1. $\dfrac{y'}{y^2}=\dfrac{1}{x}$, $\quad -\dfrac{1}{y}=\ln x + c \qquad c=-1$

$y=\dfrac{1}{1-\ln x}$, $\; 0<x<e$.

2. $y_h' + \dfrac{3x}{1+x^2}\cdot y_h = 0$, $\quad y_h'\cdot\dfrac{1}{y_h}=\dfrac{-3x}{1+x^2}$

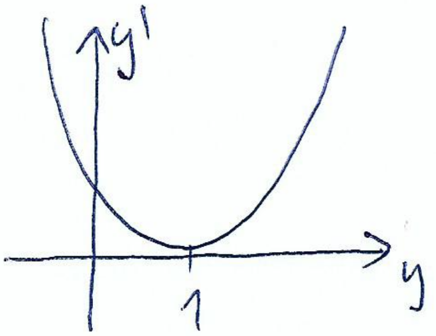
$\ln|y_h| = -\dfrac{3}{2}\displaystyle\int\dfrac{2x}{1+x^2}\,dx = -\dfrac{3}{2}\ln(1+x^2)+c$

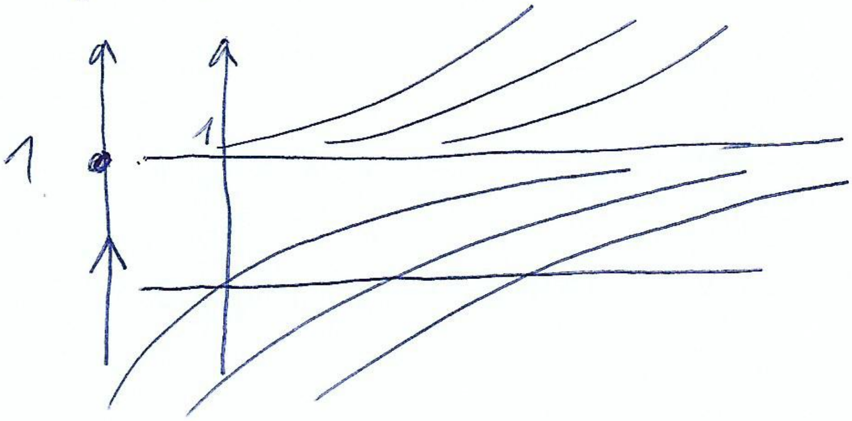
$y_h = c\cdot(1+x^2)^{-3/2}$, $\quad y = y_p + y_h =$

$= \dfrac{1}{1+x^2}+\dfrac{c}{(1+x^2)^{3/2}}$, $\quad c = 1$,

$y(x)=\dfrac{1}{1+x^2}+\dfrac{1}{(1+x^2)^{3/2}}$, $\; -\infty<x<\infty$.

3.





$y\equiv 1$: semi-stable equilibrium solution.

4. Let X_1 and X_2 be the two rolls.

$$P(|X_1 - X_2| \leq 2) = P(X_1 - X_2 = 0) +$$
$$+ P(X_1 - X_2 = \pm 1) + P(X_1 - X_2 = \pm 2) =$$
$$= \frac{1}{36}\left(6 + 2 \cdot 5 + 2 \cdot 4\right) = \frac{2}{3}$$

5. event A: we have flipped the fair coin

event B: all the 3 flips are heads

$$P(A|B) = \frac{\left(\frac{1}{2}\right)^3 \cdot \frac{1}{2}}{\left(\frac{1}{2}\right)^3 \cdot \frac{1}{2} + 1^3 \cdot \frac{1}{2}} = \frac{1}{9}$$ by Bayes' theorem.

6. We have n experiments, each happens with probability 0.01, independently;

a) $\lambda = np = 1$, $P(X \geq 1) = 1 - P(X = 0) \approx 1 - e^{-1} = 0.632$;

b) $P(X \geq 1) = 1 - (1 - 0.01)^{100} = 0,634$

7. X be the points of a random student. Then $X \sim N(65, 17^2)$

a) $P(60 \le X \le 70) = P\left(\frac{60-65}{17} \le Z \le \frac{70-65}{17}\right)$

$= \Phi\left(\frac{5}{17}\right) - \Phi\left(-\frac{5}{17}\right) = 0.231$

b) $P(60 \le X \le 70 \mid X \ge 50) =$

$= \dfrac{P(60 \le X \le 70)}{1 - P(X < 50)} = 0.285.$

8. $\left.\begin{aligned} (D+1)x + y &= 0 \quad (*) \\ 2Dx + (D+1)y &= 0 \end{aligned}\right\}$ $\Delta = \begin{vmatrix} D+1 & 1 \\ 2D & D+1 \end{vmatrix} = D^2 + 1$

$\left.\begin{aligned} x'' + x &= 0 \\ y'' + y &= 0 \end{aligned}\right\}$ $\begin{aligned} x &= c_1 \cos t + c_2 \sin t \\ y &= c_3 \cos t + c_4 \sin t \end{aligned}$ Plugging into $(*)$:

$c_1 + c_2 = -c_3$, $c_2 - c_1 = -c_4$, therefore

$x(t) = c_1 \cos t + c_2 \sin t$, $y(t) = (-c_1 - c_2)\cos t + (c_1 - c_2)\sin t$

9. Let X_i be the i^{th} roll with the die.

Then $S_{30} = X_1 + \dots + X_{30}$.

$P(90 \le S_{30} \le 120) \approx P\left(\frac{90-105}{\sqrt{30}\cdot\sqrt{\frac{35}{12}}} \le Z \le \frac{120-105}{\sqrt{30}\cdot\sqrt{\frac{35}{12}}}\right) =$

$= 2\Phi\left(\frac{15}{\sqrt{30}\cdot\sqrt{\frac{35}{12}}}\right) - 1 = 0.891$

(Exact val. 0.9025)