Asymptotic Growth in Open Chemical Reaction Networks: Dynamics and Thermodynamics

October 15, 2024

Shesha G. M. Srinivas[*], Francesco Avanzini[*^], Massimiliano Esposito[*]

*Department of Physics and Material Sciences, University of Luxembourg
^Department of Chemical Sciences, University of Padova

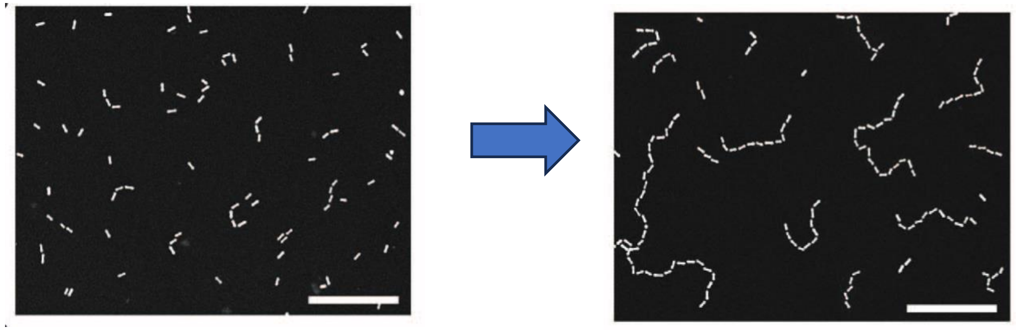
Phys. Rev. E, **109**, 064153 (2024)

shesha.marehalli@uni.lu

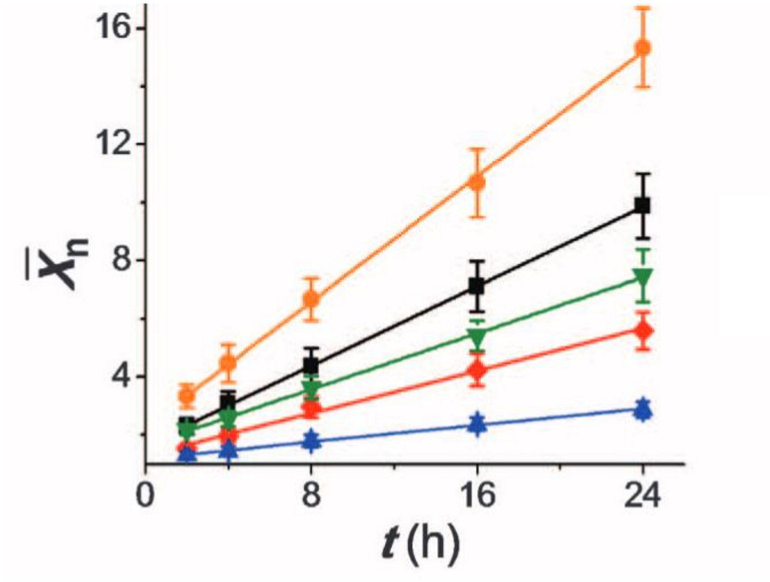
Outline

- Motivation and Problem Statement
- Formalism: Dynamics and Thermodynamics of open CRNs
- Results
- Prospects

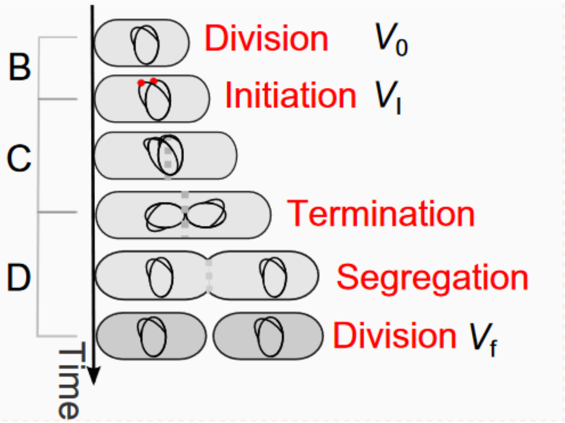
Growth is everywhere



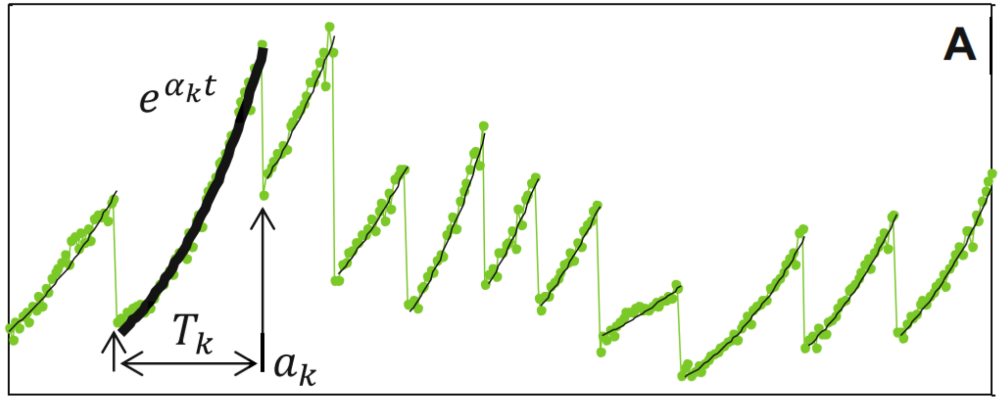
Polymerization



Science, **329**, 197 (2010)



E-coli cell cycle *Sci. Adv.*, **4**,eaau3324(2018)



Protein count *Eur. Phys. J. E* **38**, 102 (2015)

Concentration Growth in the Literature

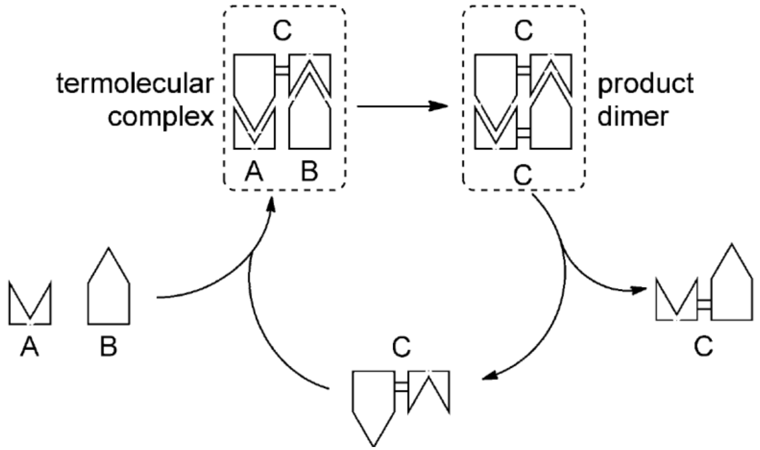
Non enzymatic Template Replication

$$A + B + C \underset{-1}{\overset{+1}{\rightleftharpoons}} ABC$$

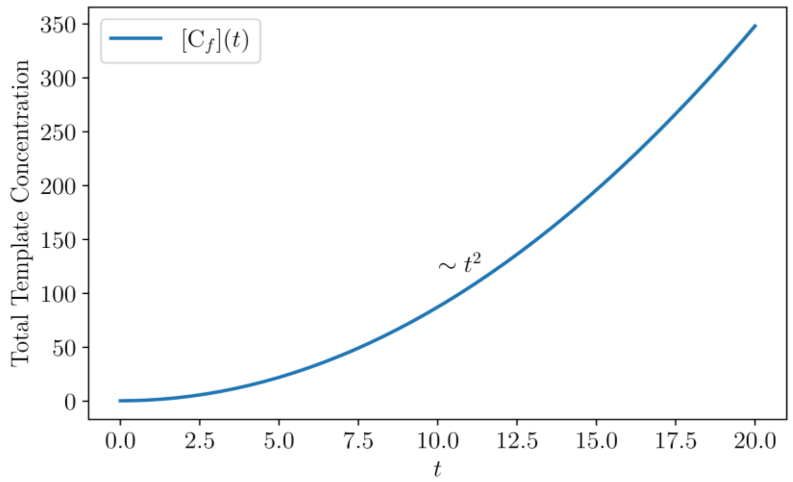
$$ABC \xrightarrow{+2} C_2$$

$$C_2 \underset{-3}{\overset{+3}{\rightleftharpoons}} 2C$$

Bulletin of Mathematical Biology **60**, 1073 (1998)



Angew. Chem. Int. Ed., **52**: 12800-12826 (2013)



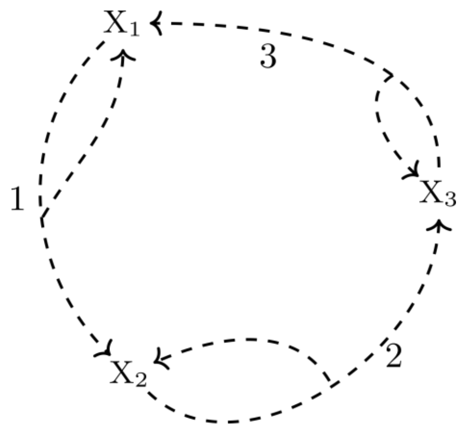
Hinshelwood Cycle

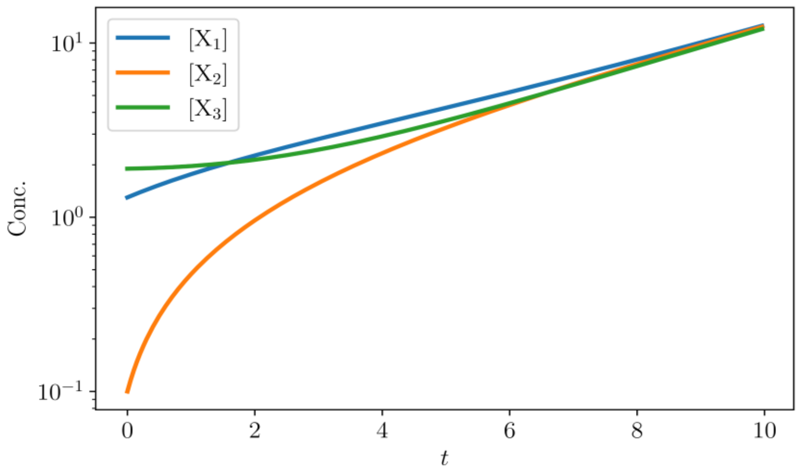
$$X_1 \xrightarrow{1} X_1 + X_2$$

$$X_2 \xrightarrow{2} X_2 + X_3$$

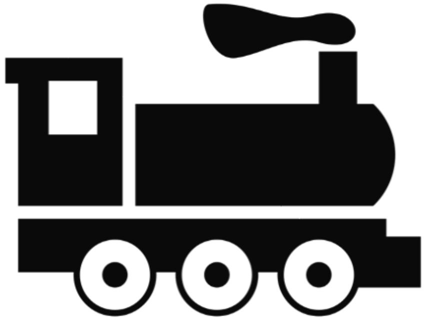
$$X_3 \xrightarrow{3} X_3 + X_1$$

Phys. Rev. Lett. 113, 028101 (2014)



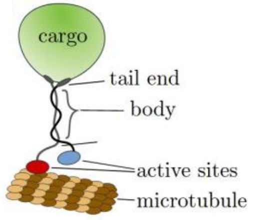


Nonequilibrium Thermodynamics



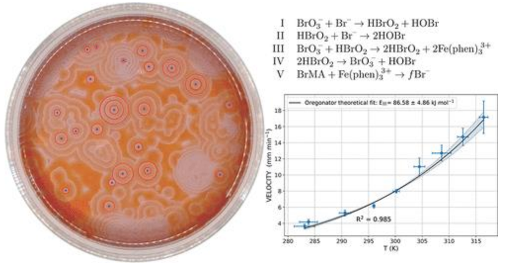
Macroscale:
Equilibrium Thermodynamics

Phys. Rev. E 92, 042133 (2015)



Mesoscale:
Stochastic Thermodynamics

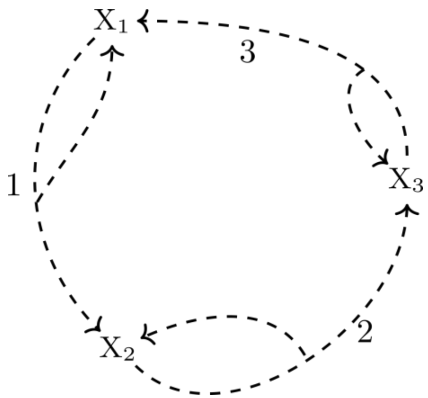
Rep. Prog. Phys. **75** 126001(2012)
Physica A, **418**, 6-16 (2015)



Macroscopic:
Nonequilibrium Thermodynamics

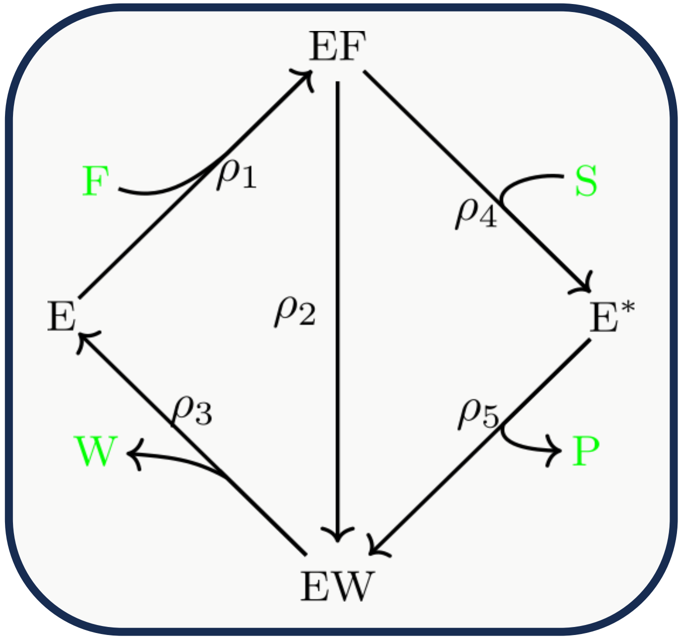
J. Chem. Phys. 141, 024117 (2014)
Phys. Rev. X 6, 041064 (2016)
J. Chem. Phys. 154, 094114 (2021)

Problem – Irreversible and coarse-grained reactions!



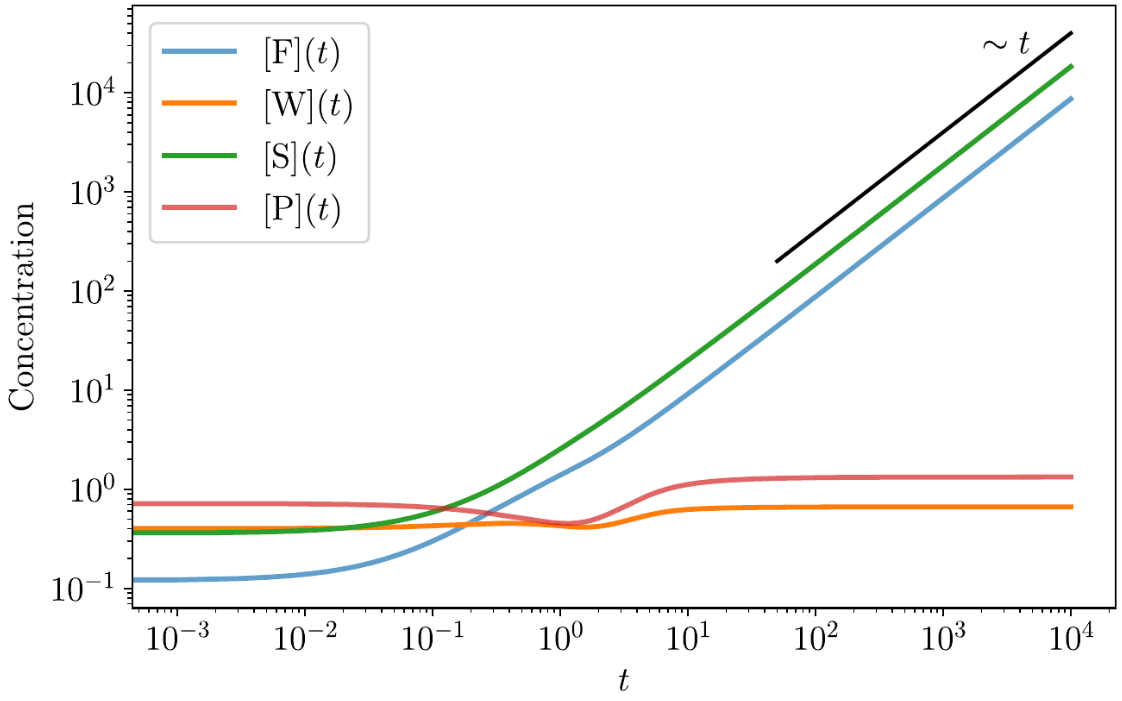
Question: Is concentration growth possible in CRNs with all reversible reactions?

Growth in open CRNs



Minimal Model of a Metabolic Network

J. Chem. Phys. 156, 014116 (2022)



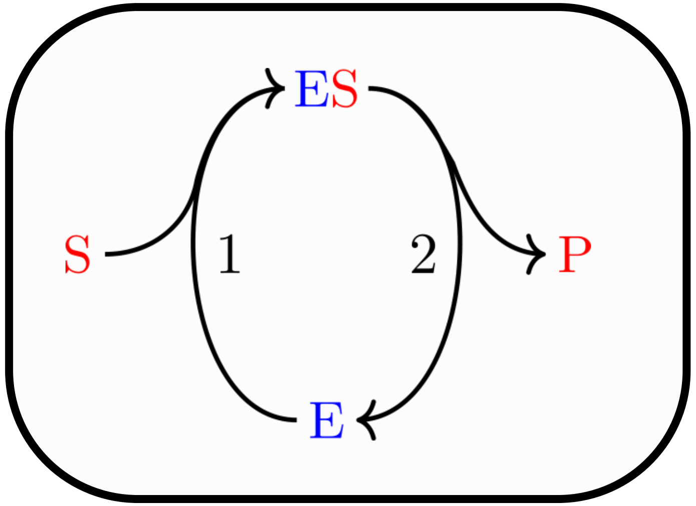
Key Questions:

- When does growth* occur?
- What is the dynamics of the growing state?
- Can we estimate the cost of growth?

Outline

- Motivation and Problem Statement
- Formalism: Dynamics and Thermodynamics of open CRNs
- Results
- Prospects

Topology of CRNs: Setup



Michealis-Menten CRN

$$\boldsymbol{\alpha} = (\mathrm{E}, \mathrm{ES}, \mathrm{S}, \mathrm{P})$$

$$\mathbb{S} = \begin{array}{c} \\ \mathrm{E} \\ \mathrm{ES} \\ \mathrm{S} \\ \mathrm{P} \end{array} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{pmatrix} -1 & 1 \\ 1 & -1 \\ -1 & 0 \\ 0 & 1 \end{pmatrix} \end{array}$$

$$\boldsymbol{\ell}^{\mathrm{S}} = \begin{pmatrix} \overset{\mathrm{E}}{0} & \overset{\mathrm{ES}}{1} & \overset{\mathrm{S}}{1} & \overset{\mathrm{P}}{1} \end{pmatrix}$$

$$\boldsymbol{\ell}^{\mathrm{E}} = \begin{pmatrix} \overset{\mathrm{E}}{1} & \overset{\mathrm{ES}}{1} & \overset{\mathrm{S}}{0} & \overset{\mathrm{P}}{0} \end{pmatrix}$$

$$\boldsymbol{\nu}_{+\rho} \cdot \boldsymbol{\alpha} \underset{-\rho}{\overset{+\rho}{\rightleftharpoons}} \boldsymbol{\nu}_{-\rho} \cdot \boldsymbol{\alpha}$$

Formal reaction

$$\mathbb{S}_\rho = \boldsymbol{\nu}_{-\rho} - \boldsymbol{\nu}_{+\rho}$$

Stoichiometric Matrix

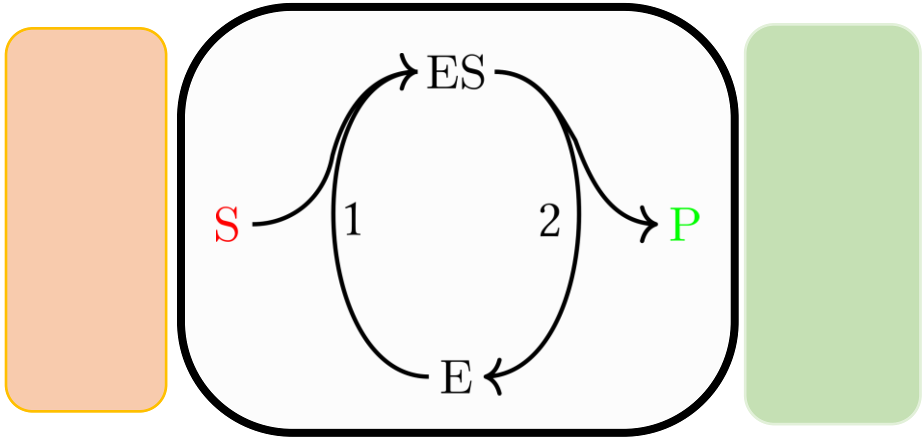
$$\boldsymbol{\ell}^\lambda \cdot \mathbb{S} = 0$$

Conservation Laws

$$\boldsymbol{\ell}^m > 0$$

Mass Conservation Law

Dynamics of CRNs : Reactions



$$\mathbb{S} = \begin{array}{c} \\ E \\ ES \\ S \\ P \end{array} \begin{pmatrix} 1 & 2 \\ -1 & 1 \\ 1 & -1 \\ \hdashline -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{array}{c} \mathbb{S}^X \\ \mathbb{S}^Y \end{array}$$

$$j_{+1} = k_{+1}[\mathrm{E}][\mathrm{S}]$$
$$j_{-1} = k_{-1}[\mathrm{ES}]$$

Concentrations

$$\boldsymbol{z} = \underbrace{(\ldots[\alpha]\ldots)^\intercal}_{(\boldsymbol{x},\boldsymbol{y})^\intercal} \Rightarrow d_t\boldsymbol{z} = \mathbb{S}\boldsymbol{j}(\boldsymbol{z}) + \boldsymbol{I} \Rightarrow$$

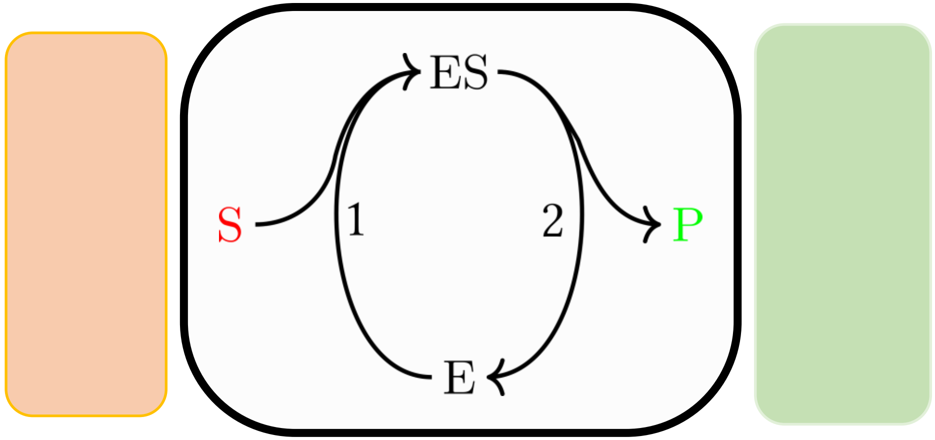
$$d_t\boldsymbol{x} = \mathbb{S}^X\boldsymbol{j}(\boldsymbol{x},\boldsymbol{y})$$ Internal Species

$$d_t\boldsymbol{y} = \mathbb{S}^Y\boldsymbol{j}(\boldsymbol{x},\boldsymbol{y}) + \boldsymbol{I}^Y$$ Exchange Species

Reaction currents

$$j_\rho(\boldsymbol{z}(t)) = j_{+\rho}(\boldsymbol{z}(t)) - j_{-\rho}(\boldsymbol{z}(t)) \Rightarrow j_{\pm\rho}(\boldsymbol{z}(t)) = k_{\pm\rho}\boldsymbol{z}^{\boldsymbol{\nu}_{\pm\rho}}$$ Ideal Dilute Solutions

Dynamics of CRNs : Reactions II



$$\mathbb{S} = \begin{array}{c} \\ E \\ ES \\ S \\ P \end{array} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{pmatrix} -1 & 1 \\ 1 & -1 \\ -1 & 0 \\ 0 & 1 \end{pmatrix} \end{array}$$

$$\boldsymbol{\ell}^{S} = \begin{array}{cccc} E & ES & S & P \\ (0 & 1 & 1 & 1) \end{array}$$

$$d_t L^{S} = I_{S} + I_{P} > 0$$

$$\boldsymbol{\ell}^{E} = \begin{array}{cccc} E & ES & S & P \\ (1 & 1 & 0 & 0) \end{array}$$

$$d_t L^{E} = 0$$

$$d_t \boldsymbol{x} = \mathbb{S}^{X} \boldsymbol{j}(\boldsymbol{x}, \boldsymbol{y})$$
$$d_t \boldsymbol{y} = \mathbb{S}^{Y} \boldsymbol{j}(\boldsymbol{x}, \boldsymbol{y}) + \boldsymbol{I}^{Y}$$

$$L^{\lambda} = \boldsymbol{\ell}^{\lambda} \cdot \boldsymbol{z} \Rightarrow$$

$$d_t L^{\lambda} = \boldsymbol{\ell}^{\lambda}_{Y} \cdot \boldsymbol{I}^{Y} \neq 0$$

$$\ell^{m} > 0 \to L^{m}(t)$$

Exchange Mechanisms

Concentration control

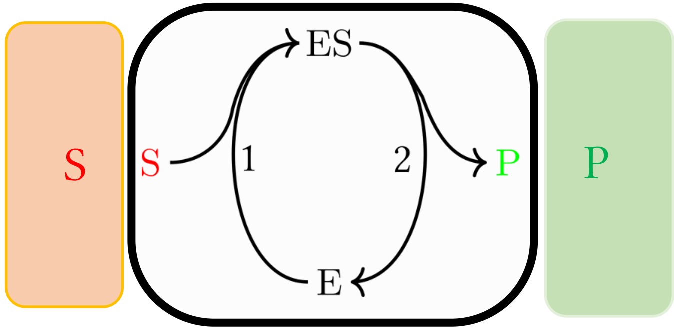
$$d_t\boldsymbol{x} = \mathbb{S}^X \boldsymbol{j}\left(\boldsymbol{x}(t), \boldsymbol{y}(0)\right)$$
$$d_t\boldsymbol{y} = \mathbb{S}^Y \boldsymbol{j}\left(\boldsymbol{x}(t), \boldsymbol{y}(0)\right) + \boldsymbol{I}^Y = 0$$

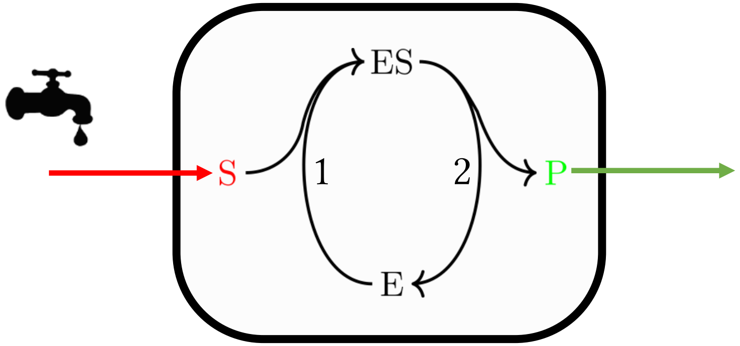
Flux control

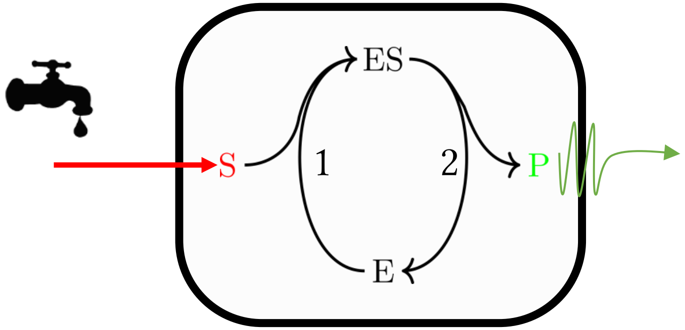
$$d_t\boldsymbol{x} = \mathbb{S}^X \boldsymbol{j}\left(\boldsymbol{x}(t), \boldsymbol{y}(t)\right)$$
$$d_t\boldsymbol{y} = \mathbb{S}^Y \boldsymbol{j}\left(\boldsymbol{x}(t), \boldsymbol{y}(t)\right) + \tilde{\boldsymbol{I}}$$

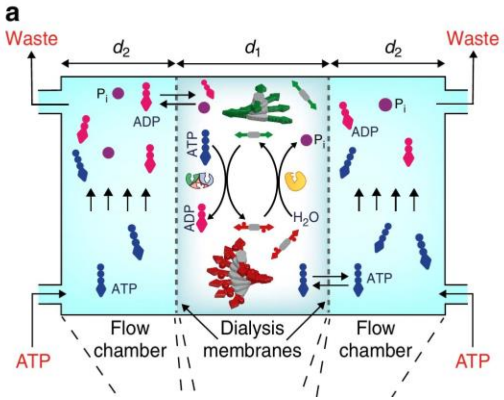
Mixed control

$$d_t\boldsymbol{x} = \mathbb{S}^X \boldsymbol{j}\left(\boldsymbol{x}(t), \boldsymbol{y}(t)\right)$$
$$d_t\boldsymbol{y} = \mathbb{S}^Y \boldsymbol{j}\left(\boldsymbol{x}(t), \boldsymbol{y}(t)\right) - \tilde{\mathbb{D}}\boldsymbol{y} + \tilde{\boldsymbol{I}}$$

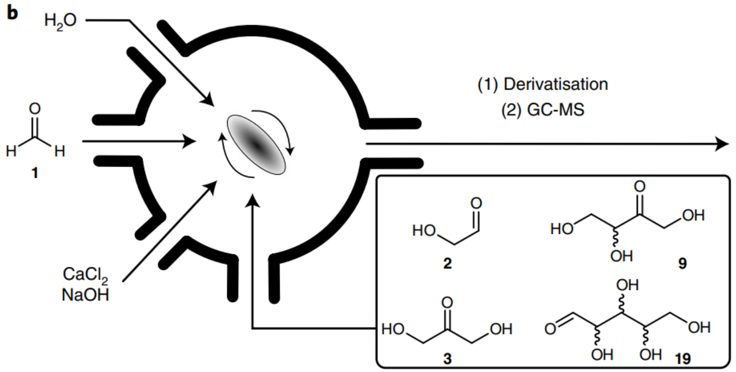








Chemostats
Nat Commun **8,** 15899 (2017)



CSTR
Nat. Chem. **14**, 623–631 (2022)
J. Chem. Phys. 148, 144902 (2018)

Thermodynamics

Phys. Rev. X 6, 041064 (2016)
J. Chem. Phys. 154, 094114 (2021)

Chemical Potentials $\boldsymbol{\mu}(\boldsymbol{z}) = \boldsymbol{\mu}^0 + RT\ \ln(\boldsymbol{z})$

Local Detailed Balance $\boldsymbol{\mu} \cdot \mathbb{S}_\rho = -RT\ln\left(\frac{j_{+\rho}(\boldsymbol{z})}{j_{-\rho}(\boldsymbol{z})}\right)$

$$T\dot{\Sigma} = \dot{w}_{\mathrm{c}} - d_t G \geq 0$$

$$T\dot{\Sigma} = RT\sum_\rho j_\rho \ln\left(\frac{j_{+\rho}}{j_{-\rho}}\right) \geq 0$$

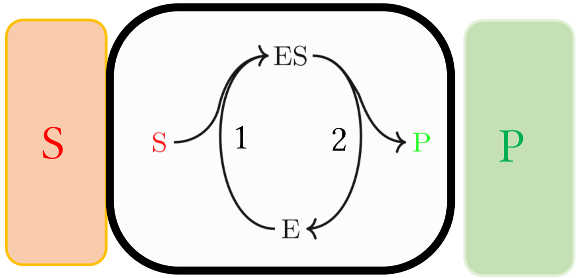
Entropy Production Rate

$$\dot{w}_{\mathrm{c}} = \sum_{\alpha\in Y} \mu_\alpha I_\alpha^Y$$

Chemical Work rate

$$G(\boldsymbol{z}) = \sum_{\alpha\in Z} (\mu_\alpha - RT)[\alpha]$$

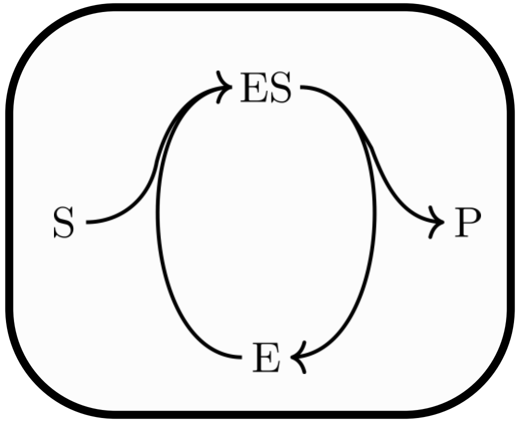
Gibbs Free Energy



$$\dot{w}_{\mathrm{c}} = \mu_{\mathrm{S}} I_{\mathrm{S}} + \mu_{\mathrm{P}} I_{\mathrm{P}}$$

Thermodynamics: Equilibria vs NESS

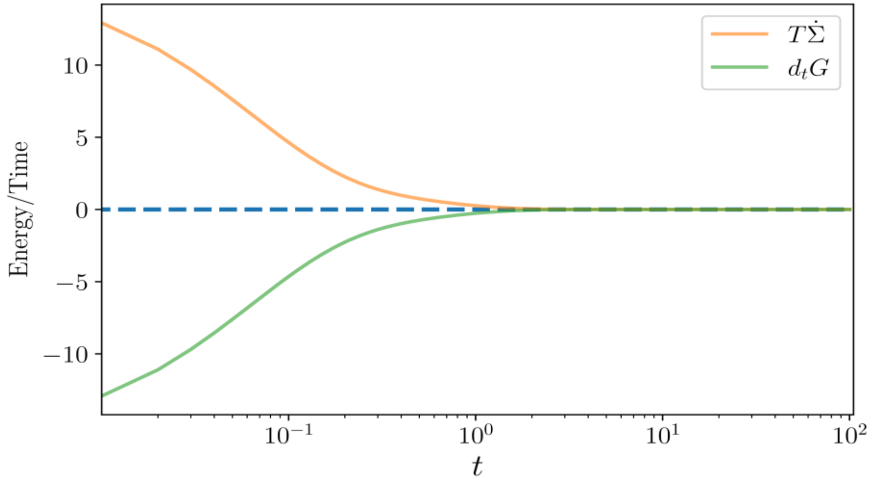
Equilibrium



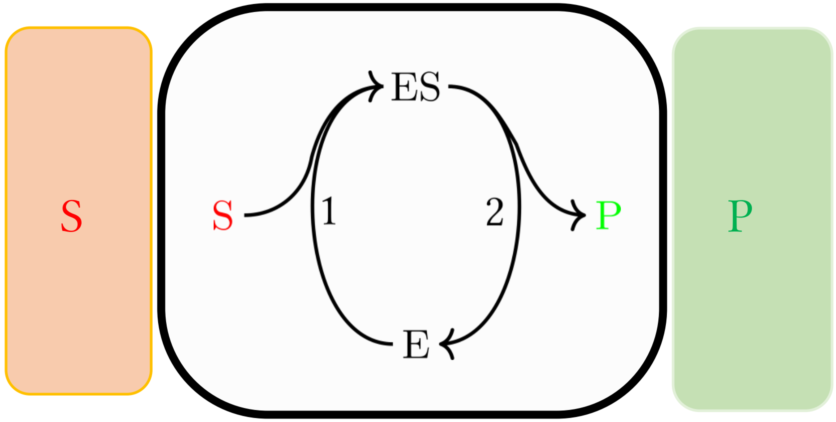
$$\boldsymbol{z}(t) \to \boldsymbol{z}_{\mathrm{eq}}$$

$$j_\rho \to 0$$

$$T\dot{\Sigma} \to 0$$



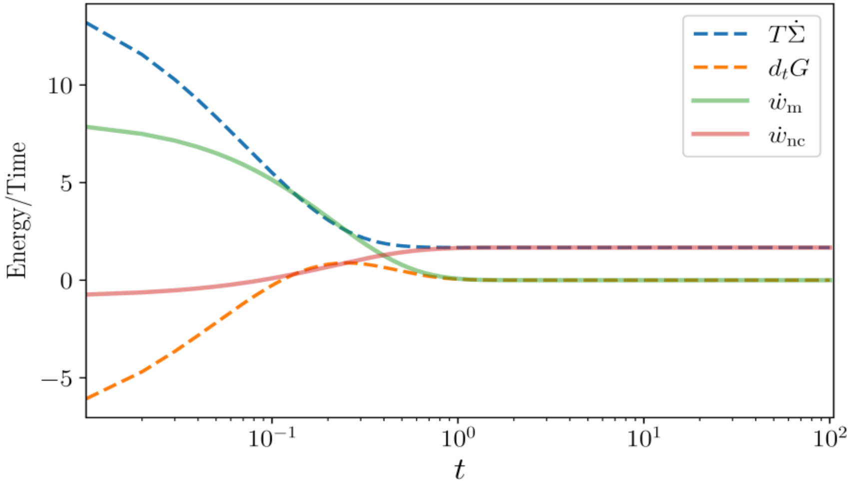
NESS



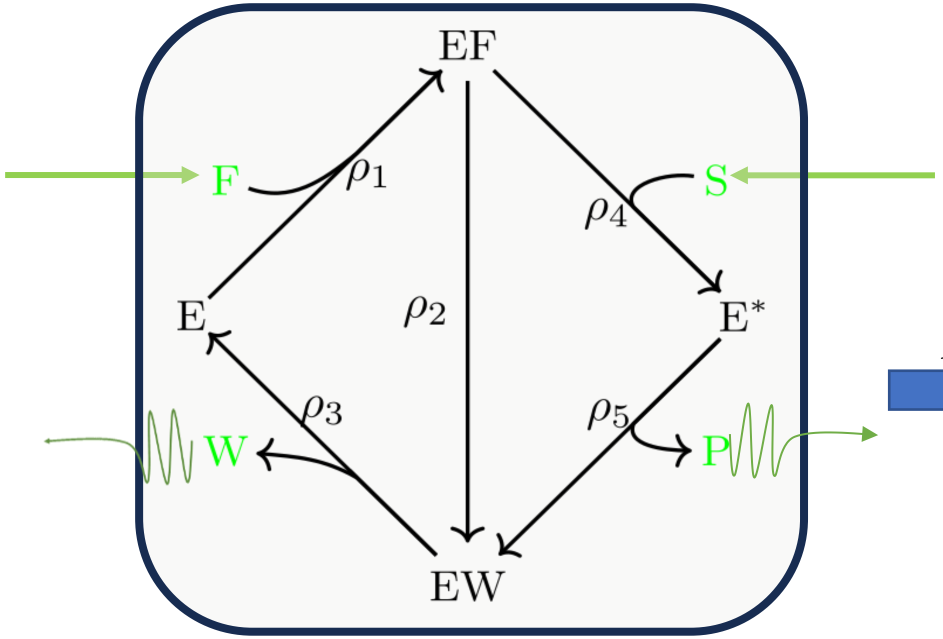
$$\boldsymbol{z}(t) \to \boldsymbol{z}_{\mathrm{ss}}$$

$$j_\rho \to \bar{j} \neq 0$$

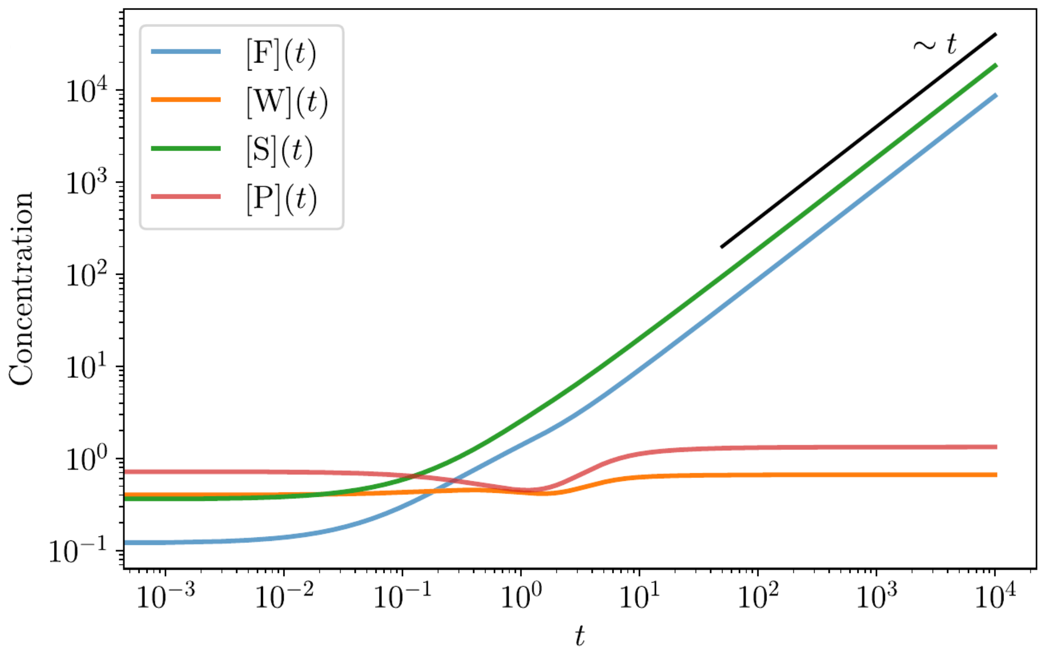
$$T\dot{\Sigma}, \dot{w}_c \to (\mu_{\mathrm{P}} - \mu_{\mathrm{S}})\bar{j} > 0$$



Growth with reversible reactions



Mixed Control



Minimal Model of a Metabolic Network

J. Chem. Phys. 156, 014116 (2022)

Growth $\Rightarrow$ $\lim_{t\to\infty} ||\boldsymbol{z}(t) - \boldsymbol{z}(0)|| = \infty$

Key Questions:

- When does growth* occur?
- What is the dynamics of the growing state?
- Can we estimate the cost of growth?

Outline

- Motivation and Problem Statement
- Formalism: Dynamics and Thermodynamics of open CRNs
- Results
- Prospects

Results: When?

Criteria for Growth:

$$\boxed{\text{Growth} \iff L^m(t) \to \infty \iff G(\boldsymbol{z}) \to \infty}$$

Closed CRN:

$$d_t L^m = 0$$

No Growth

CSTR

$$d_t L^m = -\boldsymbol{\ell}^m \cdot \mathbb{D}\boldsymbol{z} + \sum_\alpha \ell^m_\alpha I_\alpha = -\epsilon L^m + \sum_\alpha \ell^m_\alpha I_\alpha$$

No Growth

Flux Control

$$d_t L^m = \sum_\alpha \ell^m_\alpha I_\alpha \geq 0$$

Growth!

Results: Systematic Analysis

	Flux Control	Mixed Control	Conc. Control
Dynamically Linear	Growth	No Growth	No Growth
Dynamically Nonlinear	Growth	Growth	No Growth*

Dynamically Linear: Unimolecular, Pseudo-unimolecular CRNs

Dynamically Nonlinear: Multimolecular CRNs

Exchange Mechanisms

Concentration control

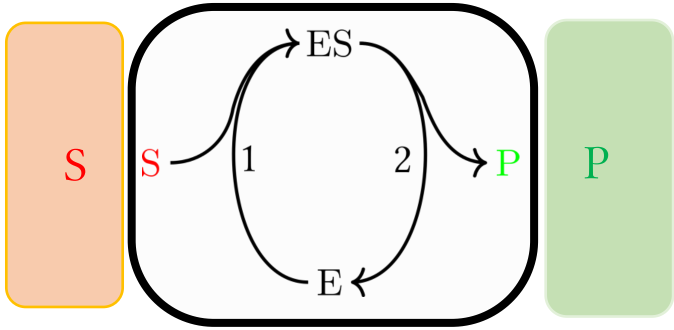
$$d_t\boldsymbol{x} = \mathbb{S}^X \boldsymbol{j}\left(\boldsymbol{x}(t), \boldsymbol{y}(0)\right)$$
$$d_t\boldsymbol{y} = \mathbb{S}^Y \boldsymbol{j}\left(\boldsymbol{x}(t), \boldsymbol{y}(0)\right) + \boldsymbol{I}^Y = 0$$

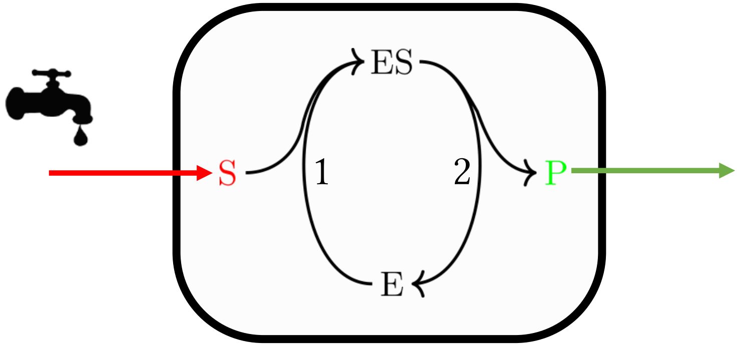
Flux control

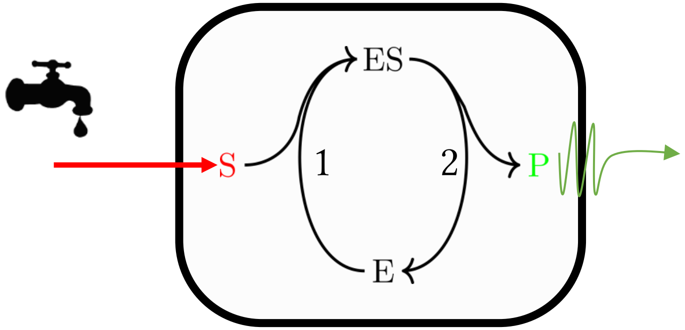
$$d_t\boldsymbol{x} = \mathbb{S}^X \boldsymbol{j}\left(\boldsymbol{x}(t), \boldsymbol{y}(t)\right)$$
$$d_t\boldsymbol{y} = \mathbb{S}^Y \boldsymbol{j}\left(\boldsymbol{x}(t), \boldsymbol{y}(t)\right) + \tilde{\boldsymbol{I}}$$

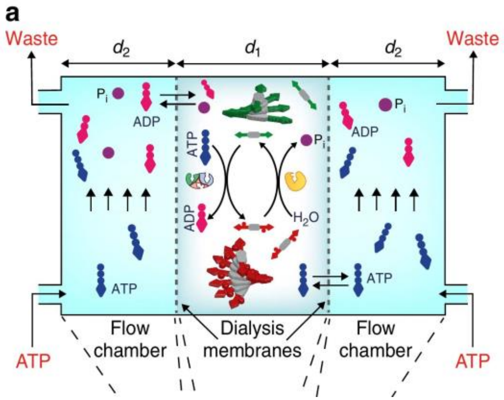
Mixed control

$$d_t\boldsymbol{x} = \mathbb{S}^X \boldsymbol{j}\left(\boldsymbol{x}(t), \boldsymbol{y}(t)\right)$$
$$d_t\boldsymbol{y} = \mathbb{S}^Y \boldsymbol{j}\left(\boldsymbol{x}(t), \boldsymbol{y}(t)\right) - \tilde{\mathbb{D}}\boldsymbol{y} + \tilde{\boldsymbol{I}}$$

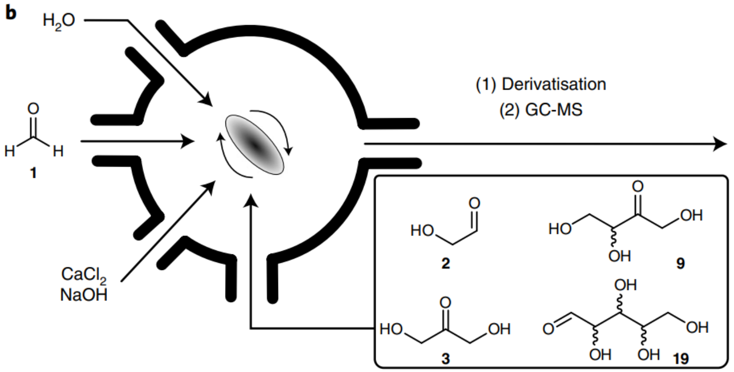








Chemostats
Nat Commun **8,** 15899 (2017)



CSTR
Nat. Chem. **14**, 623–631 (2022)
J. Chem. Phys. 148, 144902 (2018)

Unimolecular CRNs

$$X_1 \underset{-1}{\overset{+1}{\rightleftharpoons}} X_2$$

$$X_2 \underset{-2}{\overset{+2}{\rightleftharpoons}} X_3$$

$$X_3 \underset{-3}{\overset{+4}{\rightleftharpoons}} X_4$$

$$X_4 \underset{-4}{\overset{+4}{\rightleftharpoons}} X_1$$

$$\boldsymbol{\ell}^{\boldsymbol{m}} = (1, 1, 1\dots)^{\intercal}$$

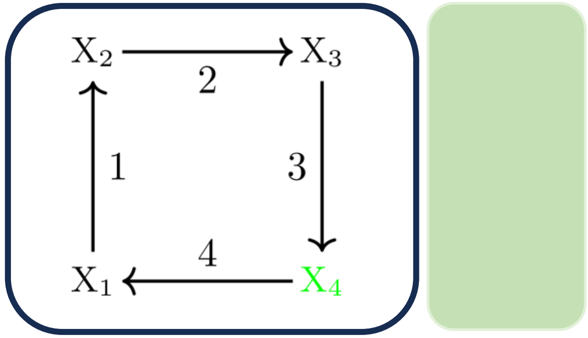
Mass Conservation Law

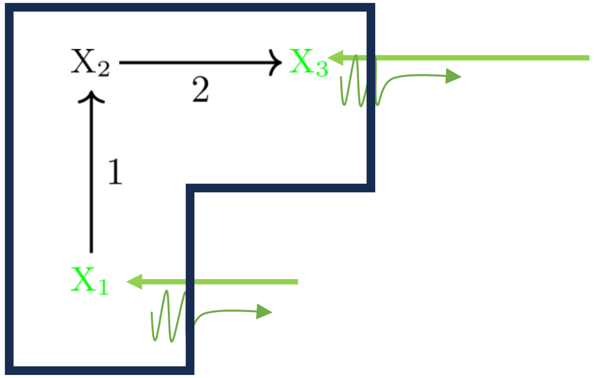
$$d_t \boldsymbol{z} = \mathbb{W} \boldsymbol{z} \xrightarrow[\lambda(\mathbb{W}) \leq 0]{\mathbb{W}\boldsymbol{\pi}^{\mathrm{eq}} = 0} \boldsymbol{z}(t) \to \boldsymbol{z}_{\mathrm{eq}} = L^m(0)\boldsymbol{\pi}^{\mathrm{eq}}$$

Closed dynamics

Universal form for open dynamics

$$d_t \boldsymbol{z} = (\mathbb{W} - \mathbb{D})\boldsymbol{z} + \boldsymbol{I}$$





Unimolecular CRNs

	Flux Control	Mixed Control	Conc. Control
Dynamically Linear	Growth	No Growth	No Growth
Dynamically Nonlinear	Growth	Growth	No Growth*

Mass Conservation

$$\boldsymbol{\ell}^{\boldsymbol{m}} = (1, 1, 1 \dots)^\intercal$$

$$X_1 \underset{-1}{\overset{+1}{\rightleftharpoons}} X_2$$

$$X_2 \underset{-2}{\overset{+2}{\rightleftharpoons}} X_3$$

$$d_t \boldsymbol{z} = \mathbb{W} \boldsymbol{z}$$

Closed dynamics

$$\xrightarrow[\lambda(\mathbb{W}) \leq 0]{\mathbb{W}\boldsymbol{\pi}^{\text{eq}} = 0}$$

$$\boldsymbol{z}(t) \to \boldsymbol{z}_{\text{eq}} = L^{m}(0)\boldsymbol{\pi}^{\text{eq}}$$

Mixed/Concentration Control

$$d_t \boldsymbol{z} = (\mathbb{W} - \mathbb{D})\boldsymbol{z} + \boldsymbol{I}$$

$$\xrightarrow[\lambda(\mathbb{W} - \mathbb{D}) < 0]{\mathbb{D} \neq \mathbb{0}}$$

$$\boldsymbol{z}(t) \to -(\mathbb{W} - \mathbb{D})^{-1}\boldsymbol{I}$$

No growth!

Unimolecular CRNs

	Flux Control	Mixed Control	Conc. Control
Dynamically Linear	Growth	No Growth	No Growth
Dynamically Nonlinear	Growth	Growth	No Growth*

$$d_t \boldsymbol{z} = (\mathbb{W} - \mathbb{D})\boldsymbol{z} + \boldsymbol{I}$$

$\mathbb{D} = \mathbb{0}$

$$d_t L^m = \sum_\alpha I_\alpha$$

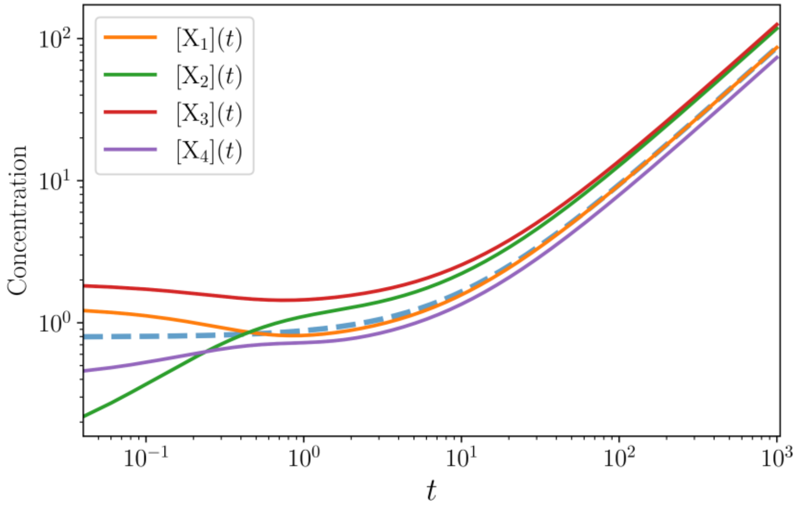
$$\boldsymbol{z}(t) = L^m(t)\boldsymbol{\pi}^{\mathrm{eq}} + \boldsymbol{c} = \mathcal{O}(t)$$

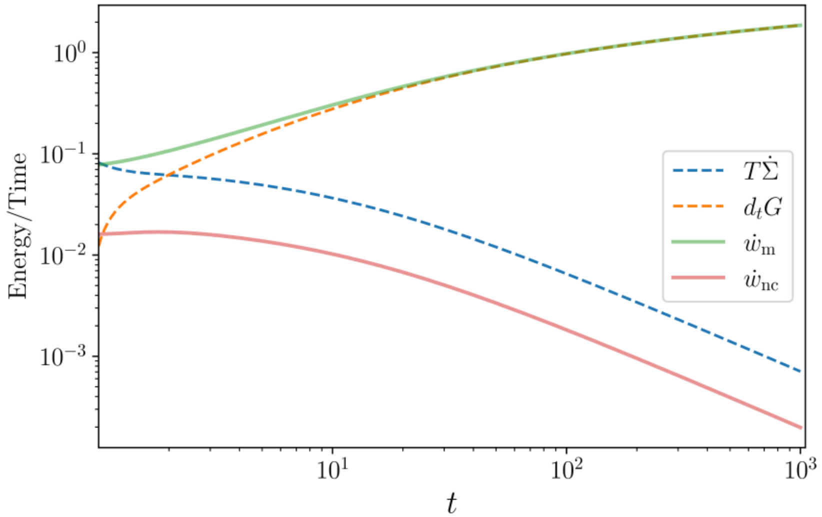
All species grow

$$d_t G \sim \dot{w}_c = \mathcal{O}(\ln(t))$$

$$T\dot{\Sigma} = \mathcal{O}(1/t)$$

Growth close to equilibrium

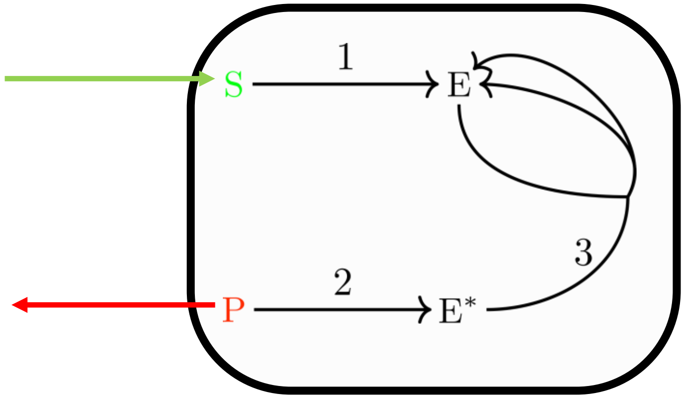


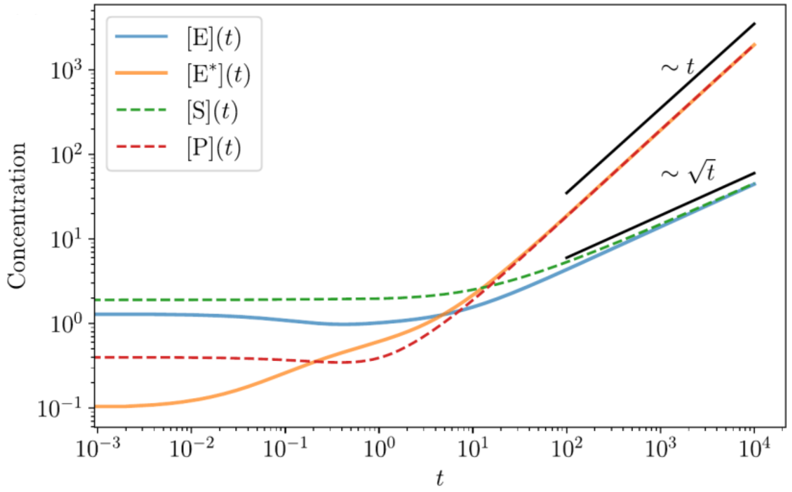


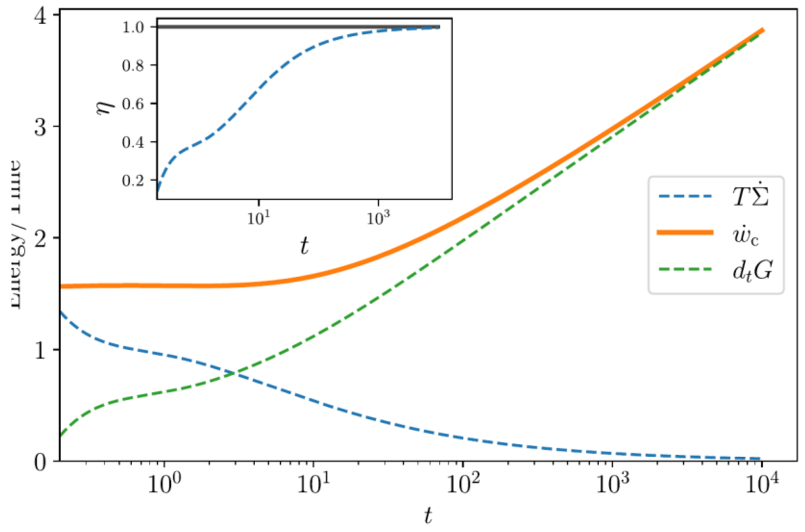
Multimolecular CRNs

	Flux Control	Mixed Control	Conc. Control
Dynamically Linear	Growth	No Growth	No Growth
Dynamically Nonlinear	Growth	Growth	No Growth*

Nonlinear Growth close to equilibrium







All species grow

$$\mathrm{S} \underset{-1}{\overset{+1}{\rightleftharpoons}} \mathrm{E}$$

$$\mathrm{P} \underset{-2}{\overset{+2}{\rightleftharpoons}} \mathrm{E}^*$$

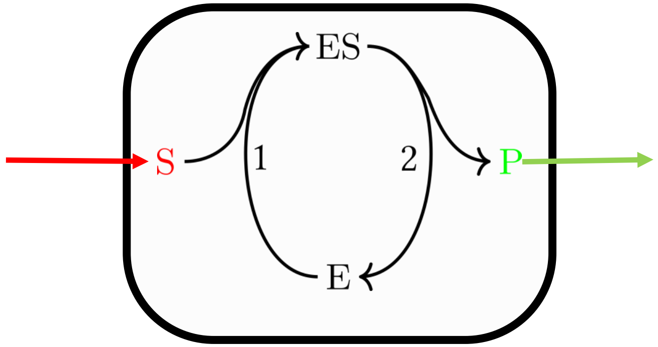
$$\mathrm{E} + \mathrm{E}^* \underset{-3}{\overset{+3}{\rightleftharpoons}} 3\mathrm{E}$$

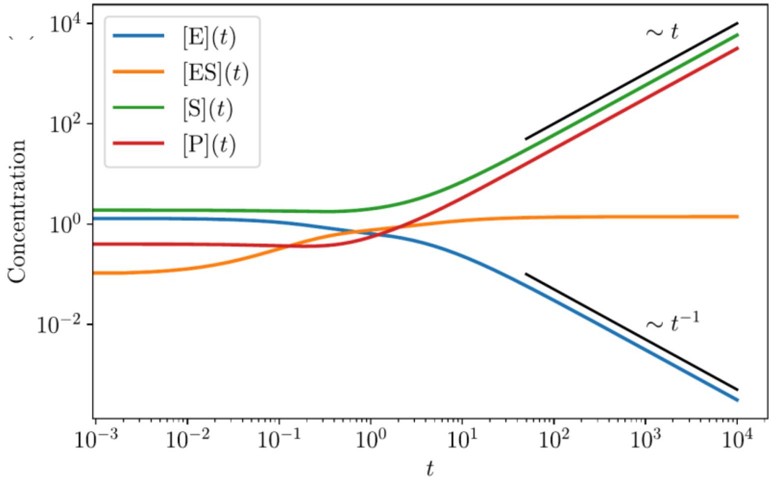
$$\boldsymbol{\ell}^m = \begin{pmatrix} \overset{\mathrm{E}}{1} & \overset{\mathrm{E}^*}{2} & \overset{\mathrm{S}}{1} & \overset{\mathrm{P}}{2} \end{pmatrix}^\intercal$$

$$\boldsymbol{z}(t) = \boldsymbol{z}_{\mathrm{eq}}(L^m(t)) + \boldsymbol{c}(t) \begin{cases} [\mathrm{E}]_{\mathrm{eq}}(t) \propto [\mathrm{S}]_{\mathrm{eq}}(t) \propto \sqrt{[\mathrm{P}]_{\mathrm{eq}}(t)} \propto \sqrt{[\mathrm{E}^*]_{\mathrm{eq}}(t)} \\ d_t G \sim \dot{w}_c = \mathcal{O}(\ln(t)) \\ T\dot{\Sigma} \to 0 \end{cases}$$

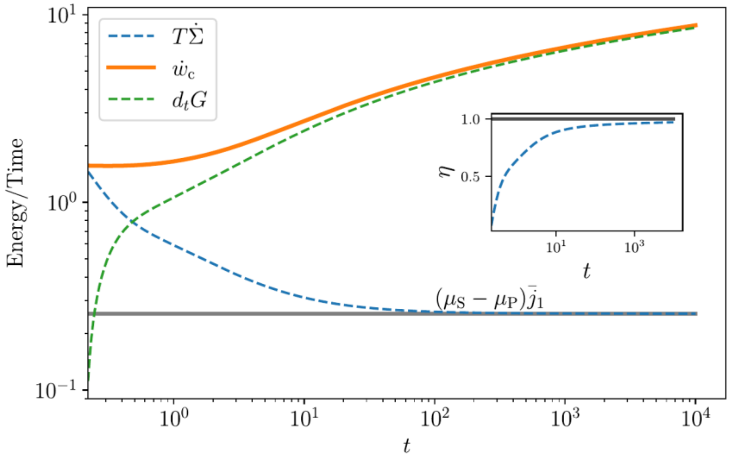
Multimolecular CRNs

	Flux Control	Mixed Control	Conc. Control
Dynamically Linear	Growth	No Growth	No Growth
Dynamically Nonlinear	Growth	Growth	No Growth*





Some species grow



Nonequilibrium growth

$$\boldsymbol{\ell}^{\mathrm{E}} = \begin{pmatrix} \overset{\mathrm{E}}{1} & \overset{\mathrm{ES}}{1} & \overset{\mathrm{S}}{0} & \overset{\mathrm{P}}{0} \end{pmatrix} \qquad d_t L^{\mathrm{E}} = 0$$

$$\boldsymbol{\ell}^{\mathrm{S}} = \begin{pmatrix} \overset{\mathrm{E}}{0} & \overset{\mathrm{ES}}{1} & \overset{\mathrm{S}}{1} & \overset{\mathrm{P}}{1} \end{pmatrix} \qquad d_t L^{\mathrm{S}} = I_{\mathrm{S}} + I_{\mathrm{P}} > 0$$

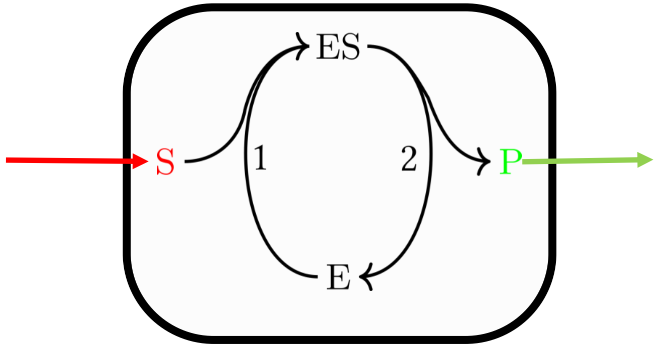
QSS Approximation

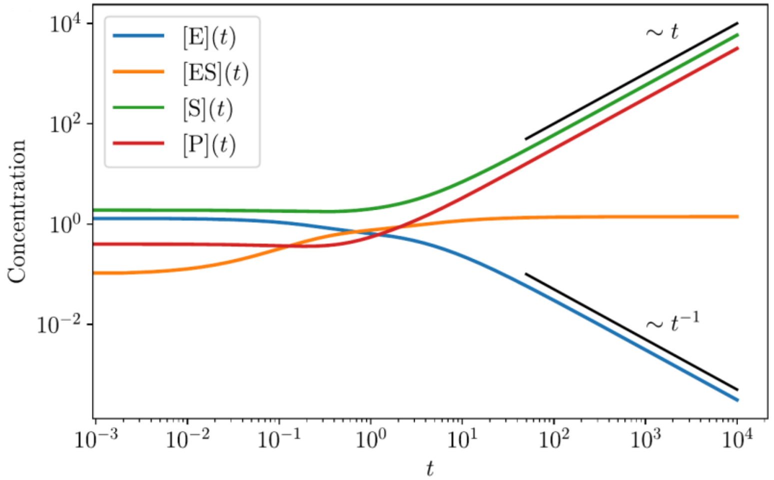
$$d_t \boldsymbol{x} = \mathbb{S}^X \bar{\boldsymbol{j}} = 0$$

$$d_t \boldsymbol{y} = \mathbb{S}^Y \bar{\boldsymbol{j}}(\boldsymbol{y}) + \boldsymbol{I}^Y(\boldsymbol{y})$$

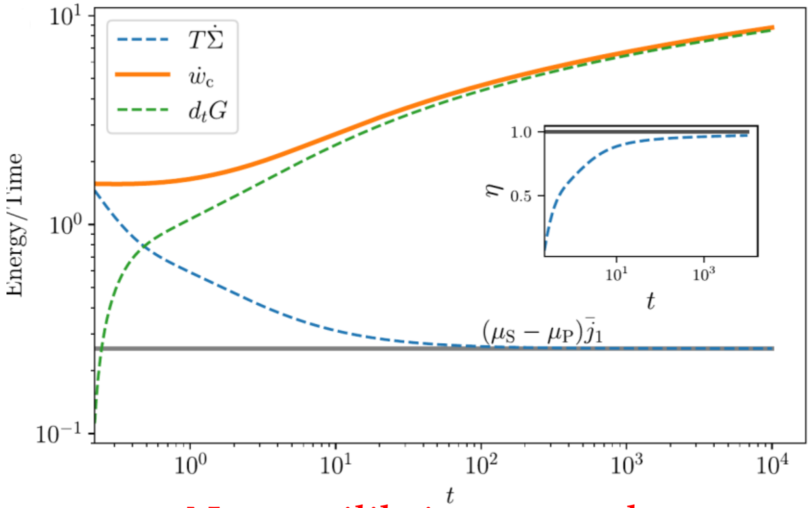
Multimolecular CRNs

	Flux Control	Mixed Control	Conc. Control
Dynamically Linear	Growth	No Growth	No Growth
Dynamically Nonlinear	Growth	Growth	No Growth*





Some species grow



Nonequilibrium growth

$$d_t \boldsymbol{x} = \mathbb{S}^X \bar{\boldsymbol{j}} = 0$$

$$d_t \boldsymbol{y} = \mathbb{S}^Y \bar{\boldsymbol{j}}(\boldsymbol{y}) + \boldsymbol{I}^Y(\boldsymbol{y})$$

QSS Approximation

$$[\mathrm{S}](t) = v_\mathrm{S} t + c_\mathrm{S}$$

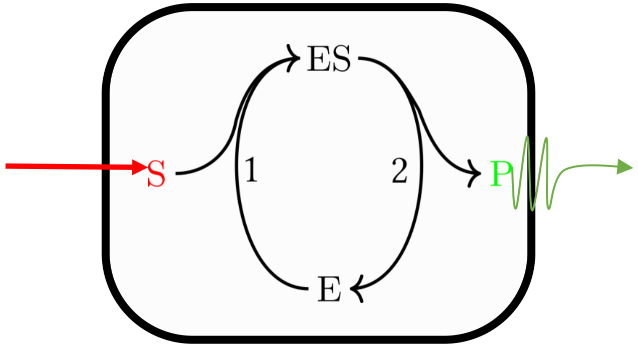
$$[\mathrm{P}](t) = v_\mathrm{P} t + c_\mathrm{S}$$

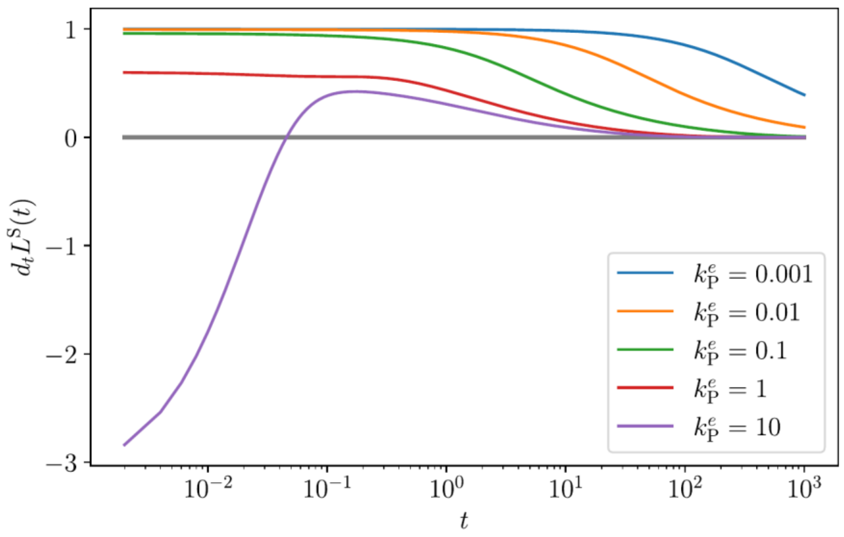
$$d_t G \sim \dot{w}_c = \mathcal{O}(\ln(t))$$

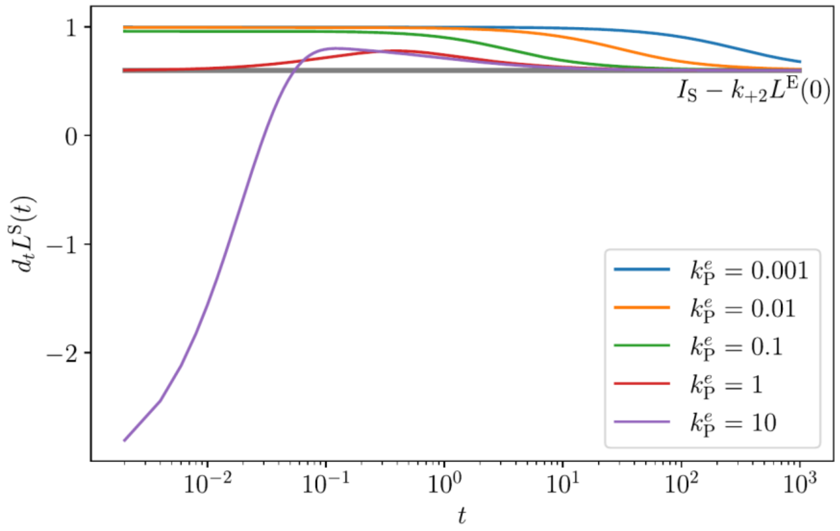
$$T\dot{\Sigma} = \mathcal{O}(1)$$

Multimolecular CRNs

	Flux Control	Mixed Control	Conc. Control
Dynamically Linear	Growth	No Growth	No Growth
Dynamically Nonlinear	Growth	Growth	No Growth*







$$d_t L^{\mathrm{E}} = 0$$

$$d_t L^{\mathrm{S}} = I_{\mathrm{S}} - k^e_{\mathrm{P}}[\mathrm{P}] > 0$$

QSS Approximation

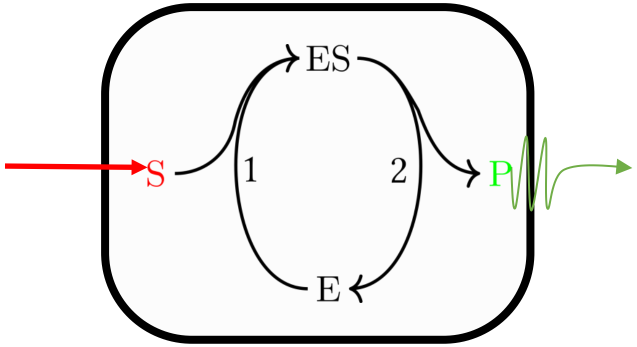
$$v_{\mathrm{S}} = I_{\mathrm{S}} - k_{+2} L^E(0)$$

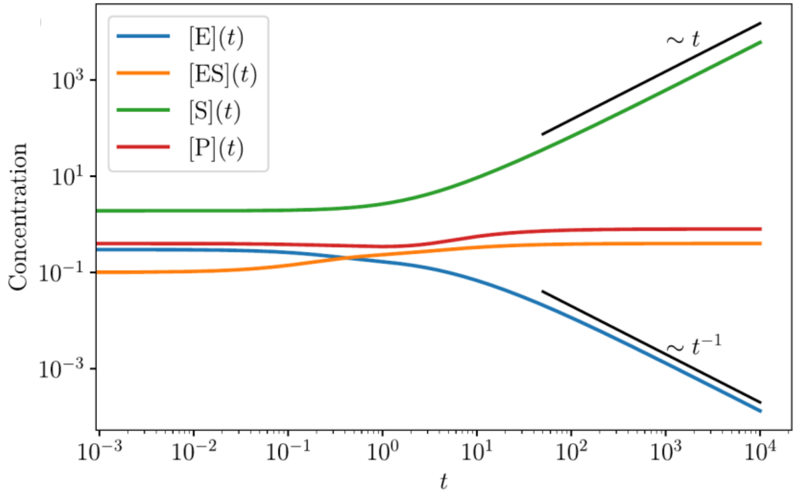
$$d_t[\mathrm{P}] = k_{+2} L^E(0) - k^e_{\mathrm{P}}[\mathrm{P}]$$

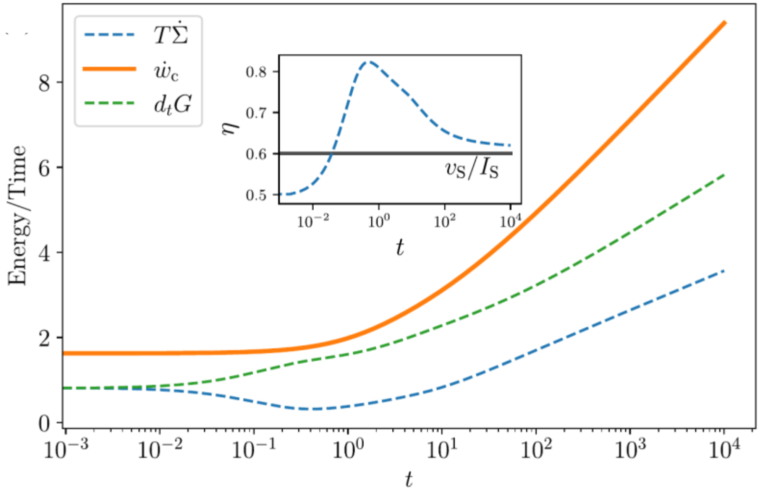
Initial Condition Dependence!

Multimolecular CRNs

	Flux Control	Mixed Control	Conc. Control
Dynamically Linear	Growth	No Growth	No Growth
Dynamically Nonlinear	Growth	Growth	No Growth*







$$d_t L^{\mathrm{E}} = 0$$

$$d_t L^{\mathrm{S}} = I_{\mathrm{S}} - k^e_{\mathrm{P}}[\mathrm{P}] > 0$$

QSS Approximation

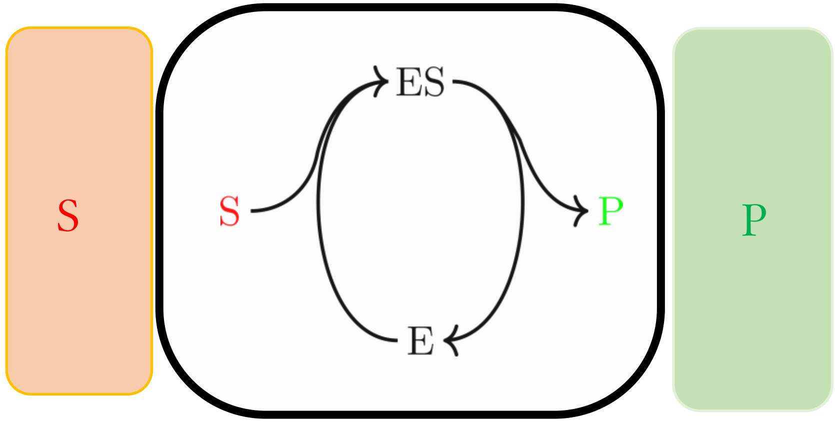
$$v_{\mathrm{S}} = I_{\mathrm{S}} - k_{+2} L^E(0)$$

$$d_t[\mathrm{P}] = k_{+2} L^E(0) - k^e_{\mathrm{P}}[\mathrm{P}]$$

$$T\dot{\Sigma} \sim d_t G \sim \dot{w}_c = \mathcal{O}(\ln(t))$$

Multimolecular CRNs

	Flux Control	Mixed Control	Conc. Control
Dynamically Linear	Growth	No Growth	No Growth
Dynamically Nonlinear	Growth	Growth	No Growth*



$$\text{Growth} \iff L^m(t) \to \infty \iff G(z) \to \infty$$

Known results: Boundedness of weakly reversible single linkage class CRNs *J Math Chem* 49 2275 (2011)

Idea: Gibbs Energy as a quasi-Lyapunov function

Numerical evidence!

Summary of Results

- Asymptotic Growth needs influx of species
- Topology of the CRN determines whether the growth is equilibrium/nonequilibrium

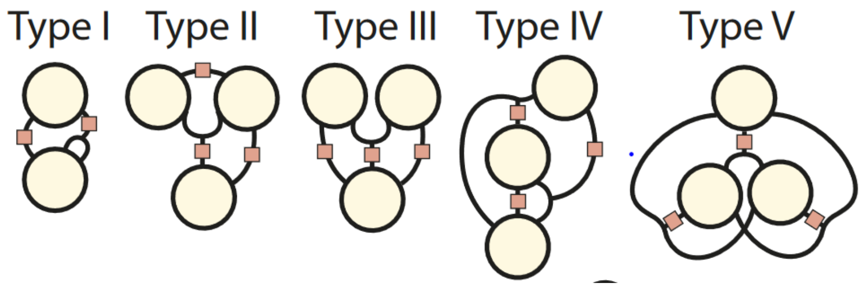
	Flux Control	Mixed Control	Conc. Control
Unimolecular CRNs	$z(t) = \mathcal{O}(t)$ $T\dot{\Sigma} = \mathcal{O}(1/t)$ $\eta \to 1$	$z(t) = \mathcal{O}(1)$	$z(t) = \mathcal{O}(1)$
Pseudo-unimolecular CRNs	$z(t) = \mathcal{O}(t)$ $T\dot{\Sigma} = \mathcal{O}(t)$ $\eta \to 0$	$z(t) = \mathcal{O}(1)$	$z(t) = \mathcal{O}(1)$
Multimolecular CRNs	$0 \leq \eta \leq 1$	$0 \leq \eta \leq 1$	

Outline

- Motivation and Problem Statement
- Formalism: Dynamics and Thermodynamics of open CRNs
- Results
- Prospective directions

Prospective Directions

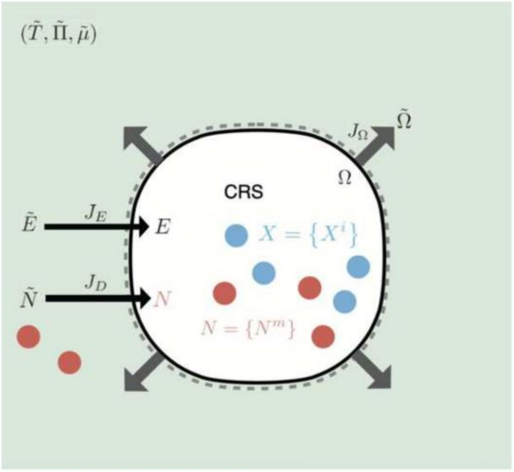
- Systematic analysis of multimolecular CRNs
- Transient Growth
- Growth with volume



PNAS, 117 , 25230 (2020)

J Math Biol. 2022 Sep 7;85(3):26

Phys. Rev. E 100, 022414 (2019)



Chemical Reaction System (CRS)

Phys. Rev. Research 4, 033191(2022)

THANK YOU!