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SOME COMMENTS ON PRIGOGINE'S THEORIES

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Objections are raised against the theories of the Brussels group because they greatly exaggerate claims for the theory, neglect other relevant theories of thermodynamics and overemphasize the role of the Poisson distribution. A necessary and sufficient condition will be given for (not necessary linear) simple birth-and-death processes to have a Poissonian stationary distribution.

Приводятся некоторые возражения относительно теорий группы Брюсселя, т.к. они слишком преувеличивают требования к теории, пренебрегают другими сюда относящимися теориями термодинамики и преувеличивают роль распределения Пуассона. Приводятся необходимые и достаточные условия (необязательно линейные) простых процессов рождения-и-смерти, соблюдающие стационарное распределение Пуассона.

INTRODUCTION

"In his 200 or so publications, Prigogine has given not only biologists, but also astronomers, chemists and meteorologists, new tools with which to build a fuller understanding of the processes governing our Universe" /1, p. 147/. The 1977 Nobel Prize for Chemistry was awarded to Ilya Romanovich Prigogine because of his contribution to the theory of nonequilibrium thermodynamics, especially to the theory of dissipative structures.

Prigogine has summarized in his Nobel lecture /2/ the results that have been obtained by him and his coworkers in Brussels and Austin.

We do not want to argue here against the central points of the theory. As a matter of fact, criticism of their theory is often refused by them /3,4,5,6/, see also Refs. /7,8,9,10,11,12/.

On the other side, Prigogine completely neglects the results of rational thermodynamics /13/. This fact produced a strange situation.

He writes /2, p. 781/: "we introduce in physics and chemistry a historical element, which until now seemed to be reserved only for sciences dealing with biological, social, and cultural phenomena." However, the existence of hysteresis phenomena is well-known in different branches of physics. These processes are explained by allowing the change of state not to depend on the instantaneous values of the state variables only, but also on their past values. The mathematical tool used in this area, that is, the theory of functional differential equations or stochastic processes different from the first order Markov processes is well known in mathematical physics.

Let us mention (rather unjustly to other research workers) only the name and work of such men as Coleman, Day and Noll (see their monographic series in the Archive for the Rational Mechanics and Analysis and their books) starting from the early sixties. Our only intention here is to mention a special doubt regarding Prigogine's theories.

POISSONIAN STATIONARY DISTRIBUTIONS

Our comment is related to the role of the Poisson distribution.

Prigogine writes (1, p. 781) about the reaction

$$A \rightarrow X \qquad 2X \rightarrow B \tag{1}$$

"it came as a great surprise ... that the stationary distribution of X ... is no longer given by the Poisson distribution." People dealing with stochastic kinetics are not so much surprised when obtaining a non-Poisson stationary distribution even in the case of first-order systems since three decades /14/.

Furthermore, it is not difficult to provide a great deal of open nonlinear reactions having a non-Poissonian stationary distribution even of the birth-and-death type specializing the results of e.g. /16/ (see /15/ and /17/ as well). Such reactions are

$$A \to X \qquad kX \to B + (k-1)X \tag{2}$$

There are other possible questions to be raised on the importance of the Poisson distribution (that, by the way, has nothing to do with the laws of large numbers, see e.g. /18/, as Prigogine thinks. About these laws and stochastic kinetics see /19/).

We give a partial answer to the following. Which of the birth-and-death type processes has Poissonian stationary distribution?

THEOREM: The necessary and sufficient condition for a simple birth-and-death type process to have a Poisson stationary distribution (if it has a non-degenerate stationary distribution at all) that it be a linear one.

Proof: It is well-known /16/ that a simple birth-and-death type process of M dimension, that is a process where the following transitions occur (with the respective infinitesimal transition probabilities):

$$\underline{n} \to \underline{n} - \underline{e}_j + \underline{e}_k \qquad \lambda_{jk}\phi_j(n_j)\Delta t + o(\Delta t) \tag{4}$$

$$\underline{n} \to \underline{n} + \underline{e}_j \qquad \nu_j \Delta t + o(\Delta t) \tag{5}$$

$$\underline{n} \to \underline{n} - \underline{e}_j \qquad \mu_j\, \phi_j(n_j)\Delta t + o(\Delta t) \tag{6}$$

$(j, k = 1, \ldots, M)$

has a unique stationary distribution if and only if the system

$$(\mu_j + \sum_k \lambda_{jk})\alpha_j - \sum_k \lambda_{kj}\alpha_k = \nu_j \tag{7}$$

$(j = 1, 2, \ldots, M)$

has a unique, finite solution in the α_js (satisfying a further technical requirement). Then the stationary distribution is

$$p(n) = \text{const.} \prod_j \alpha_j^{n_j} / [\phi_j(1) \ldots \phi_j(n_j)] \tag{8}$$

$(n_1, \ldots, n_M = 0, 1, \ldots)$

Now it can be seen by induction from this explicit formula that it is of the form

$$\pi(n) = \exp\left(-\sum_j \gamma_j\right) \prod_j \gamma_j^{n_j}/n_j! \tag{9}$$

($\gamma_1, \ldots, \gamma_M$ are positive real numbers) if and only if the functions ϕ_j $(j = 1, 2, \ldots, M)$ are homogeneous linear functions, that is if we have the stochastic model of a compartment system (a special case of the first order reaction!) at hand.

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