7. vizsga megoldásvázlata

5. (d)

6. $\mathbf{v} = (3,1,2), \qquad \mathbf{a} = (1,1,1)$
$\mathbf{v}_{||} = \frac{\mathbf{av}}{\mathbf{aa}}\mathbf{a} = \frac{6}{3}(1,1,1) = (2,2,2)$
$\mathbf{v}_{\perp} = \mathbf{v} - \mathbf{v}_{||} = (1,-1,0)$

7.

$$\left(\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ 3 & 9 & 6 & 15 \\ 4 & 5 & 6 & 6 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ 1 & 3 & 2 & 5 \\ 4 & 5 & 6 & 6 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 3 & 2 & 5 \\ 2 & 1 & 3 & 0 \\ 4 & 5 & 6 & 6 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 3 & 2 & 5 \\ 0 & -5 & -1 & -10 \\ 0 & -7 & -2 & -14 \end{array}\right) \sim$$
$$\left(\begin{array}{ccc|c} 1 & 3 & 2 & 5 \\ 0 & 1 & \frac{1}{5} & 2 \\ 0 & -7 & -2 & -14 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 3 & 2 & 5 \\ 0 & 1 & \frac{1}{5} & 2 \\ 0 & 0 & -\frac{3}{5} & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 3 & 2 & 5 \\ 0 & 1 & \frac{1}{5} & 2 \\ 0 & 0 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 3 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right) \sim$$
$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

Tehát $x = -1, y = 2, z = 0$.

8. $1 - i = \sqrt{2}(\cos(-45°) + i\sin(-45°))$, így

$$(1-i)^{12} = (\sqrt{2})^{12}(\cos(12\cdot(-45°)) + i\sin(12\cdot(-45°))) = 64(\cos(180°) + i\sin(180°)) = -64$$

9.

$$f(x) = \frac{x^2}{5}\cos\left(\frac{5x^2}{2}\right) = \frac{x^2}{5}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}\left(\frac{5x^2}{2}\right)^{2n} = \frac{x^2}{5}\sum_{n=0}^{\infty}\frac{(-25)^n}{4^n(2n)!}x^{4n} = \sum_{n=0}^{\infty}\frac{(-25)^n}{5\cdot 4^n(2n)!}x^{4n+2},$$

ami minden $x \in \mathbb{R}$ esetén konvergens.

10. A parciális deriváltak:
$f'_x(x,y) = e^{2xy} + xe^{2xy}2y$, ami a P pontban: $f'_x(2,0) = 1 + 0 = 1$
$f'_y(x,y) = xe^{2xy}2x$, ami a P pontban: $f'_y(2,0) = 2\cdot 1\cdot 4 = 8$
így a gradiens: $\operatorname{grad} f(P) = (1,8)$
$\mathbf{e} = \frac{\mathbf{v}}{|\mathbf{v}|} = \left(\frac{4}{5}, -\frac{3}{5}\right)$
$f'_{\mathbf{v}}(P) = \langle \operatorname{grad} f(P), \mathbf{e}\rangle = \frac{4}{5} - \frac{24}{5} = -4$
Az iránymenti derivált maximuma a gradiens vektor hossza:
$|\operatorname{grad} f(P)| = \sqrt{1^2 + 8^2} = \sqrt{65} \approx 8{,}06$.

11. A két görbe metszéspontja: $3x = x^2 - 4$ egyenlet megoldása, azaz $x_1 = -1$ és $x_2 = 4$. Így a kiszámítandó integrál:

$$\int_{-1}^{4}\int_{x^2-4}^{3x} 2xy\,\mathrm{d}y\,\mathrm{d}x = \int_{-1}^{4}\left[xy^2\right]_{y=x^2-4}^{3x}\mathrm{d}x = \int_{-1}^{4} x(3x)^2 - x(x^2-4)^2\,\mathrm{d}x = \int_{-1}^{4} -x^5 + 17x^3 - 16x\,\mathrm{d}x =$$
$$= \left[-\frac{x^6}{6} + 17\frac{x^4}{4} - 8x^2\right]_{-1}^{4} = -\frac{2048}{3} + 1088 - 128 - \left(-\frac{1}{6} + \frac{17}{4} - 8\right) = \frac{1125}{4} = 281{,}25$$