + Bu_k \\
y_k &= Cx_k + Du_k
\end{align*}
\]

\[
Y_d = GX_d + HU_d
\]

- Recursive subspace-based techniques.
Check if the model behaves similarly (within an error range of acceptance) as the reality does.

This step contains simulation of the estimated model based on experimental data gathered on the real system.
Permanent magnet synchronous motor parameter identification for motor control purposes:

\[
\begin{align*}
\mathbf{f} \left( \begin{bmatrix} i_d \\ i_q \\ \omega \end{bmatrix} \right) &= \begin{bmatrix} -\frac{R_i d_n p \omega}{L_q} i_q \\ -\frac{R_i q_n p \omega}{L_d} i_d - \frac{n_p \omega K_q}{L_q} \\ \frac{3n_p}{2j_{mot}} \left( i_q K_g + i_q i_d (L_d - L_q) \right) - \frac{b_{mot}}{j_{mot}} \omega \end{bmatrix} \\
\mathbf{g} \left( \begin{bmatrix} i_d \\ i_q \\ \omega \end{bmatrix} \right) &= \begin{bmatrix} \frac{1}{L_d} & 0 & 0 \\ 0 & \frac{1}{L_q} & 0 \\ 0 & 0 & -\frac{1}{j_{mot}} \end{bmatrix}
\end{align*}
\]
APPLICATION IN TKP

Electromechanical steering system model identification.
Nonlinear white box model identification for simulation purposes.
Models are sent to customers.
Linear white box model for controller and estimator design

\[
\dot{x} = A(\theta)x + B(\theta)u \\
y = C(\theta)x + D(\theta)u
\]
Dynamic equations:

\[ j_{up} \omega_{stw}(t) = T_{drv}(t) - T_{stc}(t) - T_{damp,up}(t) \]
\[ j_{dn} \omega_{pin}(t) = T_{stc}(t) + i_{gear} T_{mot}(t) + T_{load}(t) - T_{damp,dn}(t) \]
\[ \dot{T}_{mot}(t) = \frac{1}{T} (T_{mot,req}(t) - T_{mot}(t)) \]

Algebraic equations:

\[ T_{stc}(t) = c_{sensor} (\varphi_{stw}(t) - \varphi_{pin}(t)) + b_{sensor} (\omega_{stw}(t) - \omega_{pin}(t)) \]
\[ T_{damp,up}(t) = b_{up} \omega_{stw}(t) \]
\[ T_{damp,dn}(t) = b_{dn} \omega_{pin}(t) \]
\[ T_{TSU} = c_{sensor} (\varphi_{stw}(t) - \varphi_{pin}(t)) \]
\[ \omega_{mot} = i_{gear} \omega_{pin} \]

APPLICATION IN TKP
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- White-box vehicle model identification
  - Linear and nonlinear vehicle model in state-space form. These models are used to simulate dangerous vehicle movements.
  - Autonomous Driving Controllers and algorithms development use these vehicle models.

\[
\dot{x} = A(\theta)x + B(\theta)u \\
y = C(\theta)x + D(\theta)u
\]
- Black-box vehicle model identification
  - Models used for stability analysis in a complete software in the loop environment.
  - Nyquist stability criterion is used to prove the vehicle level stability.
- Permanent magnet synchronous motor parameter identification for motor control purposes:
  - Electro-mechanical steering system model identification.
    - White-box linear and nonlinear
  - White-box vehicle model identification
  - Black-box vehicle model identification

APPLICATION IN TKP
WHERE A MATHEMATICIAN CAN ENTER INTO THIS PICTURE

- Development and maintenance of the numerical optimization tools (bypassing matlab toolboxes)
- Performing identification, controller design and optimization.
- Processing the obtained simulation results.
- Analytical investigation of every aspects of the system:
  - Feasibility studies for controller and function design.
  - Mechanical change effect on the system behavior.
- Seeking for better optimization tools and methods.
- Model-based controller design for the steering system, autonomous driving.
THANKS FOR YOUR ATTENTION!
Equations of the nonlinear system

\[
\dot{x} = f(x, u) \\
y = g(x, u)
\]

LPV model

\[
\dot{x} = A_{LPV}(\theta)x + B_{LPV}(\theta)u \\
y = C_{LPV}(\theta)x + D_{LPV}(\theta)u
\]

Rational LPV model (LPV/LFR)

\[
\hat{M}(\theta) = \begin{bmatrix}
D_{zw} & C_z & D_{zu} \\
B_w & A_0 & B_0 \\
D_{yw} & C_0 & D_0
\end{bmatrix}
\]

\[
\Delta_p = \text{diag}(p_1 I_{r_1}, \ldots, p_{n_p} I_{r_{n_p}})
\]

\[
\begin{bmatrix}
A_{LPV}(\theta) & B_{LPV}(\theta) \\
C_{LPV}(\theta) & D_{LPV}(\theta)
\end{bmatrix} = \begin{bmatrix}
A_0 & B_0 \\
C_0 & D_0
\end{bmatrix} + \begin{bmatrix}
B_w \\
D_{yw}
\end{bmatrix}\Delta_p(I - D_{zw}\Delta_p)^{-1}\begin{bmatrix}
C_z \\
D_{zu}
\end{bmatrix}
\]
11 operating points are selected: \( \left[ \frac{\pi}{8} : \frac{\pi}{16} : \frac{6\pi}{8} \right] \); 
4x2 MIMO black-box local LTI models are estimated by using a subspace-based technique; 
The estimated models are validated locally (BFT %); 
A white-box 2x2 MIMO LPV model is estimated by using a frequency-domain interpolation method based on the 1,3,5,7,9,11th working points;
LPV MODEL IDENTIFICATION

Concatenation of the black-box LTI models.

Forzen LPV model.

\[
\min_{\theta} \sum_{i=1}^{N} \left\| G_{i_{BB}}^i(j\omega) - G_{LPV}(j\omega, p_i, \theta) \right\|_{\infty}^2
\]
GLOBAL VALIDATION

Real and analytical model outputs - BFT = 73.76 %

Real and analytical model outputs - BFT = 93.22 %

Time (s)
• L. Ljung: System Identification: Theory for the user
• P. Van Overschee, B. de Moor: Subspace identification for linear systems
• R. Tóth: Identification and Modeling of Linear Parameter-Varying Systems
• D. Luenberger: Optimization by vector space methods
• K. Pelckmans: Lecture Notes for a course on system identification.
• R. Pintelon, J. Shoukens: System identification: A frequency domain approach
THANKS FOR YOUR ATTENTION!