5. vizsga megoldásvázlata

5. (d)

6. $\mathbf{v} = (2, 5, -5)$, $\mathbf{a} = (2, -1, 1)$

$$\mathbf{v}_{||} = \frac{\mathbf{a}\cdot\mathbf{v}}{\mathbf{a}\cdot\mathbf{a}}\mathbf{a} = \frac{-6}{6}(2,-1,1) = (-2,1,-1)$$
$$\mathbf{v}_{\perp} = \mathbf{v} - \mathbf{v}_{||} = (2,5,-5) - (-2,1,-1) = (4,4,-4)$$

7.
$$\left[\begin{array}{cccc|c} 2 & 8 & 8 & 0 & 24 \\ 1 & 3 & 2 & 1 & 9 \\ 3 & 6 & 0 & 6 & 18 \\ 3 & 7 & 2 & 5 & 21 \end{array}\right] \overset{s_1/2}{\sim} \left[\begin{array}{cccc|c} 1 & 4 & 4 & 0 & 12 \\ 1 & 3 & 2 & 1 & 9 \\ 3 & 6 & 0 & 6 & 18 \\ 3 & 7 & 2 & 5 & 21 \end{array}\right] \underset{\substack{s_3-3s_1 \\ s_4-3s_1}}{\overset{s_2-s_1}{\sim}} \left[\begin{array}{cccc|c} 1 & 4 & 4 & 0 & 12 \\ 0 & -1 & -2 & 1 & -3 \\ 0 & -6 & -12 & 6 & -18 \\ 0 & -5 & -10 & 5 & -15 \end{array}\right] \overset{s_2/(-1)}{\sim}$$
$$\left[\begin{array}{cccc|c} 1 & 4 & 4 & 0 & 12 \\ 0 & 1 & 2 & -1 & 3 \\ 0 & -6 & -12 & 6 & -18 \\ 0 & -5 & -10 & 5 & -15 \end{array}\right] \underset{s_4+5s_2}{\overset{s_3+6s_2}{\sim}} \left[\begin{array}{cccc|c} 1 & 4 & 4 & 0 & 12 \\ 0 & 1 & 2 & -1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \overset{s_1-4s_2}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & -4 & 4 & 0 \\ 0 & 1 & 2 & -1 & 3 \end{array}\right]$$

Tehát z és v szabad paraméter. $x = 4z - 4v$ és $y = 3 - 2z + v$.

8.
$$f'_x(x,y) = \ln(xy^2) + x\frac{1}{xy^2}y^2 = \ln(xy^2) + 1 \qquad f'_x(1,-1) = 1$$
$$f'_y(x,y) = x\frac{1}{xy^2}x\cdot 2y = \frac{2x}{y} \qquad f'_y(1,-1) = -2$$

$\text{grad} f(1,-1) = (1,-2)$. Az iránymenti derivált minimuma ennek hosszának ellentettje, azaz $-|\text{grad}(1,-1)| = -\sqrt{1+(-2)^2} = -\sqrt{5} \approx -2{,}24$.

9. A két görbe metszéspontja: $x^2 + 2 = -3x$, amiből $x_1 = -2$ és $x_2 = -1$. Így az integrál:

$$\int_{-2}^{-1}\int_{x^2+2}^{-3x} x^2 y \,\mathrm{d}y\,\mathrm{d}x = \int_{-2}^{-1}\left[x^2\frac{y^2}{2}\right]_{x^2+2}^{-3x}\mathrm{d}x = \int_{-2}^{-1} x^2\frac{(-3x)^2}{2} - x^2\frac{(x^2+2)^2}{2}\,\mathrm{d}x =$$
$$= \int_{-2}^{-1} -\frac{x^6}{2} + \frac{5}{2}x^4 - 2x^2\,\mathrm{d}x = \left[-\frac{x^7}{14} + \frac{1}{2}x^5 - \frac{2}{3}x^3\right]_{-2}^{-1} = \frac{37}{21} \approx 1{,}76$$

10.
$$\sum_{n=1}^{\infty}\frac{2^{2n-1}-3^n}{3^{2n+1}} = \sum_{n=1}^{\infty}\frac{2^{2n}/2 - 3^n}{3\cdot 3^{2n}} = \sum_{n=1}^{\infty}\frac{1}{6}\left(\frac{4}{9}\right)^n - \sum_{n=1}^{\infty}\frac{1}{3}\left(\frac{1}{3}\right)^n =$$
$$= \frac{1}{6}\cdot\frac{\frac{4}{9}}{1-\frac{4}{9}} - \frac{1}{3}\cdot\frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{1}{6}\cdot\frac{\frac{4}{9}}{\frac{5}{9}} - \frac{1}{3}\cdot\frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{6}\cdot\frac{4}{5} - \frac{1}{3}\cdot\frac{1}{2} = -\frac{1}{30} \approx 0,0333$$

11.
$$x\cos(3x) = x\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}(3x)^{2n} = \sum_{n=0}^{\infty}\frac{(-1)^n\cdot 3^{2n}}{(2n)!}x^{2n+1} = x - \frac{9}{2}x^3 + \frac{27}{8}x^5 - \dots$$

Mivel a $\cos x$ hatványsora minden $x \in \mathbb{R}$-re konvergens, így ez is, tehát a konvergenciasugár végtelen.