**A3 examination test, 2020. Jan. 14.**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | $\sum$ | test1 | test2 | $\sum\sum$ | grade |
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**Name:** **Neptun-code:**

1. Solve the following initial value problem: $xy' = y^2, y(1) = 1$.
2. The function $y(x) = \frac{1}{1+x^2}$ is a solution of the differential equation $y' + \dfrac{3x}{1+x^2} \cdot y = \dfrac{x}{(1+x^2)^2}$. By the above or otherwise solve the initial value problem $y' + \dfrac{3x}{1+x^2} \cdot y = \dfrac{x}{(1+x^2)^2}, y(0) = 2$.
3. Give the equilibrium solutions of the following differential equation, characterize them from the stability point of view and sketch the graphs of some of the non-equilibrium solutions: $y' = (y-1)^2$.
4. If we throw two dice what is the probability that the distance of the two throw results does not exceed 2?
5. We have found two coins in a fountain. One is fair but the other has heads on both sides. We flip one of them three times and find that all the flips resulted in heads. What is the probability that we flipped the fair coin?
6. The frequency of the AB negative blood type in a certain population is 1%. If we choose 100 people in random then what is the

   a) approximate probability (using Poisson distribution) that at least one of them has blood type AB negative?

   b) accurate probability that at least one of them has blood type AB negative?
7. The percentage points of the students learning Math A3 follows normal distribution with mean 65 and standard deviation 17.

   (a) What is the probability that a randomly chosen student receives grade 3 (that is, his/her points are between 60 and 70, inclusive) ?

   (b) Among those who passed (that is, those who have at least 50 points) what is the frequency of those getting a 3?
8. Solve the following differential equation system: $x' + x = -y,\ 2x' = -y' - y$.
9. In many popular children games one has to throw a dice and then move forward by the number of eyes seen. After $n$ rounds, we are at position $S_n$. If you play such a game, find the probability that your position is between 90 and 120 (inclusive) after 30 rounds. The standard deviation of one throw with the die is $\sqrt{35/12}$.