

- C. 1. $\int \frac{\sin x}{1+\cos^2 x} dx$ 2. $\int_{-\pi}^1 x e^{-x} dx$
 3. $y = x - 1$ és $y = \log_2 x$ közti terület. 4. $\int_{-\infty}^2 \frac{2}{(x-2)^2} dx$

Megoldások vázlatosa:

C1. $y = \cos x$ helyettesítéssel:

$$\begin{aligned} \int \frac{\sin x}{1+\cos^2 x} dx &= \int \frac{\sin x}{1+y^2} \cdot \frac{dy}{-\sin x} && (2 \text{ pont}) \\ &= -\int \frac{1}{1+y^2} = -\arctan y && (2 \text{ pont}) \\ &= -\arctan(\cos x) && (1 \text{ pont}) \end{aligned}$$

C2.

$$\begin{aligned} \int x e^{-x} &= x \cdot \frac{e^{-x}}{-1} - \int 1 \cdot \frac{e^{-x}}{-1} = -(x+1)e^{-x} && (2 \text{ pont}) \\ \int_{-\pi}^1 x e^{-x} dx &= [-(x+1)e^{-x}]_{-\pi}^1 && (1 \text{ pont}) \\ &= (1-\pi)e^{\pi} - \frac{2}{e} && (2 \text{ pont}) \end{aligned}$$

C3.

$$\begin{aligned} x-1 &= \log_2 x = \frac{\log x}{\log 2} \\ x &= 1 \text{ és } x=2 && (1 \text{ pont}) \\ \int \left(x-1 - \frac{\log x}{\log 2}\right) &= \frac{x^2}{2} - x - \frac{1}{\log 2} \int \log x \\ \int \log x &= \int 1 \cdot \log x = x \log x - \int x \cdot \frac{1}{x} \\ &= x(\log x - 1) && (1 \text{ pont}) \\ \int \left(x-1 - \frac{\log x}{\log 2}\right) &= \frac{x^2}{2} - x - \frac{x(\log x - 1)}{\log 2} && (1 \text{ pont}) \\ \int_1^2 \left(x-1 - \frac{\log x}{\log 2}\right) dx &= \left[\frac{x^2}{2} - x - \frac{x(\log x - 1)}{\log 2}\right]_1^2 && (1 \text{ pont}) \\ &= \frac{1}{\log 2} - \frac{3}{2} && (1 \text{ pont}) \end{aligned}$$

C4.

$$\int \frac{2}{(x-2)^2} = 2 \int (x-2)^{-2} = 2 \cdot \frac{(x-2)^{-1}}{-1} = \frac{2}{2-x} \quad (2 \text{ pont})$$

$$\int_a^b \frac{2}{(x-2)^2} dx = \left[\frac{2}{2-x} \right]_a^b = \frac{2}{2-b} - \frac{2}{2-a} \quad (1 \text{ pont})$$

$$\lim_{a \rightarrow -\infty} \frac{2}{2-a} = 0 \qquad \lim_{b \rightarrow 2^-} \frac{2}{2-b} = +\infty \quad (1 \text{ pont})$$

$$\int_{-\infty}^2 \frac{2}{(x-2)^2} dx = +\infty \quad (1 \text{ pont})$$