Adjon meg f-hez F primitív függvényt, melyre $F(0)=0$.

$$\begin{aligned} f(x) &= 3\sqrt[3]{x}\cdot x^2; \quad f(x)=3e^{2x+2}; \quad f(x)=\frac{5}{(2x+1)^6}; \\ f(x) &= 3(x-1)^4; \qquad f(x)=2x^5+3x^4+2x^2-x+2 \end{aligned}$$

Kiszámítandók az alábbi határozatlan integrálok:

$$\int (1+x)^{10}; \quad \int \frac{(x+2)^3}{x}; \quad \int \frac{x^3-1}{x-1}; \quad \int \frac{x^4-1}{x+1};$$
$$\int \sqrt{x^2+2x+1}; \quad \int \left(\frac{x^3}{\sqrt{x}}+\frac{\sqrt{x}}{x^3}\right); \quad \int \frac{(2x+1)^2}{2x^2};$$
$$\int 2\sin 3x; \quad \int 3\cos 2x; \quad \int \sin 2x\cdot\cos 2x; \quad \int \left(\cos^2 2x-\sin^2 2x\right);$$
$$\int 2\sin^2 x; \quad \int \cos^2 2x; \quad \int \sin 3x\cdot\sin 2x; \quad \int \cos 2x\cdot\cos 3x; \quad \int \sin 2x\cdot\cos 3x$$

$$\int e^{2x-1}; \quad \int e^{2-3x}; \quad \int 2^{2x-1}; \quad \int 3^{2-2x}; \quad \int \frac{1}{2x+1}; \quad \int \sqrt{x^3\cdot\sqrt{x}}; \quad \int \sqrt{x\cdot\sqrt[3]{x}}$$

$$\int \frac{1}{x\log x}; \quad \int \frac{x-3}{x^2-6x+27}; \quad \int \frac{e^{3x}}{e^{3x}+5}; \quad \int \cot 2x; \quad \int x\sin x^2; \quad \int \frac{e^x}{\sqrt{1-e^{2x}}};$$
$$\int \frac{e^{\tan x}}{\cos^2 x}; \quad \int \frac{\log x}{\sqrt{x}}; \quad \int x^4\log x; \quad \int (2x-1)\sin 2x; \quad \int (3x+2)\cos 3x;$$
$$\int (x^2+2x)e^{-x}; \quad \int (2x^2+x+1)\sin 4x; \quad \int x^5 e^{x^2}; \quad \int (2-x^2)\cos 2x$$

$$\int \frac{2}{(x-1)(x+2)}, \quad \int \frac{3x}{(x+1)(x-3)}, \quad \int \frac{2x+1}{(x-1)(x-3)}, \quad \int \frac{2}{x(x+2)},$$
$$\int \frac{3x-2}{x^2+x-6}, \quad \int \frac{x(2x+2)}{(x^2+1)(x^2-9)}, \quad \int \frac{x^3-4x}{(x-2)(x^2+6x+8)}, \quad \int \frac{3x+4}{(2x+3)(2-x)}$$

Megoldások vázlata:

$$\begin{aligned} \int \left(3\sqrt[3]{x}\cdot x^2\right) &= 3\int x^{7/3} = 3\cdot\frac{x^{10/3}}{10/3} = 0.9x^3\sqrt[3]{x} \\ F(x) &= 0.9\cdot x^3\cdot\sqrt[3]{x} \\ & \quad \text{ez valóban 0 nullában} \end{aligned}$$

$$\int 3e^{2x+2} = 3\int \exp(2x+2) = 3\cdot\frac{\exp(2x+2)}{2} = \frac{3e^{2x+2}}{2}$$

de ez sohasem lehet nulla, ezért

$$F(x) = \frac{3e^{2x+2}}{2} - \frac{3e^{2\cdot 0+2}}{2} = \frac{3e^2}{2}(e^{2x}-1)$$

$$\int \frac{5}{(2x+1)^6} = 5\int (2x+1)^{-6} = 5\cdot\frac{(2x+1)^{-5}}{-5\cdot 2} = \frac{-1}{2\left(2x+1\right)^5}$$

de ez sohasem lehet nulla, ezért

$$F(x) = \frac{-1}{2\left(2x+1\right)^5} - \frac{-1}{2\left(2\cdot 0+1\right)^5} = \frac{1}{2}\left(1-\frac{1}{\left(2x+1\right)^5}\right)$$

$$\int 3(x-1)^4 = 3\int (x-1)^4 = \frac{3\left(x-1\right)^5}{5}$$

$$F(x) = \frac{3\left(x-1\right)^5}{5} - \frac{3\left(0-1\right)^5}{5}$$

$$= \frac{3x^5}{5} - 3x(x-1)(x^2-x+1)$$

$$\int \left(2x^5+3x^4+2x^2-x+2\right)$$

$$F(x) = \frac{1}{3}x^6 + \frac{3}{5}x^5 + \frac{2}{3}x^3 - \frac{1}{2}x^2 + 2x$$

—

$$\int (1+x)^{10} = \frac{(1+x)^{11}}{11} + \text{konstans}$$

$$\int \frac{(x+2)^3}{x} = \int \frac{x^3+6x^2+12x+8}{x}$$

$$= \int \left(x^2+6x+12\right) + \int \frac{8}{x}$$

$$= \frac{x^3}{3} + 3x^2 + 12x + 8\log x + \text{konstans}$$

$$\int \frac{x^3-1}{x-1} = \int \left(x^2+x+1\right) = \frac{x^3}{3} + \frac{x^2}{2} + x + \text{konstans}$$

$$\int \frac{x^4-1}{x+1} = \int \frac{(x^2-1)(x^2+1)}{x+1}$$

$$= \int (x-1)(x^2+1) = \int \left(x^3-x^2+x-1\right)$$

$$= \frac{x^4}{4} - \frac{x^3}{3} + \frac{x^2}{2} - x + \text{konstans}$$

$$
\begin{aligned}
\int \sqrt{x^2+2x+1} &= \int \sqrt{(x+1)^2} = \int |x+1| \\
\int |x+1| &= \int (x+1) = \frac{(x+1)^2}{2} + \text{konstans}_1, \\
\text{ha } x &\geq -1 \\
\int |x+1| &= \int -(x+1) = \frac{-(x+1)^2}{2} + \text{konstans}_2, \\
\text{ha } x &\leq -1 \\
\text{Összevonva} &: \\
\int \sqrt{x^2+2x+1} &= \frac{|x+1|(x+1)}{2} + \text{konstans}
\end{aligned}
$$

$$
\begin{aligned}
\int \left(\frac{x^3}{\sqrt{x}} + \frac{\sqrt{x}}{x^3} \right) &= \int x^{2.5} + \int x^{-2.5} \\
&= \frac{x^{3.5}}{3.5} + \frac{x^{-1.5}}{-1.5} + \text{konstans} \\
& \frac{6x^5 - 14}{21\sqrt{x^3}} + \text{konstans}
\end{aligned}
$$

$$
\begin{aligned}
\int \frac{(2x+1)^2}{2x^2} &= \int \frac{4x^2+4x+1}{2x^2} \\
&= \int \left(2 + \frac{2}{x} + \frac{1}{x^2} \right) \\
&= 2x + \log x^2 - \frac{1}{x} + \text{konstans}
\end{aligned}
$$

A most következő feladatokhoz felhasználjuk az ismert azonosságokat:

$$
\begin{aligned}
\cos(\alpha+\beta) &= \cos\alpha\cos\beta - \sin\alpha\sin\beta \\
\sin(\alpha+\beta) &= \sin\alpha\cos\beta + \cos\alpha\sin\beta
\end{aligned}
$$

Ha β helyére $-\beta$ kerül, akkor ezek így írhatók:

$$
\begin{aligned}
\cos(\alpha-\beta) &= \cos\alpha\cos\beta + \sin\alpha\sin\beta \\
\sin(\alpha-\beta) &= \sin\alpha\cos\beta - \cos\alpha\sin\beta
\end{aligned}
$$

Ha $\beta = \alpha$, akkor ezekből megkapjuk a következőket:

$$
\begin{aligned}
\cos 2\alpha &= \cos^2\alpha - \sin^2\alpha \\
\sin 2\alpha &= 2\sin\alpha\cos\alpha
\end{aligned}
$$

Ha $\alpha + \beta = \frac{\pi}{2}$, akkor ezt nyerjük:

$$
\cos^2\alpha + \sin^2\alpha = 1
$$

Összeadásokkal, kivonásokkal ezeket kapjuk:

$$\begin{aligned}
\cos 2\alpha &= 1-2\sin^2\alpha \\
&= 2\cos^2\alpha - 1 \\
2\cos^2 &= 1+\cos 2\alpha \\
2\sin^2\alpha &= 1-\cos 2\alpha \\
2\cos\alpha\cos\beta &= \cos(\alpha-\beta)+\cos(\alpha+\beta) \\
2\cos\alpha\sin\beta &= \sin(\alpha+\beta)-\sin(\alpha-\beta) \\
2\sin\alpha\sin\beta &= \cos(\alpha-\beta)-\cos(\alpha+\beta)
\end{aligned}$$

Az alábbi képletekben a konstans tagokat – az egyszerűbb képletírás érdekében – elhagyjuk:

$$\begin{aligned}
\int 2\sin 3x &= 2\int \sin 3x = 2\cdot\frac{\cos 3x}{-3} = \frac{-2}{3}\cos 3x \\
\int 3\cos 2x &= 3\int \cos 2x = 3\cdot\frac{\sin 2x}{2} = \frac{3}{2}\sin 2x \\
\int \sin 2x\cos 2x &= \frac{1}{2}\int 2\sin 2x\cos 2x = \frac{1}{2}\int \sin 4x = \frac{1}{2}\cdot\frac{\cos 4x}{-4} = \frac{-1}{8}\cos 4x \\
\int \left(\cos^2 2x - \sin^2 2x\right) &= \int \cos 4x = \frac{\sin 4x}{4} \\
\int 2\sin^2 x &= \int (1-\cos 2\alpha) = x - \frac{\sin 2\alpha}{2} \\
\int \cos^2 2x &= \int (1+\cos 4x) = x + \frac{\sin 4x}{4} \\
\int \sin 3x\sin 2x &= \frac{1}{2}\int (\cos x - \cos 5x) = \frac{\sin x}{2} - \frac{\sin 5x}{10} \\
\int \cos 2x\cos 3x &= \frac{1}{2}\int (\cos(-x)+\cos 5x) = \frac{1}{2}\int(\cos x + \cos 5x) = \frac{\sin x}{2} + \frac{\sin 5x}{10} \\
\int \sin 2x\cos 3x &= \frac{1}{2}\int (\sin 5x - \sin x) = \frac{\cos 5x}{2\cdot(-5)} - \frac{\cos x}{2\cdot(-1)} = \frac{\cos x}{2} - \frac{\cos 5x}{10}
\end{aligned}$$

$$\begin{aligned}
\int e^{2x-1} &= \frac{e^{2x-1}}{2} = \frac{e^{2x}}{2e} \\
\int e^{2-3x} &= \frac{e^{2-3x}}{-3} = \frac{-e^2}{3e^{3x}} \\
\int 2^{2x-1} &= \frac{1}{2}\int 4^x = \frac{1}{2}\int e^{(\log 4)x} = \frac{1}{2}\cdot\frac{e^{(\log 4)x}}{\log 4} = \frac{4^x}{2\log 4} = \frac{2^{2x}}{4\log 2} \\
\int 3^{2-2x} &= 9\int 9^{-x} = 9\int e^{-(\log 9)x} = \frac{9e^{-(\log 9)x}}{-\log 9} = \frac{-9^{1-x}}{\log 3^2} = \frac{-3^{2-2x}}{2\log 3}
\end{aligned}$$

$$\begin{aligned}
\int \frac{1}{2x+1} &= \frac{\log|2x+1|}{2} = \log\sqrt{|2x+1|} \\
\int \sqrt{x^3\cdot\sqrt{x}} &= \int x^{7/4} = \frac{x^{11/4}}{11/4} = \frac{4\cdot\sqrt[4]{x^{11}}}{11} \\
\int \sqrt{x\cdot\sqrt[3]{x}} &= \int x^{2/3} = \frac{x^{5/3}}{5/3} = \frac{3\cdot\sqrt[3]{x^5}}{5}
\end{aligned}$$

A most következő feladatok megoldása helyettesítése integrálással történik. Ennek az a lényege, hogy az $\int f(x)$ alakú feladatot úgy próbáljuk megoldani, hogy $f(x)$-et $g'(h(x))\cdot h'(x)$ alakú szorzattá bontjuk, és így $\int f(x) = g(h(x))$.

$$\begin{aligned}
\int \frac{1}{x\log x} &= \log|\log x| \\
\text{mert } \frac{1}{x\log x} &= \frac{1}{\log x}\cdot\frac{1}{x} = \log'(\log x)\cdot\log'(x)
\end{aligned}$$

$$\begin{aligned}
\int \frac{x-3}{x^2-6x+27} &= \int \frac{x-3}{(x-3)^2+18} \\
&= \frac{1}{2}\int \frac{1}{(x-3)^2+18}\cdot(2(x-3)) \\
&= \frac{1}{2}\int \log'((x-3)^2+18)\cdot\left((x-3)^2+18\right)' \\
&= \frac{1}{2}\log((x-3)^2+18) \\
&= \frac{1}{2}\log(x^2-6x+27) = \log\sqrt{x^2-6x+27}
\end{aligned}$$

$$\begin{aligned}
\int \frac{e^{3x}}{e^{3x}+5} &= \frac{1}{3}\int (e^{3x}+5)^{-1}\cdot(3e^{3x}) \\
&= \frac{1}{3}\int (e^{3x}+5)^{-1}\cdot(e^{3x}+5)' \\
&= \frac{1}{3}\cdot\log(e^{3x}+5) = \log\sqrt[3]{e^{3x}+5}
\end{aligned}$$

$$\begin{aligned}\int \cot 2x &= \int \frac{\cos 2x}{\sin 2x} = \int (\sin 2x)^{-1} \cdot \frac{(\sin 2x)'}{2} \\ &= \frac{1}{2} \log|\sin 2x| = \log\sqrt{|\sin 2x|}\end{aligned}$$

$$\begin{aligned}\int x \sin x^2 &= \frac{1}{2} \int \sin(x^2) \cdot (2x) \\ &= \frac{1}{2} \int (-\cos'(x^2)) \cdot (x^2)' \\ &= \frac{1}{2}(-\cos x^2) = \frac{-\cos x^2}{2}\end{aligned}$$

$$\begin{aligned}\int \frac{e^x}{\sqrt{1-e^{2x}}} &= \int \frac{1}{\sqrt{1-(e^x)^2}} \cdot e^x \\ &= \int \big(\arcsin'(e^x)\big) \cdot (e^x)' = \arcsin(e^x)\end{aligned}$$

$$\int \frac{e^{\tan x}}{\cos^2 x} = \int (\exp(\tan x)) \cdot (\tan' x) = \exp(\tan x) = e^{\tan x}$$

A most következő feladatok megoldása parciális integrálással történik. Ennek az a lényege, hogy az $\int f(x)$ alakú feladatot úgy próbáljuk megoldani, hogy $f(x)$-et $g'(x) \cdot h(x)$ alakban szorzattá bontjuk, és most

$$\int f(x) = \int g'(x) \cdot h(x) = g(x) \cdot h(x) - \int g(x) \cdot h'(x)$$

$$\begin{aligned}\int \frac{\log x}{\sqrt{x}} &= 2 \int \frac{1}{2\sqrt{x}} \cdot \log x = 2 \int \left(\sqrt{x}\right)' \cdot \log x \\ &= 2\left(\sqrt{x}\log x - \int \sqrt{x} \cdot \log' x\right) \\ &= 2\left(\sqrt{x}\log x - \int \frac{\sqrt{x}}{x}\right) = 2\left(\sqrt{x}\log x - \int x^{-1/2}\right) \\ &= 2(\sqrt{x}\log x - \frac{x^{1/2}}{1/2}) = 2\sqrt{x}\log x - 4\sqrt{x} = 2\sqrt{x}\,(\ln x - 2)\end{aligned}$$

$$
\begin{aligned}
\int \log x &= \int 1 \cdot \log x = \int x' \cdot \log x \\
&= x \log x - \int x \log' x = x \log x - \int 1 \\
&= x \log x - x = x(\log x - 1)
\end{aligned}
$$

$$
\begin{aligned}
\int x \log x &= \frac{1}{2} \int \left(x^2\right)' \cdot \log x \\
&= \frac{1}{2} \left(x^2 \log x - \int x^2 \log' x \right) = \frac{1}{2} \left(x^2 \log x - \int x \right) \\
&= \frac{1}{2} \left(x^2 \log x - \frac{x^2}{2} \right) = \frac{x^2(2 \log x - 1)}{4}
\end{aligned}
$$

$$
\begin{aligned}
\int x^2 \log x &= \frac{1}{3} \int (x^3)' \cdot \log x \\
&= \frac{1}{3} \left(x^3 \log x - \int x^3 \log' x \right) \\
&= \frac{x^3(3 \log x - 1)}{9}
\end{aligned}
$$

$$
\begin{aligned}
\int x^3 \log x &= \frac{x^4(4 \log x - 1)}{16} \\
\int x^4 \log x &= \frac{x^5(5 \log x - 1)}{25} \\
\int x^{1000} \log x &= \frac{x^{1001}(1001 \log x - 1)}{1000000}
\end{aligned}
$$

$$
\begin{aligned}
\int (2x - 1) \sin 2x &= 2 \int x \sin 2x - \int \sin 2x \\
&= 2 \cdot \left(\frac{x \cos 2x}{-2} - \int 1 \cdot \frac{\cos 2x}{-2} \right) - \int \sin 2x \\
&= -x \cos 2x + \frac{\sin 2x}{2} - \frac{\cos 2x}{-2} \\
&= -x \cos 2x + \frac{\sin 2x + \cos 2x}{2}
\end{aligned}
$$

$$
\begin{aligned}
\int (3x+2)\cos 3x &= 3\int x\cos 3x + 2\int \cos 3x \\
\int \cos 3x &= \frac{\sin 3x}{3} \\
\int x\cos 3x &= x\cdot\frac{\sin 3x}{3} - \int \frac{\sin 3x}{3} \\
& \frac{x\sin 3x}{3} - \frac{\cos 3x}{-3\cdot 3} \\
\int (3x+2)\cos 3x &= 3\left(\frac{x\sin 3x}{3} - \frac{\cos 3x}{-3\cdot 3}\right) + 2\frac{\sin 3x}{3} \\
&= x\sin 3x + \frac{2\sin 3x + \cos 3x}{3}
\end{aligned}
$$

$$
\begin{aligned}
\int (x^2+2x)e^{-x} &= \int x^2 e^{-x} + 2\int xe^{-x} \\
\int xe^{-x} &= x\frac{e^{-x}}{-1} - \int \frac{e^{-x}}{-1} = -(x+1)e^{-x} \\
\int x^2 e^{-x} &= x^2\frac{e^{-x}}{-1} - \int 2x\frac{e^{-x}}{-1} \\
\int (x^2+2x)e^{-x} &= -(x^2+4x+4)e^{-x}
\end{aligned}
$$

$$\begin{aligned}
\int (2x^2+x+1)\sin 4x &= 2\int x^2 \sin 4x + \int x\sin 4x + \int \sin 4x \\
\int \sin 4x &= \frac{\cos 4x}{-4} \\
\int x\sin 4x &= x\frac{\cos 4x}{-4} - \int \frac{\cos 4x}{-4} = \frac{-4x\cos 4x + \sin 4x}{16} \\
\int x^2 \sin 4x &= x^2\frac{\cos 4x}{-4} - \int 2x\frac{\cos 4x}{-4} \\
\int x\cos 4x &= x\frac{\sin 4x}{4} - \int \frac{\sin 4x}{4} = \frac{4x\sin 4x + \cos 4x}{16} \\
\int x^2 \sin 4x &= \frac{-x^2\cos 4x}{4} + \frac{4x\sin 4x + \cos 4x}{32} \\
\int (2x^2+x+1)\sin 4x &= 2\left(\frac{-x^2\cos 4x}{4} + \frac{4x\sin 4x + \cos 4x}{32}\right) + \frac{-4x\cos 4x + \sin 4x}{16} - \frac{\cos 4x}{4} \\
&= \frac{-x^2\cos 4x}{2} + \frac{4x\sin 4x + \cos 4x}{16} + \frac{-4x\cos 4x + \sin 4x}{16} - \frac{\cos 4x}{4}
\end{aligned}$$

$$\begin{aligned}
\int x^5 e^{x^2} &= \frac{1}{2}\int (x^2)^2 \cdot \left(e^{x^2}\cdot 2x\right) \\
y &= x^2 \text{ esetén } \int x^5 e^{x^2} = \frac{1}{2}\int y^2 e^y \\
\int e^y &= e^y \\
\int ye^y &= ye^y - \int e^y = (y-1)e^y \\
\int y^2 e^y &= y^2 e^y - \int 2ye^y = y^2e^y - 2(y-1)e^y = (y^2-2y+2)e^y \\
\int x^5 e^{x^2} &= \frac{1}{2}(y^2-2y+2)e^y = \frac{(x^4-2x^2+2)e^{x^2}}{2}
\end{aligned}$$

$$\begin{aligned}
\int (2-x^2)\cos 2x &= (2-x^2)\frac{\sin 2x}{2} - \int (-2x)\frac{\sin 2x}{2} \\
&= \left(1-\frac{x^2}{2}\right)\sin 2x + \left(x\frac{\cos 2x}{-2} - \int \frac{\cos 2x}{-2}\right) \\
&= \left(1-\frac{x^2}{2}\right)\sin 2x - \frac{x}{2}\cos 2x + \frac{\sin 2x}{4} \\
&= \frac{(4-2x^2)\sin 2x - 2x\cos 2x + \sin 2x}{4}
\end{aligned}$$

A most következő megoldások parciális törtekre bontással történnek;

$$\begin{aligned}
\frac{2}{(x-1)(x+2)} &= \frac{A}{x-1}+\frac{B}{x+2}=\frac{A(x+2)+B(x-1)}{(x-1)(x+2)} \\
2 &= A(x+2)+B(x-1) \\
0 &= A+B;\quad 2=2A-B;\quad 2=3A;\quad \frac{2}{3}=A;\quad \frac{-2}{3}=B \\
\int\frac{2}{(x-1)(x+2)} &= \frac{2\log|x-1|}{3}-\frac{2\log|x+2|}{3}=\frac{\log(x-1)^2-\log(x+2)^2}{3}
\end{aligned}$$

$$\begin{aligned}
\frac{3x}{(x+1)(x-3)} &= \frac{A}{x+1}+\frac{B}{x-3}=\frac{A(x-3)+B(x+1)}{(x+1)\,(x-3)} \\
3x &= A(x-3)+B(x+1)=-3A+B+(A+B)x \\
0 &= -3A+B;\quad 3=A+B;\quad A=\frac{3}{4};\quad B=\frac{9}{4} \\
\int\frac{3x}{(x+1)(x-3)} &= \frac{3\log|x+1|+9\log|x-3|}{4}
\end{aligned}$$

$$\begin{aligned}
\frac{2x+1}{(x-1)(x-3)} &= \frac{A}{x-1}+\frac{B}{x-3}=\frac{A(x-3)+B(x-1)}{(x-1)(x-3)} \\
1+2x &= -3A-B+(A+B)x \\
1 &= -3A-B;\quad 2=A+B;\quad A=\frac{-3}{2};\quad B=\frac{7}{2} \\
\int\frac{2x+1}{(x-1)(x-3)} &= \frac{7\log|x-1|-3\log|x-1|}{2}
\end{aligned}$$

$$\begin{aligned}
\frac{2}{x(x+2)} &= \frac{A}{x}+\frac{B}{x+2}=\frac{A(x+2)+Bx}{x(x+2)} \\
2 &= A(x+2)+Bx;\quad 2=2A;\quad 0=A+B;\quad A=1;\quad B=-1 \\
\int\frac{2}{x(x+2)} &= \log|x|-\log|x+2|
\end{aligned}$$

$$
\begin{aligned}
x^2+x-6 &= 0; \quad x=-3 \text{ vagy } 2\\
x^2+x-6 &= (x+3)(x-2)\\
\frac{3x-2}{(x+3)(x-2)} &= \frac{A}{x+3}+\frac{B}{x-2}=\frac{A(x-2)+B(x+3)}{(x-3)(x+2)}\\
-2+3x &= A(x-2)+B(x+3)\\
-2 &= -2A+3B; \quad 3=A+B; \quad A=\frac{11}{5}; \quad B=\frac{4}{5}\\
\int\frac{3x-2}{x^2+x-6} &= \frac{4\log|x-2|+11\log|x+3|}{5}
\end{aligned}
$$

$$
\begin{aligned}
\frac{x(2x+2)}{(x^2+1)(x^2-9)} &= \frac{Ax+B}{x^2+1}+\frac{Cx+D}{x^2-9}\\
&= \frac{(Ax+B)(x^2-9)+(Cx+D)(x^2+1)}{(x^2+1)(x^2-9)}\\
x(2x+2) &= (Ax+B)(x^2-9)+(Cx+D)(x^2+1)
\end{aligned}
$$

$$
\begin{aligned}
0\cdot(2\cdot 0+2) &= (A\cdot 0+B)(0^2-9)+(C\cdot 0+D)(0^2+1)\\
1\cdot(2\cdot 1+2) &= (A\cdot 1+B)(1^2-9)+(C\cdot 1+D)(1^2+1)\\
2\cdot(2\cdot 2+2) &= (A\cdot 2+B)(2^2-9)+(C\cdot 2+D)(2^2+1)\\
(-1)\cdot(2\cdot(-1)+2) &= (A\cdot(-1)+B)((-1)^2-9)+(C\cdot(-1)+D)((-1)^2+1)
\end{aligned}
$$

$$
\begin{aligned}
0 &= -9B+D\\
4 &= -8A-8B+2C+2D\\
12 &= -10A-5B+10C+5D\\
0 &= 8A-8B-2C+2D
\end{aligned}
$$

$$A=\frac{-1}{5}, \quad B=\frac{1}{5}, \quad C=\frac{1}{5}, \quad D=\frac{9}{5}$$

$$\frac{x(2x+2)}{(x^2+1)(x^2-9)}=\frac{1-x}{5(x^2+1)}+\frac{x+9}{5\,(x^2-9)}$$

$$
\begin{aligned}
\int\frac{1-x}{5(x^2+1)} &= \frac{-1}{5}\int\frac{x-1}{x^2+1}\\
\int\frac{x-1}{x^2+1} &= \int\frac{x}{x^2+1}+\int\frac{1}{x^2+1}\\
\int\frac{1-x}{5(x^2+1)} &= \frac{\log(x^2+1)}{2}-\arctan x
\end{aligned}
$$

$$
\begin{aligned}
\frac{x+9}{x^2-9} &= \frac{x+9}{(x-3)\,(x+3)} = \frac{a}{x-3} + \frac{b}{x+3} \\
&= \frac{a(x+3)+b(x-3)}{(x-3)\,(x+3)} \\
9+x &= 3a-3b+(a+b)x \\
9 &= 3a-3b; \quad 1 = a+b; \quad a=2; \quad b=-1
\end{aligned}
$$

$$
\begin{aligned}
\frac{x+9}{x^2-9} &= \frac{2}{x-3} - \frac{1}{x+3} \\
\int \frac{x+9}{x^2-9} &= 2\log|x-3| - \log|x+3|
\end{aligned}
$$

$$
\int \frac{x(2x+2)}{(x^2+1)(x^2-9)} = \frac{2\arctan x - \log(x^2+1) + 2\log(x-3)^2 - \log(x+3)^2}{10}
$$

$$
\begin{aligned}
x^2+6x+8 &= 0 \\
x &= -2 \text{ vagy } -4 \\
x^2+6x+8 &= (x+2)(x+4) \\
x^3-4x &= x(x^2-4) = x\,(x-2)\,(x+2) \\
\frac{x^3-4x}{(x-2)(x^2+6x+8)} &= \frac{x\,(x-2)\,(x+2)}{(x-2)(x+2)(x+4)} = \frac{x}{x+4} = 1 - \frac{1}{x+4}
\end{aligned}
$$

$$
\int \frac{x^3-4x}{(x-2)(x^2+6x+8)} = x - \log(x+4)^4
$$

$$
\begin{aligned}
\frac{3x+4}{(2x+3)(2-x)} &= \frac{A}{2x+3} + \frac{B}{2-x} = \frac{A(2-x)+B(2x+3)}{(2x+3)(2-x)} \\
4+3x &= 2A+3B+(-A+2B)x \\
A &= \frac{-1}{7}; \quad B = \frac{10}{7} \\
\int \frac{3x+4}{(2x+3)(2-x)} &= \frac{\frac{-\log|2x+3|}{2} + \frac{\log|2-x|}{-1}}{7} = \frac{-\log|2x+3|}{14} + \frac{-\log|2-x|}{7}
\end{aligned}
$$