Keressük meg a következő függvények primitív függvényét!

$$\sqrt{\sqrt[3]{\sqrt[5]{x}}}$$
$$1+2x+3x^2+...+99x^{98}$$
$$(6+66x)^{666}$$
$$\frac{x^3+8}{x+2}$$
$$\frac{\sqrt{x}}{\sqrt[3]{x^2}}-\frac{\sqrt[4]{x^5}}{\sqrt[5]{x^4}}$$
$$\frac{1}{\sqrt[3]{x^2-12x+36}}$$
$$\sqrt[e]{\exp(2x)}$$
$$x\cos x^2$$
$$\frac{x-1}{x^2-2x+7}$$
$$\frac{e^{2x}}{1+3e^{2x}}$$
$$6^{9x}$$

Megoldásokhoz induló ötletek:

$$\begin{aligned}
\sqrt{\sqrt[3]{\sqrt[5]{x}}} &= x^{1/30} \\
(k+1)x^k &= \left(x^{k+1}\right)' \\
x^3+8 &= (x+2)(x^2-2x+4) \\
x^2-12x+36 &= (x-6)^2 \\
\sqrt[e]{\exp(2x)} &= e^{2x/e} \\
x &= (x^2)'/2 \\
(x^2-2x+7)' &= 2(x-1) \\
(\log(1+3e^{2x}))' &= ? \\
6 &= \exp(\log 6)
\end{aligned}$$

Végeredmények:

$$\frac{x^{31/30}}{31/30} = \frac{30\ x\ \sqrt[30]{x}}{31}$$

$$x + x^2 + ... + x^{99} = \begin{cases} \frac{x^{100}-x}{x-1} & \text{ha} \quad x \neq 1 \\ 99 & \text{ha} \quad x = 1 \end{cases}$$

$$\int (6+66x)^{666} = \frac{(6+66x)^{667}}{66}$$

$$\int \frac{x^3+8}{x+2} = \int (x^2-2x+4) = \frac{x^3}{3} - x^2 + 4x$$

$$\int \left(\frac{\sqrt{x}}{\sqrt[3]{x^2}} - \frac{\sqrt[4]{x^5}}{\sqrt[5]{x^4}} \right) = \int \left(x^{-1/6} - x^{9/4} \right)$$

$$= \frac{x^{5/6}}{5/6} - \frac{x^{13/4}}{13/4} = \frac{6x^{5/6}}{5} - \frac{4x^{13/4}}{13}$$

$$\frac{6\sqrt[6]{x^5}}{5} - \frac{4x^3\sqrt[4]{x}}{13}$$

$$\int \frac{1}{\sqrt[3]{x^2-12x+36}} = \int (x-6)^{-2/3} = \frac{(x-6)^{1/3}}{1/3} = 3\sqrt[3]{x-6}$$

$$\int \sqrt[e]{\exp(2x)} = \int e^{2x/e} = \frac{e^{2x/e}}{2/e} = \frac{1}{2}\exp\left(\frac{2x}{e} - 1\right)$$

$$\int x \cos x^2 = \frac{1}{2} \int (x^2)' \cos x^2 = \frac{\sin x^2}{2}$$

$$\int \frac{x-1}{x^2-2x+7} = \int (x^2-2x+7)^{-1}(x^2-2x+7)'$$

$$= \frac{\log|x^2-2x+7|}{2} = \frac{\log(x^2-2x+7)}{2}$$

$$= \log\sqrt{x^2-2x+7}$$

$$\int \frac{e^{2x}}{1+3e^{2x}} = \frac{1}{6} \int (1+3e^{2x})^{-1}(1+3e^{2x})'$$

$$= \frac{1}{6}\log(1+3e^{2x}) = \log\sqrt[6]{1+3e^{2x}}$$

$$\int 6^{9x} = \int e^{(\log 6)9x} = \frac{e^{(\log 6)9x}}{(\log 6)9} = \frac{6^9}{\log 6^9}$$