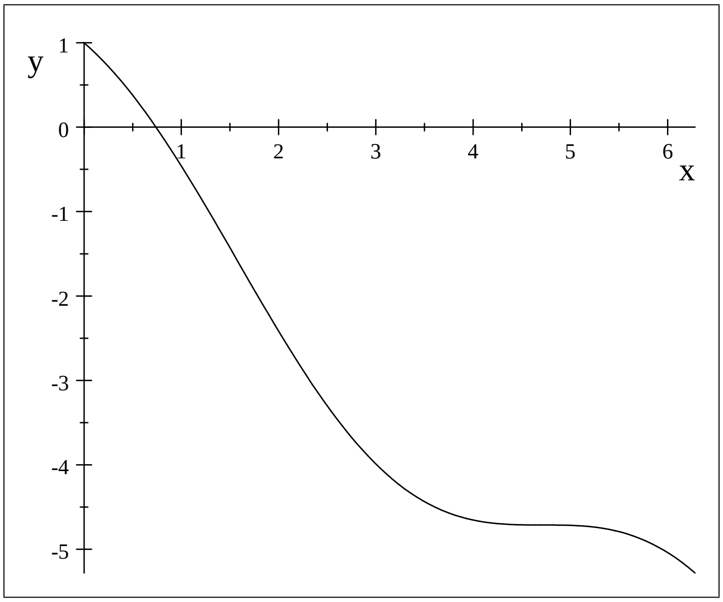
Vázoljuk a következő függvényeket, melyek mindegyikét a $(0, 2\pi)$ intervallumban értelmezzük! Írjuk fel az értékkészletüket is!

$$
\begin{aligned}
f_1(x) &= \cos x - x \\
f_2(x) &= 2\sin x + \sin 2x \\
f_3(x) &= 2\sin x + \cos 2x \\
f_4(x) &= x - \sin x - \pi
\end{aligned}
$$

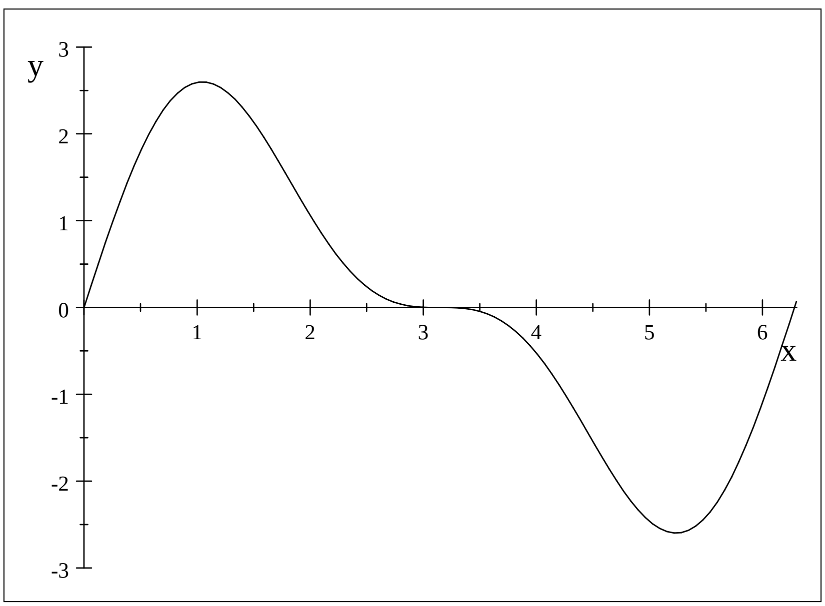
Megoldási vázlatok:

$$
\begin{aligned}
\frac{\partial}{\partial x}(\cos x - x) &= -\sin x - 1 \le 0 \\
\frac{\partial}{\partial x}(-\sin x - 1) &= -\cos x \\
-\cos x &< 0 \text{ ha } 0 < x < \frac{\pi}{2} \\
-\cos x &= 0 \text{ ha } x = \frac{\pi}{2} \\
-\cos x &> 0 \text{ ha } \frac{\pi}{2} < x < \pi
\end{aligned}
$$



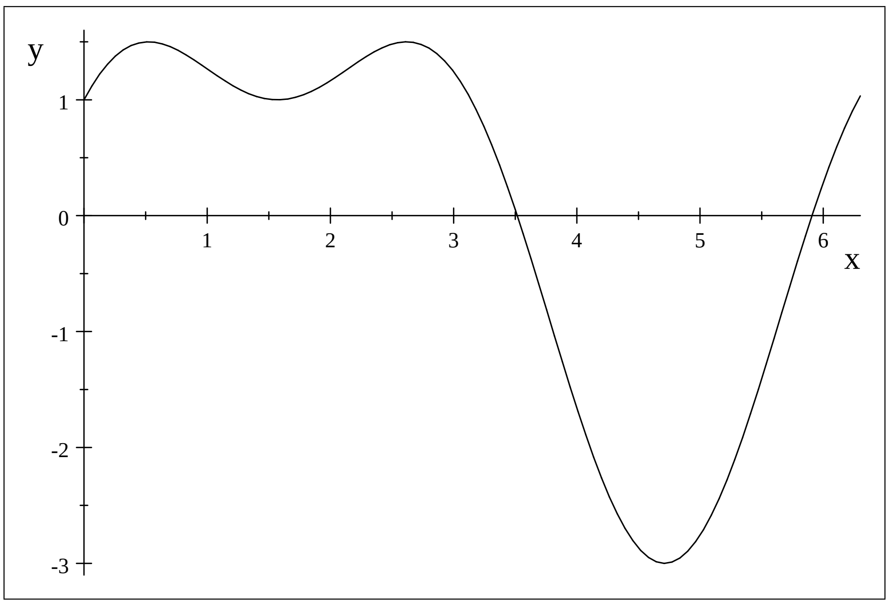
$$
\begin{aligned}
f_1(x) &= \cos x - x \\
R_{f_1} &= (1 - 2\pi, 1)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial}{\partial x}\left(2\sin x+\sin 2x\right) &= 4\cos^2 x+2\cos x-2\\
4\cos^2 x+2\cos x-2 &= 0\\
4t^2+2t-2 &= 0\\
t &= \cos x=\frac{1}{2}\text{ vagy } t=\cos x=-1\\
x &= \frac{\pi}{3}\text{ vagy } \pi \text{ vagy } \frac{5\pi}{3}\\
f_2(\frac{\pi}{3}) &= 2\sin\frac{\pi}{3}+\sin\frac{2\pi}{3}=\frac{3\sqrt{3}}{2}\\
f_2(\pi) &= 0\\
f_2(\frac{5\pi}{3}) &= 2\sin\frac{5\pi}{3}+\sin\frac{10\pi}{3}=\frac{-3\sqrt{3}}{2}\\
\frac{\partial}{\partial x}\left(4\cos^2 x+2\cos x-2\right) &= -2\sin x-8\sin x\cos x\\
-2\sin x-8\sin x\cos x &= -2\sin x\left(1+4\cos x\right)
\end{aligned}
$$



$$
\begin{aligned}
f_2(x) &= 2\sin x+\sin 2x\\
D_{f_2} &= \left(\frac{-3\sqrt{3}}{2},\frac{3\sqrt{3}}{2}\right)
\end{aligned}
$$

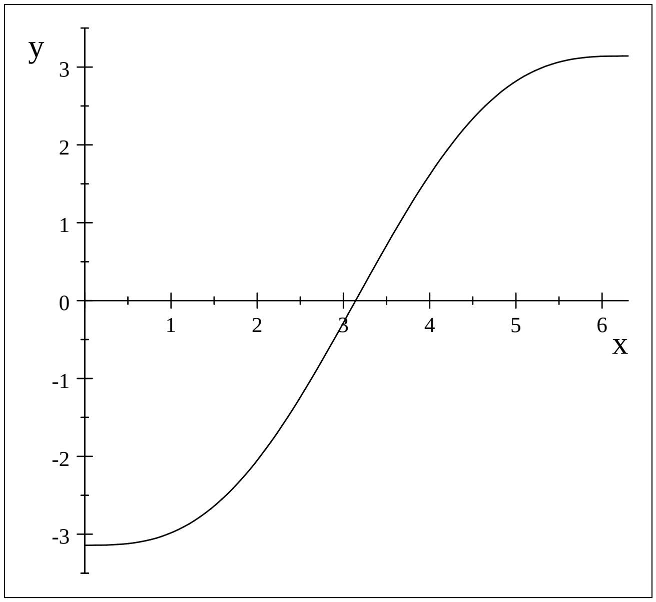
$$\begin{aligned}
\frac{\partial}{\partial x}(2\sin x + \cos 2x) &= 2\cos x(1 - 2\sin x) \\
2\cos x(1 - 2\sin x) &= 0 \\
\cos x &= 0 \text{ vagy } \sin x = \frac{1}{2} \\
x &= \frac{\pi}{6} \text{ vagy } \frac{\pi}{2} \text{ vagy } \frac{5\pi}{6} \text{ vagy } \frac{3\pi}{2} \\
f_3(\frac{\pi}{6}) &= 2\sin\frac{\pi}{6} + \cos\frac{\pi}{3} = \frac{3}{2} \\
f_3(\frac{\pi}{2}) &= 2\sin\frac{\pi}{2} + \cos\pi = 1 \\
f_3(\frac{5\pi}{6}) &= 2\sin\frac{5\pi}{6} + \cos\frac{5\pi}{3} = \frac{3}{2} \\
f_3(\frac{3\pi}{3}) &= 2\sin\frac{3\pi}{2} + \cos 3\pi = -3
\end{aligned}$$



$$\begin{aligned}
f_3(x) &= 2\sin x + \cos 2x \\
D_{f_3} &= [-3, \frac{3}{2}]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial x}(x - \sin x - \pi) &= 1 - \cos x \geq 0 \\
\frac{\partial}{\partial x}(1 - \cos x) &= \sin x
\end{aligned}$$

$x - \sin x - \pi$



$$\begin{aligned} f_4(x) &= x - \sin x - \pi \\ D_{f_4} &= (-\pi, +\pi) \end{aligned}$$

Megjegyzések:
1. f_1 zérushelye: $x = 0.739$; inflexiós pontjai: $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

2. f_2 inflexiós pontja: π
3. f_3 inflexiós pontjai: $1.0030, 2.1386, 3.\,776\,5, 5.\,648\,3$, mert

$$
\begin{aligned}
\frac{\partial^2}{\partial x^2}\left(2\sin x+\cos 2x\right) &= -2\left(\sin x+2\cos^2 x-2\sin^2 x\right)\\
\sin x+2\cos^2 x-2\sin^2 x &= 0\\
t+2(1-t^2)-2t^2 &= 0\\
t &= \sin x=\frac{1-\sqrt{33}}{8}\text{ vagy }\frac{1+\sqrt{33}}{8}\\
\arcsin\frac{1-\sqrt{33}}{8} &= -0.634\,87\\
\arcsin\frac{1+\sqrt{33}}{8} &= 1.0030\\
\pi-\arcsin\frac{1+\sqrt{33}}{8} &= 2.1386\\
2\pi+\arcsin\frac{1-\sqrt{33}}{8} &= 5.\,648\,3\\
\pi-\arcsin\frac{1-\sqrt{33}}{8} &= 3.\,776\,5
\end{aligned}
$$

4. f_1 zérushelye és inflexiós pontja: π.