4. vizsga megoldásvázlata

5. (b)

6. $$\mathbf{b} = \overrightarrow{AB} = (0,2,1), \qquad \mathbf{c} = \overrightarrow{AC} = (1,-2,-2)$$

$\mathbf{b}\times\mathbf{c} = (-2,1,-2)$, melynek hossza: $|\mathbf{b}\times\mathbf{c}| = \sqrt{2^2+1^2+2^2} = 3$. A háromszög területe ennek a fele, azaz $\frac{3}{2}$.

7. $$\begin{bmatrix} 2 & 3 & 1 & 0 \\ 3 & 4 & 2 & 1 \\ 4 & 5 & 3 & 2 \\ 3 & 5 & 1 & -1 \end{bmatrix} \overset{o_1 \leftrightarrow o_3}{\sim} \begin{bmatrix} 1 & 3 & 2 & 0 \\ 2 & 4 & 3 & 1 \\ 3 & 5 & 4 & 2 \\ 1 & 5 & 3 & -1 \end{bmatrix} \underset{\substack{s_3-3s_1 \\ s_4-s_1}}{\overset{s_2-2s_1}{\sim}} \begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -2 & -1 & 1 \\ 0 & -4 & -2 & 2 \\ 0 & 2 & 1 & -1 \end{bmatrix} \underset{o_3-2o_1}{\overset{o_2-3o_1}{\sim}}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -2 & -1 & 1 \\ 0 & -4 & -2 & 2 \\ 0 & 2 & 1 & -1 \end{bmatrix} \underset{s_4+s_2}{\overset{s_3-2s_2}{\sim}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -2 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \underset{o_3+o_4}{\overset{o_2+2o_4}{\sim}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Mivel kettő darab egyes maradt, így a rang 2, és így két lineárisan független vektor választható ki.

8. $$f'_x(x,y) = e^{x^2(1-y)}2x(1-y) \qquad f'_x(2,1) = 0$$
$$f'_y(x,y) = e^{x^2(1-y)}x^2\cdot(-1) \qquad f'_y(2,1) = -4$$

Így $\operatorname{grad} f(P) = (0,-4)$, és $\mathbf{e} = (\cos 150°, \sin 150°) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

$$f'_\alpha(P) = (0,-4)\cdot\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right) = -2$$

9. Polárkoordinátákat használunk, a határok: $0 \le r \le \sqrt{2}$ és $0 \le \varphi \le \frac{\pi}{4}$.

$$\int_0^{\sqrt{2}}\int_0^{\frac{\pi}{4}} \big((r\cos\varphi)(r\sin\varphi)^2 + r\cos\varphi\big)\, r\,\mathrm{d}\varphi\,\mathrm{d}r = \int_0^{\sqrt{2}}\int_0^{\frac{\pi}{4}} r^4\cos\varphi\sin^2\varphi + r^2\cos\varphi\,\mathrm{d}\varphi\,\mathrm{d}r =$$

$$\int_0^{\sqrt{2}}\left[r^4\frac{\sin^3\varphi}{3} + r^2\sin\varphi\right]_0^{\frac{\pi}{4}}\mathrm{d}r = \int_0^{\sqrt{2}}\frac{r^4}{6\sqrt{2}} + \frac{r^2}{\sqrt{2}}\,\mathrm{d}r = \left[\frac{r^5}{30\sqrt{2}} + \frac{r^3}{3\sqrt{2}}\right]_0^{\sqrt{2}} = \frac{2}{15} + \frac{2}{3} = \frac{4}{5}$$

10. $$a_n = \frac{\sqrt{n^2+1}-n}{\sqrt{n^2-1}-n} = \frac{\left(\sqrt{n^2+1}-n\right)\frac{\sqrt{n^2+1}+n}{\sqrt{n^2+1}+n}}{\left(\sqrt{n^2-1}-n\right)\frac{\sqrt{n^2-1}+n}{\sqrt{n^2-1}+n}} = \frac{\frac{n^2+1-n^2}{\sqrt{n^2+1}+n}}{\frac{n^2-1-n^2}{\sqrt{n^2-1}+n}} =$$

$$= \frac{\frac{1}{\sqrt{n^2+1}+n}}{\frac{-1}{\sqrt{n^2-1}+n}} = -\frac{\sqrt{n^2-1}+n}{\sqrt{n^2+1}+n} = -\frac{\sqrt{1-\frac{1}{n^2}}+1}{\sqrt{1+\frac{1}{n^2}}+1} \to -1$$

11. $$\frac{x}{x^2-9} = -\frac{x}{9}\cdot\frac{1}{1-\frac{x^2}{9}} = -\frac{x}{9}\sum_{n=0}^{\infty}\left(\frac{x^2}{9}\right)^n = \sum_{n=0}^{\infty}\frac{-1}{9^{n+1}}x^{2n+1} = -\frac{x}{9} - \frac{x^3}{81} - \frac{x^5}{729} - \dots,$$

ha $\left|\frac{x^2}{9}\right| < 1$, azaz $x^2 < 9$, azaz $|x| < 3$, így a konvergenciasugár 3.