The averaging method is a good tool for studying limit cycles of differential systems.

In this talk, we first present an efficient symbolic program using Maple for computing the averaged functions of any order for continuous differential systems. The program allows us to systematically analyze zero-Hopf bifurcations of polynomial differential systems using symbolic computation methods. Then we study the number of limit cycles that may bifurcate from an equilibrium of an autonomous system of differential equations.

The system in question is assumed to be of dimension n, have a zero-Hopf equilibrium at the origin, and consist only of homogeneous terms of order m. Denote by $H_k(n,m)$ the maximum number of limit cycles of the system that can be detected by using the averaging method of order k. We prove that $H_1(n,m)\leq(m-1)\cdot m^{n-2}$ and $H_k(n,m)\leq(km)^{n-1}$ for generic $n\geq3$, $m\geq2$ and $k>1$.The exact numbers of $H_k(n,m)$ or tight bounds on the numbers are determined by computing the mixed volumes of some polynomial systems obtained from the averaged functions. A number of examples are presented to demonstrate the effectiveness of the proposed algorithmic approach.