$$\lim_{x\to 1} \frac{x^n-1}{x-1}\frac{x^{n-1}-1}{x^2-1}\dots\frac{x^{n-k+1}-1}{x^k-1} = ?$$

A trükk az, hogy

$$\lim_{x\to 1} \frac{x^m-1}{x-1} = \lim_{x\to 1}\left(x^{m-1}+x^{m-2}+\dots+1\right) = m$$

Bővítgetjük az eredeti törtet $x-1$-gyel:

$$\begin{aligned}
\lim_{x\to 1} \frac{x^n-1}{x-1}\frac{x^{n-1}-1}{x^2-1}\dots\frac{x^{n-k+1}-1}{x^k-1} &= \\
\lim_{x\to 1} \frac{x^n-1}{x-1}\frac{(x-1)(x^{n-1}-1)}{(x-1)(x^2-1)}\dots\frac{(x-1)(x^{n-k+1}-1)}{(x-1)(x^k-1)} &= \\
n\cdot\frac{n-1}{2}\cdot\dots\cdot\frac{n-k+1}{k} &= \binom{n}{k}
\end{aligned}$$