

A3 (exam)

1. $y' \cdot \frac{2y}{1+y^2} = 2x \quad \ln(1+y^2) = x^2 + c$

$c = \ln 5 \quad 1+y^2 = 5e^{x^2} \quad y = -\sqrt{5e^{x^2} - 1}$

2. $y' + \frac{1}{x}y = x^2 \quad y_h' + \frac{1}{x} \cdot y_h = 0 \quad y_h = \frac{c}{x}$

$y_p = c(x) \cdot \frac{1}{x} \quad y_p = x^4/4 \quad y = \frac{x^3}{4} + \frac{c}{x}$

3. $y = c_1 e^x + c_2 x e^x$

$y(0) = 0 : c_1 = 0$

$y'(0) = 1 : c_2 = 1$

$y = x e^x$

4. A: 4 red, 8 black, B: 2 red, 3 black

$p = P(rb) + P(br) = \frac{4}{12} \cdot \frac{3}{5} + \frac{8}{12} \cdot \frac{2}{5} = \frac{7}{15} = 0.466$

5. $P(5 \text{ defect}) = \binom{300}{5} \cdot 0.01^5 \cdot 0.99^{295} \quad (= 0.1009)$

with Poisson distribution: $\lambda = 300 \cdot 0.01 = 3$

$P(5 \text{ defect}) \approx \frac{3^5}{5!} \cdot e^{-3} = 0.1008$

6. Let the income of a random family be X .

$P(2 < X < 3) = P\left(-\frac{1}{2} < \frac{X-2.5}{1} < \frac{1}{2}\right) =$

$= 2\Phi(0.5) - 1 = 0.3830$

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7. ~~7.~~ $X \sim \text{Exp}(2)$

a) $P(X > 2) = 1 - (1 - e^{-2/2}) = \frac{1}{e}$

b) $P(X > 4 | X > 2) =$ (by memoryless)
 $= P(X > 2) = 1/e.$

8. Let X_i be the weight of the i^{th} person,

$$S_{100} = X_1 + \dots + X_{100} \quad P(S_{100} \leq 7250) =$$

$$= P\left(\frac{S_{100} - 7000}{\sqrt{100} \cdot 10} \leq \frac{250}{100}\right) \approx \text{by CLT}$$

$$\approx \Phi(2.5) = 0.984,$$

9. $2Dx - (D+1)y = 0$
 $2(D+1)x - Dy = 0$ $\Delta = \begin{vmatrix} 2D & -(D+1) \\ 2(D+1) & -D \end{vmatrix} = 4D+2$

$$(4D+2)x = 0 \quad 4x' + 2x = 0 \quad x = c_1 e^{-t/2}$$

$$(4D+2)y = 0 \quad 4y' + 2y = 0 \quad y = c_2 e^{-t/2}$$

Plugging these back to the d.e.: $c_2 = -2c_1$

$$x = c_1 e^{-t/2}, \quad y = -2c_1 e^{-t/2}$$

From the initial conditions:

$$x = e^{-t/2}, \quad y = -2e^{-t/2}.$$