A. 1. $\int \frac{1}{x^2+x-20}$ 2. $\int_{\pi/6}^{\pi/4} 2-\tan^2 x\ dx$

3. $y=x$ és $y=\sqrt[4]{\pi x^3}$ közti terület. 4. $\int_{-\infty}^{2} xe^{4x}\ dx$

Megoldások vázlata:

A1.

$$\begin{aligned}
x^2+x-20 &= (x+5)(x-4) && (\ 1 \text{ pont}\ )\\
\frac{1}{x^2+x-20} &= \frac{A}{x+5}+\frac{B}{x-4} = \frac{A(x-4)+B(x+5)}{(x+5)(x-4)}\\
1 &= -4A+5B\\
0 &= A+B\\
A &= -\frac{1}{9},\quad B=\frac{1}{9} && (\ 1 \text{ pont}\ )\\
\frac{1}{x^2+x-20} &= \frac{1}{9}\left(\frac{1}{x-4}-\frac{1}{x+5}\right) && (\ 1 \text{ pont}\ )\\
\int \frac{1}{x^2+x-20} &= \frac{1}{9}\log\left|\frac{x-4}{x+5}\right| && (\ 2 \text{ pont}\ )
\end{aligned}$$

A2.

$$\begin{aligned}
\int \left(2-\tan^2 x\ \right) &= \int \left(3-(1+\tan^2 x)\right) && (\ 1 \text{ pont}\ )\\
&= 3x-\tan x && (\ 1 \text{ pont}\ )\\
\int_{\pi/6}^{\pi/4} 2-\tan^2 x\ dx &= [3x-\tan x]_{\pi/6}^{\pi/4} && (\ 1 \text{ pont}\ )\\
&= \frac{\pi}{4}+\frac{\sqrt{3}}{3}-1 && (\ 2 \text{ pont}\ )
\end{aligned}$$

A3.

$$\begin{aligned}
x &= \sqrt[4]{\pi x^3},\quad x\ge 0\\
x^4 &= \pi x^3 \qquad x=0 \quad \text{és} \quad x=\pi && (\ 1 \text{ pont}\ )\\
\int \left(\sqrt[4]{\pi x^3}-x\right) &= \sqrt[4]{\pi}\int x^{3/4}-\frac{x^2}{2} = \sqrt[4]{\pi}\cdot\frac{x^{7/4}}{7/4}-\frac{x^2}{2}\\
&= \frac{8\sqrt[4]{\pi}x^{7/4}-7x^2}{14} && (\ 2 \text{ pont}\ )\\
\int_0^{\pi} \left(\sqrt[4]{\pi x^3}-x\right)dx &= \left[\frac{8\sqrt[4]{\pi}x^{7/4}-7x^2}{14}\right]_0^{\pi} = \frac{\pi^2}{14} && (\ 2 \text{ pont}\ )
\end{aligned}$$

A4.

$$\begin{aligned}
\int xe^{4x} &= x \cdot \frac{e^{4x}}{4} - \int \frac{e^{4x}}{4} \\
&= \frac{(4x-1)e^{4x}}{16} && \text{(2 pont)} \\
\int_a^2 xe^{4x+1}\ dx &= \left[\frac{(4x-1)e^{4x}}{16}\right]_a^2 \\
&= \frac{7e^8}{16} - \frac{(4a-1)e^{4a}}{16} && \text{(2 pont)} \\
\lim_{a\to-\infty}(4a-1)e^{4a} &= \lim_{a\to-\infty}\frac{4a-1}{e^{-4a}} \\
&= \lim_{a\to-\infty}\frac{4}{-4e^{-4a}} = -\lim_{a\to-\infty} e^{4a} = 0 \\
\int_{-\infty}^2 xe^{4x}\ dx &= \frac{7e^8}{16} && \text{(1 pont)}
\end{aligned}$$