## 5. vizsga megoldásvázlata

5. (c)

6. $\left.\begin{array}{l}\overrightarrow{AB}=(-1,3,1)\\ \overrightarrow{AC}=(2,1,2)\end{array}\right\}\overrightarrow{AB}\times\overrightarrow{AC}=(5,4,-7),$ így a sík egyenlete $5x+4y-7z=19$.

7.
$$\left[\begin{array}{cccc|c}2&1&3&2&2\\3&2&5&3&3\\2&4&0&6&8\\4&7&3&7&12\end{array}\right]\sim\left[\begin{array}{cccc|c}1&2&0&3&4\\3&2&5&3&3\\2&1&3&2&2\\4&7&3&7&12\end{array}\right]\sim\left[\begin{array}{cccc|c}1&2&0&3&4\\0&-4&5&-6&-9\\0&-3&3&-4&-6\\0&-1&3&-5&-4\end{array}\right]\sim$$
$$\left[\begin{array}{cccc|c}1&2&0&3&4\\0&1&-3&5&4\\0&-3&3&-4&-6\\0&-4&5&-6&-9\end{array}\right]\sim\left[\begin{array}{cccc|c}1&2&0&3&4\\0&1&-3&5&4\\0&0&-6&11&6\\0&0&-7&14&7\end{array}\right]\sim\left[\begin{array}{cccc|c}1&2&0&3&4\\0&1&-3&5&4\\0&0&1&-3&-1\\0&0&-7&14&7\end{array}\right]\sim$$
$$\left[\begin{array}{cccc|c}1&2&0&3&4\\0&1&-3&5&4\\0&0&1&-3&-1\\0&0&0&-7&0\end{array}\right]\sim\left[\begin{array}{cccc|c}1&2&0&0&4\\0&1&-3&0&4\\0&0&1&0&-1\\0&0&0&1&0\end{array}\right]\sim\left[\begin{array}{cccc|c}1&0&0&0&2\\0&1&0&0&1\\0&0&1&0&-1\\0&0&0&1&0\end{array}\right]$$

Tehát $x=2, y=1, z=-1, w=0$.

8.
$$\frac{z_1}{z_2}=\frac{5+i}{3-2i}=\frac{5+i}{3-2i}\cdot\frac{3+2i}{3+2i}=\frac{15+3i+10i+2i^2}{9+4}=\frac{13+13i}{13}=1+i$$
$$\frac{z_1}{z_2}+\overline{z_1}+|z_1|=1+i+(5-i)+\sqrt{26}=6+\sqrt{26}\approx 11{,}1$$

9.
$$f'_x(x,y)=\ln(y^2)\qquad\qquad f'_x(3,1)=0$$
$$f'_y(x,y)=x\tfrac{1}{y^2}2y=\tfrac{2x}{y}\qquad\qquad f'_y(3,1)=6$$

A $\mathbf{v}$ vektort normalizáljuk: $\mathbf{e}=\frac{\mathbf{v}}{|\mathbf{v}|}=\frac{(2,-1)}{\sqrt{4+1}}=\left(\frac{2}{\sqrt5},-\frac{1}{\sqrt5}\right)$. Tehát:
$$f'_{\mathbf{e}}(3,1)=(0,6)\cdot\left(\tfrac{2}{\sqrt5},-\tfrac{1}{\sqrt5}\right)=-\tfrac{6}{\sqrt5}\approx 2{,}683$$

10. $F(x,y,\lambda)=x^2+xy+y^2+\lambda(x+2y-6)$ Lagrange-függvénnyel:
$$F'_x(x,y,\lambda)=2x+y+\lambda\quad\Rightarrow\quad 2x+y+\lambda=0$$
$$F'_y(x,y,\lambda)=x+2y+2\lambda\quad\Rightarrow\quad x+2y+2\lambda=0$$
$$F'_\lambda(x,y,\lambda)=x+2y-6\quad\Rightarrow\quad x+2y=6,$$

Az utolsó kettőből $\lambda=-3$, amiből $x+y=3$, és így $x=0, y=3$. $(0,3,-3)$ az egyetlen stacionárius pont. A Hesse-determináns:
$$\begin{vmatrix}2&1&1\\1&2&2\\1&2&0\end{vmatrix}=0+2+2-2-0-8=-6<0,$$

azaz a stacionárius pont feltételes lokális minimumhely, értéke: $f(0,3)=9$.

11.
$$\sum_{n=1}^{\infty}\frac{3^n-2^{2n+3}}{7^{n-1}}=\sum_{n=1}^{\infty}\frac{3^n}{7^n/7}-\sum_{n=1}^{\infty}\frac{(2^2)^n\cdot 8}{7^n/7}=\sum_{n=1}^{\infty}7\left(\frac{3}{7}\right)^n-\sum_{n=1}^{\infty}56\left(\frac{4}{7}\right)^n=$$
$$=7\frac{\frac{3}{7}}{1-\frac{3}{7}}-56\frac{\frac{4}{7}}{1-\frac{4}{7}}=7\cdot\frac{3}{4}-56\cdot\frac{4}{3}=-\frac{833}{12}\approx -69{,}417$$