3. vizsga megoldásvázlata

5. (b)

6. $\mathbf{b} = \overrightarrow{AB} = (1,0,2), \qquad \mathbf{c} = \overrightarrow{AC} = (1,-1,-1), \qquad \mathbf{d} = \overrightarrow{AD} = (-1,1,4)$

A vegyes szorzatuk: $\mathbf{bcd} = (\mathbf{b}\times\mathbf{c})\cdot\mathbf{d} = (2,3,-1)\cdot(-1,1,4) = -3$. A tetraéder térfogata hatoda ennek az abszolút értékének, azaz $\frac{1}{2}$ a tetraéder térfogata.

7. $$\left[\begin{array}{cccc|c} 3 & 2 & 1 & 0 & 1 \\ 1 & 1 & 2 & 3 & 2 \\ 2 & 3 & 1 & 4 & 3 \\ -1 & 1 & 0 & 4 & p \end{array}\right] \overset{s_1\leftrightarrow s_2}{\sim} \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 2 \\ 3 & 2 & 1 & 0 & 1 \\ 2 & 3 & 1 & 4 & 3 \\ -1 & 1 & 0 & 4 & p \end{array}\right] \underset{\substack{s_3-2s_1\\ s_4+s_1}}{\overset{s_2-3s_1}{\sim}} \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 2 \\ 0 & -1 & -5 & -9 & -5 \\ 0 & 1 & -3 & -2 & -1 \\ 0 & 2 & 2 & 7 & p+2 \end{array}\right] \overset{s_2\leftrightarrow s_3}{\sim}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 2 \\ 0 & 1 & -3 & -2 & -1 \\ 0 & -1 & -5 & -9 & -5 \\ 0 & 2 & 2 & 7 & p+2 \end{array}\right] \underset{s_4-2s_2}{\overset{s_3+s_2}{\sim}} \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 2 \\ 0 & 1 & -3 & -2 & -1 \\ 0 & 0 & -8 & -11 & -6 \\ 0 & 0 & 8 & 11 & p+4 \end{array}\right] \overset{s_4+s_3}{\sim} \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 2 \\ 0 & 1 & -3 & -2 & -1 \\ 0 & 0 & -8 & -11 & -6 \\ 0 & 0 & 0 & 0 & p-2 \end{array}\right]$$

Ha $p \neq 2$, akkor nincs megoldás. Ha $p = 2$, akkor végtelen sok megoldás van.

8. $$f'_x(x,y) = \frac{-2y(x-3)-(1-2xy)}{(x-3)^2} = \frac{6y-1}{(x-3)^2} \qquad f'_x(2,1) = 5$$

$$f'_y(x,y) = \frac{-2x}{x-3} \qquad f'_y(2,1) = 4$$

Felhasználva, hogy $f(2,1) = 3$, az érintősík egyenlete: $z = 5(x-2)+4(y-1)+3$.

9. Felcseréljük az integrálás sorrendjét:

$$\int_0^1 \int_{2x}^2 \sin\left(y^2\right)\,\mathrm{d}y\,\mathrm{d}x = \int_0^2 \int_0^{y/2} \sin\left(y^2\right)\,\mathrm{d}x\,\mathrm{d}y = \int_0^2 \left[x\sin\left(y^2\right)\right]_{x=0}^{y/2}\,\mathrm{d}y =$$

$$= \int_0^2 \frac{y}{2}\sin\left(y^2\right)\,\mathrm{d}y = \int_0^2 \frac{2y\sin\left(y^2\right)}{4}\,\mathrm{d}y = \left[-\frac{\cos\left(y^2\right)}{4}\right]_{y=0}^2 = \frac{1-\cos 4}{4} \approx 0{,}413$$

10. $$\sum_{n=1}^\infty \frac{2^{2n+1}-5^n}{2^{3n-2}} = \sum_{n=1}^\infty \frac{2\cdot(2^2)^n-5^n}{(2^3)^n/4} = \sum_{n=1}^\infty 8\left(\frac{1}{2}\right)^n - \sum_{n=1}^\infty 4\left(\frac{5}{8}\right)^n =$$

$$= \frac{8\cdot\frac{1}{2}}{1-\frac{1}{2}} - \frac{4\cdot\frac{5}{8}}{1-\frac{5}{8}} = 8 - \frac{20}{3} = \frac{4}{3} \approx 1{,}33$$

11. $$\lim_{n\to\infty}\sqrt[n]{|a_n|} = \lim_{n\to\infty}\sqrt[n]{\frac{1}{(n+1)3^n}} = \lim_{n\to\infty}\frac{1}{\sqrt[n]{(n+1)3^n}} = \lim_{n\to\infty}\frac{1}{\sqrt[n]{n+1}\cdot 3} = \frac{1}{3},$$

így a konvergenciasugár 3, így a hatványsor $(-6,0)$ intervallumban konvergens.

$x=0$: $\displaystyle\sum_{n=0}^\infty \frac{(0+3)^n}{(n+1)3^n} = \sum_{n=0}^\infty \frac{1}{n+1}$, ami divergens.

$x=-6$: $\displaystyle\sum_{n=0}^\infty \frac{(-6+3)^n}{(n+1)3^n} = \sum_{n=0}^\infty \frac{(-3)^n}{(n+1)3^n} = \sum_{n=0}^\infty \frac{(-1)^n}{n+1}$, Leibniz-sor, konvergens.

A konvergenciaintervallum: $[-6,0)$.