B. 1. $\int \frac{1}{x^2-x-12}$ 2. $\int_{\pi/4}^{\pi/3} \pi + \tan^2 x \; dx$

3. $y = x$ és $y = \sqrt{\frac{ex}{2}}$ közti terület. 4. $\int_{-\infty}^{0} x e^{\sqrt{2}x} \; dx$

Megoldások vázlata:

B1.

$$
\begin{aligned}
x^2 - x - 12 &= (x+3)(x-4) && (\text{ 1 pont })\\
\frac{1}{x^2-x-12} &= \frac{A}{x+3} + \frac{B}{x-4} = \frac{A(x-4)+B(x+3)}{(x+5)(x-4)} \\
1 &= -4A + 3B \\
0 &= A + B \\
A &= -\frac{1}{7}, \quad B = \frac{1}{7} && (\text{ 1 pont })\\
\frac{1}{x^2-x-12} &= \frac{1}{7}\left(\frac{1}{x-4} - \frac{1}{x+3}\right) && (\text{ 1 pont })\\
\int \frac{1}{x^2-x-12} &= \frac{1}{7}\log\left|\frac{x-4}{x+3}\right| && (\text{ 2 pont })
\end{aligned}
$$

B2.

$$
\begin{aligned}
\int \left(\pi + \tan^2 x\ \right) &= \int \left((\pi-1) + (1+\tan^2 x)\right) && (\text{ 1 pont })\\
&= (\pi-1)x + \tan x && (\text{ 1 pont })\\
\int_{\pi/4}^{\pi/3} \pi + \tan^2 x \; dx &= \left[(\pi-1)x + \tan x\ \right]_{\pi/4}^{\pi/3} && (\text{ 1 pont })\\
&= \frac{\pi(\pi-1)}{12} + \sqrt{3} - 1 && (\text{ 2 pont })
\end{aligned}
$$

B3.

$$
\begin{aligned}
x &= \sqrt{\frac{ex}{2}}, \quad x \geq 0 \\
x^2 &= \frac{ex}{2} \qquad x = 0 \quad \text{és} \quad x = \frac{e}{2} && (\text{ 1 pont })\\
\int \left(\sqrt{\frac{ex}{2}} - x\right) &= \sqrt{\frac{e}{2}} \int x^{1/2} - \frac{x^2}{2} = \\
&= \frac{\sqrt{8e}x^{3/2} - 3x^2}{6} && (\text{ 2 pont })\\
\int_0^{e/2} \left(\sqrt{\frac{ex}{2}} - x\right) dx &= \left[\frac{\sqrt{8e}x^{3/2} - 3x^2}{6}\right]_0^{e/2} = \frac{e^2}{24} && (\text{ 2 pont })
\end{aligned}
$$

B4.

$$\int xe^{\sqrt{2}x} = \left(x \cdot \frac{e^{\sqrt{2}x}}{\sqrt{2}} - \int \frac{e^{\sqrt{2}x}}{\sqrt{2}}\right)$$

$$= \frac{\left(\sqrt{2}x - 1\right)e^{\sqrt{2}x}}{2} \qquad \text{(2 pont)}$$

$$\int_a^0 xe^{\sqrt{2}x}\ dx = \left[\frac{\left(\sqrt{2}x - 1\right)e^{\sqrt{2}x}}{2}\right]_a^0$$

$$= \frac{-1}{2} - \frac{\left(\sqrt{2}a - 1\right)e^{\sqrt{2}a}}{2} \qquad \text{(2 pont)}$$

$$\lim_{a\to-\infty} \left(\sqrt{2}a - 1\right)e^{\sqrt{2}a} = \lim_{a\to-\infty} \frac{\sqrt{2}a - 1}{e^{-\sqrt{2}a}}$$

$$= \lim_{a\to-\infty} \frac{\sqrt{2}}{-\sqrt{2}e^{-\sqrt{2}a}} = -\lim_{a\to-\infty} e^{\sqrt{2}a} = 0$$

$$\int_{-\infty}^0 xe^{\sqrt{2}x}\ dx = \frac{-1}{2} \qquad \text{(1 pont)}$$