Végezzen függvényvizsgálatot a következő függvényeken! A vizsgálat tartalmazza az értelmezési tartomány, a zérushelyek és a derivált zérushelyeinek megtalálását, az értelmezési tartomány határában a függvény határértékei létezésének és értékének felderítését, a szélsőértékhelyek megkeresését, a függvény értékkészletének megadását, továbbá egy vázlatot a függvény grafikonjára.

$$\sqrt{x} - x$$
$$\arcsin x - \frac{\pi}{2}x$$
$$\frac{1}{x} + \sqrt{2x}$$
$$\log(1 - x^2)$$
$$e^{\sqrt{x}}$$
$$\log(1 + \sqrt{x})$$

Megoldások vázlata:

$$\begin{aligned} f(x) &= \sqrt{x} - x \\ x &\geq 0 \\ D_f &= [0, +\infty) \end{aligned}$$

$$\begin{aligned} \sqrt{x} - x &= 0 \\ x &= 0 \text{ vagy } 1 \end{aligned}$$

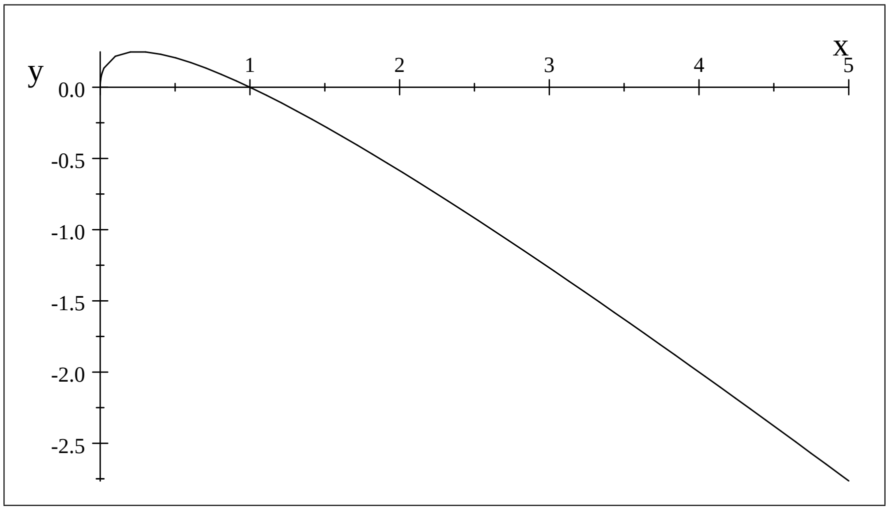
$$\begin{aligned} \frac{\partial}{\partial x}\left(\sqrt{x} - x\right) &= \frac{1}{2\sqrt{x}} - 1 \\ \frac{1}{2\sqrt{x}} - 1 &= 0 \\ x &= \frac{1}{4} \end{aligned}$$

$$\frac{\partial}{\partial x}\left(\frac{1}{2\sqrt{x}} - 1\right) = \frac{-1}{4\sqrt{x^3}} < 0$$

Maximum van 0.25-nél, $f(0.25) = 0.25$.

$$\begin{aligned} \lim_{x \to 0^+} \left(\sqrt{x} - x\right) &= 0 \\ \lim_{x \to +\infty} \left(\sqrt{x} - x\right) &= -\infty \end{aligned}$$

$$R_f = (-\infty, \frac{1}{4}]$$



$$
\begin{aligned}
f(x) &= \arcsin x - \pi x \\
-1 &\leq x \leq +1 \\
D_f &= [-1, +1]
\end{aligned}
$$

$$
\begin{aligned}
\arcsin x - \frac{\pi}{2}x &= 0 \\
x &= -1 \text{ vagy} \\
x &= +1
\end{aligned}
$$

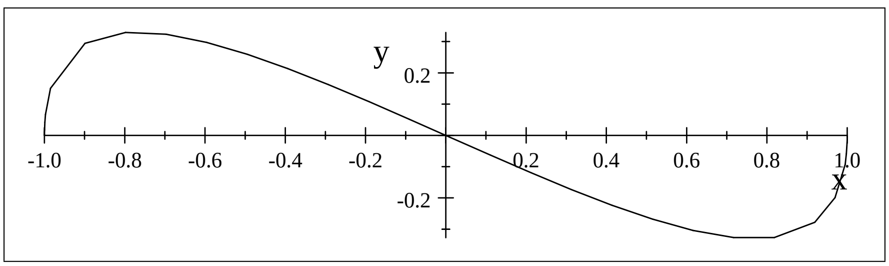
$$
\begin{aligned}
\frac{\partial}{\partial x}\left(\arcsin x - \frac{\pi}{2}x\right) &= \frac{1}{\sqrt{1-x^2}} - \frac{\pi}{2} \\
\frac{1}{\sqrt{1-x^2}} - \frac{\pi}{2} &= 0 \\
x &= \frac{-\sqrt{\pi^2-4}}{\pi} \text{ vagy} \\
x &= \frac{-\sqrt{\pi^2-4}}{\pi}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{1-x^2}} - \frac{\pi}{2}\right) &= \frac{x}{\sqrt{(1-x^2)^3}} \\
\frac{x}{\sqrt{(1-x^2)^3}} &< 0 \text{ ha } x < 0 \\
\frac{x}{\sqrt{(1-x^2)^3}} &= 0 \text{ ha } x = 0 \\
\frac{x}{\sqrt{(1-x^2)^3}} &> 0 \text{ ha } x > 0
\end{aligned}
$$

Maximumhely van a $\frac{-\sqrt{\pi^2-4}}{\pi}$ helyen, minimumhely van a $\frac{+\sqrt{\pi^2-4}}{\pi}$ helyen, inflexiós pont az origóban.

$$\begin{aligned}\lim_{x\to -1}\left(\arcsin x - \frac{\pi}{2}x\right) &= 0\\ \lim_{x\to +1}\left(\arcsin x - \frac{\pi}{2}x\right) &= 0\end{aligned}$$

$$\begin{aligned}R_f &= (f(\frac{+\sqrt{\pi^2-4}}{\pi}), \frac{-\sqrt{\pi^2-4}}{\pi})\\ &= \left(\arcsin\frac{\sqrt{\pi^2-4}}{\pi} - \frac{\sqrt{\pi^2-4}}{2}, \frac{\sqrt{\pi^2-4}}{2} - \arcsin\frac{\sqrt{\pi^2-4}}{\pi}\right)\end{aligned}$$



$$\begin{aligned}f(x) &= \frac{1}{x} + \sqrt{2x}\\ x &> 0\\ D_f &= (0, +\infty)\end{aligned}$$

$$\begin{aligned}\frac{1}{x} + \sqrt{2x} &= 0\\ &\text{nincs pozitív zérushely}\end{aligned}$$

$$\begin{aligned}\frac{\partial}{\partial x}\left(\frac{1}{x} + \sqrt{2x}\right) &= \frac{1}{\sqrt{2x}} - \frac{1}{x^2}\\ \frac{1}{\sqrt{2x}} - \frac{1}{x^2} &= 0\\ \sqrt{2x} &= x^2\\ 2x &= x^4\\ x &= \sqrt[3]{2}\end{aligned}$$

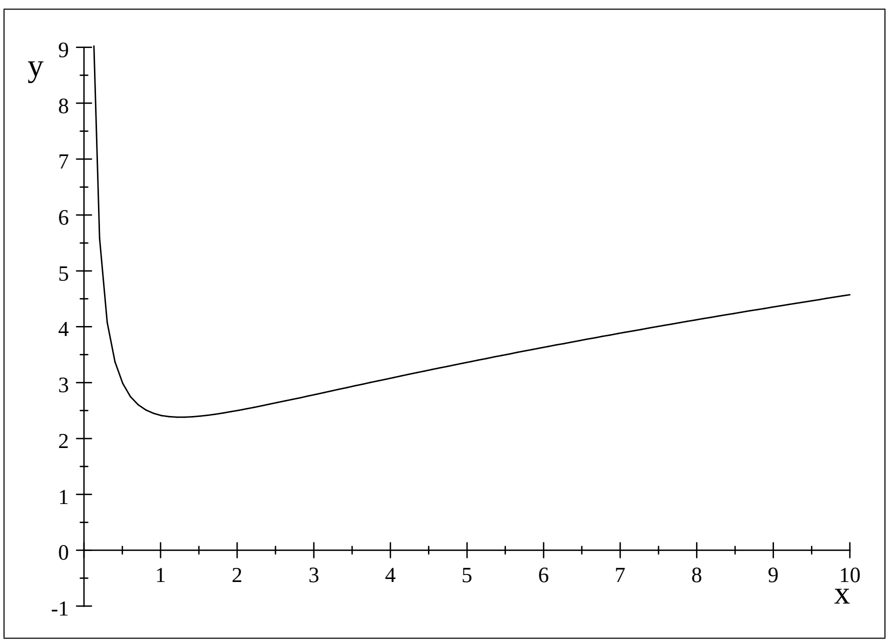
,

$$
\begin{aligned}
\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{2x}}-\frac{1}{x^2}\right) &= \frac{2}{x^3}-\frac{1}{2\sqrt{2x^3}} \\
\frac{2}{x^3}-\frac{1}{2\sqrt{2x^3}} &= 0 \text{ ha } x=\sqrt[3]{32}=2\sqrt[3]{4} \\
\frac{2}{x^3}-\frac{1}{2\sqrt{2x^3}} &< 0 \text{ ha } x>\sqrt[3]{32}=2\sqrt[3]{4} \\
\frac{2}{x^3}-\frac{1}{2\sqrt{2x^3}} &> 0 \text{ ha } 0<x<\sqrt[3]{32}=2\sqrt[3]{4}
\end{aligned}
$$

Tehát az $x=\sqrt[3]{2}$ helyen minimum van. $f(\sqrt[3]{2})=\frac{1}{\sqrt[3]{2}}+\sqrt{2\sqrt[3]{2}}=\frac{3}{\sqrt[3]{2}}$

$$
\begin{aligned}
\lim_{x\to 0^+}\left(\frac{1}{x}+\sqrt{2x}\right) &= +\infty \\
\lim_{x\to +\infty}\left(\frac{1}{x}+\sqrt{2x}\right) &= +\infty
\end{aligned}
$$

$$R_f=[\frac{3}{\sqrt[3]{2}},+\infty)$$



$$
\begin{aligned}
f(x) &= \log(1-x^2) \\
-1 &< x<+1 \\
D_f &= (-1,+1)
\end{aligned}
$$

$$\begin{aligned} \log(1-x^2) &= 0 \\ x &= 0 \end{aligned}$$

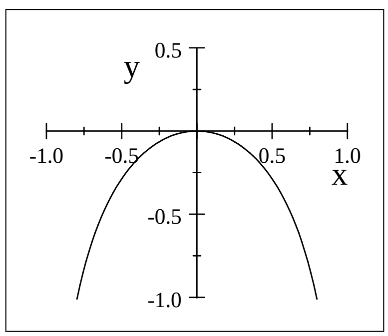
$$\begin{aligned} \frac{\partial}{\partial x}\left(\log\left(1-x^2\right)\right) &= \frac{-2x}{1-x^2} \\ \frac{-2x}{1-x^2} &= 0 \\ x &= 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial}{\partial x}\left(\frac{-2x}{1-x^2}\right) &= \frac{-2(1+x^2)}{(1-x^2)^2} \\ \frac{-2(1+x^2)}{(1-x^2)^2} &< 0 \end{aligned}$$

Tehát $x = 0$-ban maximumhely van; $\log(1-0^2) = 0$.

$$\begin{aligned} \lim_{x\to -1^+} \log(1-x^2) &= -\infty \\ \lim_{x\to +1^-} \log(1-x^2) &= -\infty \end{aligned}$$

$$R_f = (-\infty, 0]$$



$$\begin{aligned} f(x) &= e^{\sqrt{x}} \\ D_f &= [0, +\infty) \end{aligned}$$

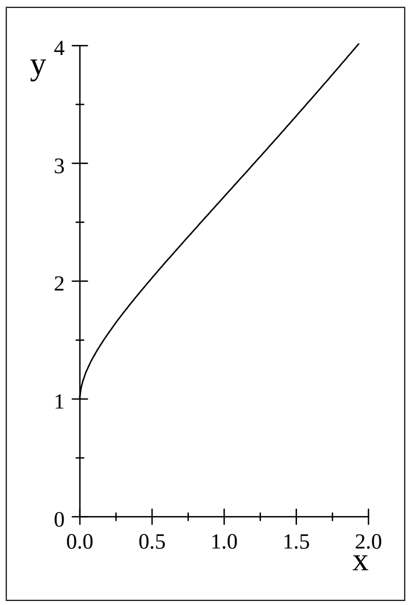
$$\begin{aligned} \frac{\partial}{\partial x}\left(e^{\sqrt{x}}\right) &= \frac{e^{\sqrt{x}}}{2\sqrt{x}} \\ \frac{e^{\sqrt{x}}}{2\sqrt{x}} &> 0 \end{aligned}$$

Sem a függvénynek, sem a deriváltjának nincs zérushelye.

$$\begin{aligned}
\lim_{x\to 0^+} e^{\sqrt{x}} &= 1 \\
\lim_{x\to +\infty} e^{\sqrt{x}} &= +\infty
\end{aligned}$$

$$R_f = [1, +\infty)$$

$$e^{\sqrt{x}}$$



$$\begin{aligned}
f(x) &= \log(1+\sqrt{x}) \\
D_f &= [0, +\infty)
\end{aligned}$$

$$\begin{aligned}
\log(1+\sqrt{x}) &= 0 \\
x &= 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial x}\left(\log(1+\sqrt{x})\right) &= \frac{1}{2\left(x+\sqrt{x}\right)} \\
\frac{1}{2\left(x+\sqrt{x}\right)} &< 0
\end{aligned}$$

$$
\begin{aligned}
\lim_{x\to 0^+} \log(1+\sqrt{x}) &= 0 \\
\lim_{x\to +\infty} \log(1+\sqrt{x}) &= +\infty
\end{aligned}
$$

$$R_f = [0, +\infty)$$

$$\log(1+\sqrt{x})$$