1. vizsga megoldásvázlata

5. (b)

6. $\mathbf{v} = (1, -1, -2)$, $\mathbf{a} = (1, 2, 3)$, így

$$\mathbf{v}_{\parallel} = \frac{\mathbf{a}\cdot\mathbf{v}}{\mathbf{a}\cdot\mathbf{a}}\mathbf{a} = \frac{-7}{14}\mathbf{a} = -\tfrac{1}{2}(1,2,3) = \left(-\tfrac{1}{2}, -1, -\tfrac{3}{2}\right),$$
$$\mathbf{v}_{\perp} = \mathbf{v} - \mathbf{v}_{\parallel} = (1,-1,-2) - \left(-\tfrac{1}{2}, -1, -\tfrac{3}{2}\right) = \left(\tfrac{3}{2}, 0, -\tfrac{1}{2}\right).$$

7. $$\begin{bmatrix} 2 & 1 & 3 & 0 \\ 4 & 3 & 5 & 2 \\ 3 & 2 & 4 & 1 \\ 3 & 1 & 5 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -2 & -4 & 2 \\ 0 & -1 & -2 & 1 \\ 0 & 1 & 2 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & -1 & -2 & 1 \\ 0 & 1 & 2 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Tehát a rang 2, így két lineárisan független vektor választható ki.

8. Mivel $1 - \sqrt{3}i = 2(\cos(-60°) + i\sin(-60°))$, így
$(1-\sqrt{3}i)^9 = 2^9(\cos(-540°) + i\sin(-540°)) = 2^9(\cos(180°) + i\sin(180°)) = -512$

9. $$f'_x(x,y) = 2xe^{xy} + x^2e^{xy}y \qquad f'_x(2,0) = 4$$
$$f'_y(x,y) = x^2e^{xy}x = x^3e^{xy} \qquad f'_y(2,0) = 8$$

A $\mathbf{v}$ vektort normalizáljuk: $\mathbf{e} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{(-1,3)}{\sqrt{1+9}} = \left(-\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right)$. Tehát:

$$f'_{\mathbf{e}}(0,3) = (4,8)\cdot\left(-\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right) = \frac{-4+24}{\sqrt{10}} = \frac{20}{\sqrt{10}} = 2\sqrt{10} \approx 6{,}32$$

10. $F(x,y,\lambda) = 6x - 4y + 5 + \lambda(x^2 + 2y^2 - 11)$ Lagrange-függvénnyel:

$$F'_x(x,y,\lambda) = 6 + 2x\lambda \quad \Rightarrow \quad x = -\tfrac{3}{\lambda}$$
$$F'_y(x,y,\lambda) = -4 + 4y\lambda \quad \Rightarrow \quad y = \tfrac{1}{\lambda}$$
$$F'_\lambda(x,y,\lambda) = x^2 + 2y^2 - 11 \quad \Rightarrow \tfrac{9}{\lambda^2} + 2\cdot\tfrac{1}{\lambda^2} = 11,$$

Az utolsóból $\lambda^2 = 1$, azaz $\lambda = \pm 1$. Két stacionárius pont van: $(-3,1,1)$ és $(3,-1,-1)$. A Hesse-determináns:

$$\begin{vmatrix} 2\lambda & 0 & 2x \\ 0 & 4\lambda & 4y \\ 2x & 4y & 0 \end{vmatrix} = 0 - 16x^2\lambda - 32y^2\lambda = -16\lambda(x^2 + 2y^2) = -16\lambda\cdot 11,$$

a $(-3,1,1)$ pontban negatív, így ez felt. lok. min.hely, értéke: $f(-3,1) = -17$;
a $(3,-1,-1)$ pontban pozitív, így ez felt. lok. max.hely, értéke: $f(3,-1) = 27$.

11. $$\sum_{n=1}^{\infty}\frac{3^{n+1} - 2^{2n-1}}{5^{n+3}} = \sum_{n=1}^{\infty}\frac{3\cdot 3^n}{5^3\cdot 5^n} - \sum_{n=1}^{\infty}\frac{2^{2n}/2}{5^3\cdot 5^n} = \sum_{n=1}^{\infty}\frac{3}{125}\left(\frac{3}{5}\right)^n - \sum_{n=1}^{\infty}\frac{1}{250}\left(\frac{4}{5}\right)^n =$$
$$= \frac{3}{125}\cdot\frac{\frac{3}{5}}{1-\frac{3}{5}} - \frac{1}{250}\cdot\frac{\frac{4}{5}}{1-\frac{4}{5}} = \frac{3}{125}\cdot\frac{3}{2} - \frac{1}{250}\cdot 4 = \frac{5}{250} = \frac{1}{50} = 0{,}02$$