

Name:

Neptun-code:

**A3 examination test, 2019. Dec. 17.**

1	2	3	4	5	6	7	8	9	$\Sigma$	test1	test2	$\Sigma\Sigma$	grade

1. (10p) Solve the following initial value problem:

$$yy' = xy^2 + x, y(0) = -2.$$

2. (10p) Solve the following differential equation:

$$xy' + y = x^3.$$

3. (10p) Solve the following initial value problem:  $y'' - 2y' + y = 0$ ,  $y(0) = 0$ ,  $y'(0) = 1$ .

4. (10p) Urn A contains 4 red and 8 black balls, whereas urn B contains 2 red and 3 black balls. If a ball is randomly selected from each urn, what is the probability that the balls will be different color?

5. (10p) A company makes electric motors. The probability that a random electric motor is defective is 0.01. What is the probability that a sample of 300 electric motors will contain exactly 5 defective motors (It is fine if you live your expression as an expression containing binomial coefficient)? Now give its approximation using Poisson distribution.

6. (10p) A new tax law is expected to benefit "middle income" families, those with incomes between \$20,000 and \$30,000. If the family income is normally distributed with mean \$25000 and variance \$10000<sup>2</sup>, what percentage of the population will benefit from the law?

7. The density function of the random variable  $X$  is  $f(x) = 2e^{-2x}$ ,  $x > 0$ . a) Give  $P(X > 2)$ ; b) Give  $P(X > 4|X > 2)$ . (Hint: the type of the distribution of  $X$  has a name)

8. A crane can lift at most 9800 kg. A shipment arrives with 49 boxes. The average weight of the boxes is 205kg and the standard deviation is 15kg. What is the approximate probability that the crane can lift the whole shipment? (Hint: use the central Limit Theorem)

9. Solve the following differential equation system:  $x' + x + y' + y = 0$ ,  $x + y' = 0$ ,  $x(0) = 5$ ,  $y(0) = 1$ .