3. vizsga megoldásvázlata

5. (c)

6.
$$\left.\begin{aligned}\mathbf{v}_1 &= (1,2,0)\\ \mathbf{v}_2 &= (3,1,1)\end{aligned}\right\}\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2 = (2,-1,-5),$$

így a sík egyenlete $2x - y - 5z = -1$. A $(2,3,1)$ pont távolsága:

$$\left|\frac{2\cdot 2 - 3 - 5\cdot 1 + 1}{\sqrt{2^2+1^2+5^2}}\right| = \left|\frac{-3}{\sqrt{30}}\right| = \frac{\sqrt{3}}{\sqrt{10}} \approx 0{,}548$$

7.
$$\left[\begin{array}{cccc|c} 3 & 4 & 2 & 5 & 9\\ 1 & 2 & 0 & 3 & 1\\ 2 & 1 & 3 & 0 & 11\end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & 0 & 3 & 1\\ 3 & 4 & 2 & 5 & 9\\ 2 & 1 & 3 & 0 & 11\end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & 0 & 3 & 1\\ 0 & -2 & 2 & -4 & 6\\ 0 & -3 & 3 & -6 & 9\end{array}\right] \sim$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 3 & 1\\ 0 & 1 & -1 & 2 & -3\\ 0 & -3 & 3 & -6 & 9\end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & 0 & 3 & 1\\ 0 & 1 & -1 & 2 & -3\\ 0 & 0 & 0 & 0 & 0\end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 7\\ 0 & 1 & -1 & 2 & -3\end{array}\right]$$

x_3 és x_4 szabad paraméter, a megoldás:
$$\begin{aligned} x_1 &= 7 - 2x_3 + x_4\\ x_2 &= -3 + x_3 - 2x_4\end{aligned} \qquad x_3, x_4 \in \mathbb{R}$$

8.
$$\frac{z_1}{z_2} = \frac{21+i}{5-3i} = \frac{21+i}{5-3i}\cdot\frac{5+3i}{5+3i} = \frac{105+63i+5i+3i^2}{25+9} = \frac{102+68i}{34} = 3+2i$$
$$\frac{z_1}{z_2} + 2\overline{z_1} + |z_2| = 3 + 2i + 2\cdot(21-i) + \sqrt{34} = 45 + \sqrt{34} \approx 50{,}83$$

9.
$$\begin{aligned} f'_x(x,y) &= ye^x + (x+2)ye^x & \qquad f'_x(0,3) &= 9\\ f'_y(x,y) &= (x+2)e^x & \qquad f'_y(0,3) &= 2\end{aligned}$$

$$\mathbf{e} = (\cos\alpha, \sin\alpha) = \left(-\tfrac{1}{2}, \tfrac{\sqrt{3}}{2}\right)$$

$$f'_\alpha(0,3) = (9,2)\cdot\left(-\tfrac{1}{2}, \tfrac{\sqrt{3}}{2}\right) = \sqrt{3} - \tfrac{9}{2} \approx -2{,}768$$

10. Ha $a, b, c > 0$ az oldalhosszak, akkor $abc = 1$, azaz $c = \frac{1}{ab}$, és így
$$f(a,b) = ab + 4ac + 2bc = ab + \tfrac{4}{b} + \tfrac{2}{a}$$
$$\begin{aligned} f'_a(a,b) &= b - \tfrac{2}{a^2} & \qquad a^2b &= 2\\ f'_b(a,b) &= a - \tfrac{4}{b^2} & \qquad ab^2 &= 4\end{aligned}$$

Összeszorozva $a^3b^3 = 8$, amiből $ab = 2$, és így $a = 1$ és $b = 2$. Ekkor $c = 1/2$. Hesse-determináns:
$$\begin{vmatrix} 4/a^3 & 1\\ 1 & 8/b^3\end{vmatrix} = \frac{32}{a^3b^3} - 1 = 3$$
a stacionárius pontban, ami lokális minimum ($4/a^3 > 0$).

11.
$$\sum_{n=0}^{\infty}\frac{5^{n+2} - 2^{3n+1}}{3^{2n}} = \sum_{n=0}^{\infty}\frac{5^2\cdot 5^n}{(3^2)^n} - \sum_{n=0}^{\infty}\frac{2\cdot(2^3)^n}{(3^2)^n} = \sum_{n=0}^{\infty} 25\left(\frac{5}{9}\right)^n - \sum_{n=0}^{\infty} 2\left(\frac{8}{9}\right)^n =$$
$$= \frac{25}{1-\frac{5}{9}} - \frac{2}{1-\frac{8}{9}} = \frac{225}{4} - 18 = 38{,}25$$