I. INTRODUCTION

The reduction in dimension of dynamic or kinetic equations describing a large system is a very important problem in many areas, such as chemical kinetics, chemical engineering, control theory, combustion, and ecology. A method for achieving this reduction is called lumping or aggregation. In this context, we refer to a lumping scheme as a method for reducing a set of differential equations to a set with fewer variables, thereby simplifying the analysis.

A singular perturbation method is employed for the determination of lumping schemes. The lumping system, which contains a single large parameter, is described by a set of first-order differential equations. These equations are obtained by substituting their explicit expressions into the original lumped kinetic equation system. The lumped kinetic equation system yields a lumped differential equation system, which can be solved using any desired accuracy for any species containing the independent variable of interest. The lumped system can be further approximated to define the dynamics of the system on a slow invariant manifold. Two simple examples are used to illustrate this approach.
(1) $x^2 + xy^2 = 1$

(2) $\phi(x, y) = x^2 + xy^2 - 1$

(3) $\frac{\partial \phi}{\partial x} = 2x + y^2$

(4) $\frac{\partial \phi}{\partial y} = 2xy$

(5) $\frac{\partial^2 \phi}{\partial x^2} = 2$

(6) $\frac{\partial^2 \phi}{\partial y^2} = 2x$

(7) $\frac{\partial^2 \phi}{\partial x \partial y} = 2y$

(8) $\frac{\partial^2 \phi}{\partial y \partial x} = 2y$

(9) $\frac{\partial^2 \phi}{\partial x \partial y} = 2x$
(29) \[ \int (0) \delta \psi \ldots ((0) \delta \phi) dx = \int (0) \psi \ldots ((0) \delta \phi) dx = (0) \phi \]

When \( A \) is a diagonal matrix, we have:

\[
\begin{pmatrix}
\delta \psi \\
\vdots \\
\delta \psi
\end{pmatrix} = \begin{pmatrix}
\delta \\
\vdots \\
\delta
\end{pmatrix}
\]

where

(30) \[ (t \phi) \psi \ldots ((t \phi) \delta \phi) dx = \int (0) \psi \ldots ((0) \delta \phi) dx = (0) \phi \]

(31) \[ (t \phi) \psi \ldots ((t \phi) \delta \phi) dx = \int (0) \psi \ldots ((0) \delta \phi) dx = (0) \phi \]

(32) \[ (t \phi) \psi \ldots ((t \phi) \delta \phi) dx = \int (0) \psi \ldots ((0) \delta \phi) dx = (0) \phi \]

where \( \phi \) is a diagonal matrix.

The solution is

(33) \[ \phi \psi \ldots ((\phi) \delta \phi) = \begin{pmatrix}
\phi \\
\vdots \\
\phi
\end{pmatrix} \]

Let us rewrite Eq. (32) as follows:

(34) \[ (0) \phi \psi \ldots ((0) \delta \phi) = (0) \phi \]

Eq. (34) states that for every \( \phi \), the above expression for \( \phi \) will be the trivial solution. However, this equation is non-trivial if \( \phi \) is non-zero and \( \psi \) is non-zero. Notice that in Eq. (34), we have some conditions that are not satisfied.

(35) \[ (\phi) \psi \ldots ((\phi) \delta \phi) = (\phi) \phi \]

where \( \phi \) is a diagonal matrix.

Equation (35) is obtained by solving Eq. (34) for \( \phi \).

Now we will solve Eq. (35) for \( \phi \), by a singular perturbation method. To obtain the solution, we need to solve for the perturbation function. The equation is:

(36) \[ (\phi) \psi \ldots ((\phi) \delta \phi) = (\phi) \phi \]

Following the same procedure, we can obtain a recursion:

(37) \[ (\phi) \psi \ldots ((\phi) \delta \phi) = (\phi) \phi \]

(38) \[ (\phi) \psi \ldots ((\phi) \delta \phi) = (\phi) \phi \]

where \( \phi \) always be obtained from the linear solution we have according to the definition above. The inverse can be:

(39) \[ (\phi) \psi \ldots ((\phi) \delta \phi) = (\phi) \phi \]

(40) \[ (\phi) \psi \ldots ((\phi) \delta \phi) = (\phi) \phi \]

(41) \[ (\phi) \psi \ldots ((\phi) \delta \phi) = (\phi) \phi \]

This gives:

(42) \[ 0 = (\phi) \psi \ldots ((\phi) \delta \phi) = (\phi) \phi \]
(36)  \[ (r^k) \phi(t) \nu + (s^k) \psi(t) \nu - (t^k) \nu = \frac{dp}{dt} \]

(37)  \[ (s^k) \nu + (s^k) \psi(t) \nu - (t^k) \nu = \frac{dp}{dt} \]

The same method can be applied to solve other similar type of equations. The simplified notation method can also be applied to similar type of equations where the simplification process is not necessary. For instance, if the simplified notation method is used, the results of the simplified notation method can be used in further calculations. For example, the simplified notation method can be used to simplify the notation of a continuous function. For example, consider the following function:

\[ f(x) = \begin{cases} 
1 & \text{if } x > 0 \\
0 & \text{if } x \leq 0 
\end{cases} \]

The simplified notation method can be used to simplify the notation of this function as follows:

\[ f(x) = 1 \quad \text{for } x > 0 \]

This simplification process is not necessary for the application of the simplified notation method. However, it is useful for further calculations. For example, if the simplified notation method is used to simplify the notation of a continuous function, the result of the simplified notation method can be used in further calculations. For example, consider the following function:

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\[ f(x) = \begin{cases} 
1 & \text{if } x > 0 \\
0 & \text{if } x \leq 0 
\end{cases} \]

The simplified notation method can be used to simplify the notation of this function as follows:

\[ f(x) = 1 \quad \text{for } x > 0 \]
\[
\begin{align*}
(55) & \quad \frac{\partial}{\partial \sigma} (\psi \sigma) = 0 = \frac{\partial}{\partial \sigma} (\psi \sigma) - \frac{\partial}{\partial \sigma} (\psi \sigma) \\
& \text{determine the form of the equations for the flow field.} \\
& \text{Eq.} (55) \text{ is obtained after substituting} \\
& \text{for} \psi \text{ in Eq. (52) and setting the coefficient} \\
& \text{of} \psi \text{ in the following} \\
& \text{equation:} \\
& \psi = 0 = \frac{\partial}{\partial \sigma} (\psi \sigma) - \frac{\partial}{\partial \sigma} (\psi \sigma) \\
& \text{from Eq. (55),} \\
& \text{we have} \\
& \psi = 0 = \frac{\partial}{\partial \sigma} (\psi \sigma) - \frac{\partial}{\partial \sigma} (\psi \sigma) \\
& \text{The solution of the set of equations obtained above can be readily} \\
& \text{obtained as} \\
& \psi = 0 = \frac{\partial}{\partial \sigma} (\psi \sigma) - \frac{\partial}{\partial \sigma} (\psi \sigma) \\
& \text{We will obtain the particle flow and stable (as) -} \\
& \text{-equation.} \\
\end{align*}
\]

\[
\begin{align*}
(56) & \quad \frac{\partial}{\partial \sigma} (\psi \sigma) = 0 = \frac{\partial}{\partial \sigma} (\psi \sigma) - \frac{\partial}{\partial \sigma} (\psi \sigma) \\
& \text{similarly for} \psi \sigma, \text{we have} \\
& \psi = 0 = \frac{\partial}{\partial \sigma} (\psi \sigma) - \frac{\partial}{\partial \sigma} (\psi \sigma) \\
& \text{The derivative of} \psi \text{ with respect to} \sigma \text{can be readily} \\
& \text{obtained as} \\
& \psi = 0 = \frac{\partial}{\partial \sigma} (\psi \sigma) - \frac{\partial}{\partial \sigma} (\psi \sigma) \\
& \text{from Eq. (55),} \\
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& \psi = 0 = \frac{\partial}{\partial \sigma} (\psi \sigma) - \frac{\partial}{\partial \sigma} (\psi \sigma) \\
& \text{We will obtain the particle flow and stable (as) -} \\
& \text{-equation.} \\
\end{align*}
\]
(17) \[(\delta')^2 y_{q} + \ldots + (\delta')^2 y_{q} + (\delta' \phi) y = (\delta ' \phi) y = 0\]

(22) \[\frac{\partial}{\partial x} y = (\delta ' \phi) y = 0\]

(89) \[\frac{\partial}{\partial x} y = (\delta ' \phi) y = 0\]

We have used the initial conditions in (18) to obtain the lumped differential equation for \(y\). We substitute the solutions of (18) into Eq. (9).

(86) \[\frac{\partial}{\partial x} y = (\delta ' \phi) y = 0\]

(93) \[\frac{\partial}{\partial x} y = (\delta ' \phi) y = 0\]

Substituting Eq. (86) into Eq. (93) yields:

(94) \[\frac{\partial}{\partial x} y = (\delta ' \phi) y = 0\]

In summary, we obtain the expressions for \(y\) and \(\delta p / (\delta x) y\) as follows:

(95) \[\frac{\partial}{\partial x} y = (\delta ' \phi) y = 0\]

and

(96) \[\frac{\partial}{\partial x} y = (\delta ' \phi) y = 0\]

The solutions are:

(97) \[0 = (\delta ' \phi) y = (\delta ' \phi) y = 0\]

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(99) \[0 = (\delta ' \phi) y = (\delta ' \phi) y = 0\]

The derivative of \(y\) with respect to \(x\) is:

(100) \[\frac{\partial}{\partial x} y = (\delta ' \phi) y = 0\]
From (72) we obtain:

\[ (t')\bar{y} - t\bar{y} = (t')z \]

The time value of \( t = 0 \) is given to the initial condition.

Under the condition (72)

\[ ((t')\bar{y})_{t=0} = (0)z \]

The value of \( t = 0 \) can be obtained by

\[ (t')\bar{y} = (t)\bar{y} = (t)z \]

Support the initial values are \( \alpha = 0 \), where \( \alpha = \frac{1}{2} \).

\[ (t')\bar{y} = (t)\bar{y} = (t)z \]

The initial value of the gain function is the first-order

\[ (t')\bar{y} = (t)\bar{y} = (t)z \]

The lumped differential equation of the first-order
The convergence rate of the expansion of $h(y)$ and $\hat{h}(y)$ can be accelerated by a special transformation. Let us define a linear partial differential operator $A$ from Eq. (49) as

\[ A \hat{h}(y) = \ldots \]

A function $\Phi(x)\varphi(z)$ is called an invariant of $A$ if

\[ d\Phi(x)\varphi(z) = \lambda(x)\varphi(z) \]

where $\lambda(x)$ is an eigenvalue of $A$ and referred to as the eigenvalue of $A$ corresponding to $\varphi(z)$. Obviously, $\lambda(x)$ is an invariant of $A$ defined by Eq. (75). The eigenfunctions of $A$ can be readily determined by solving the partial differential equation

\[ \frac{d\Phi(x)\varphi(z)}{dx} = \lambda(x)\varphi(z) \]

The corresponding characteristic equation of Eq. (78) is

\[ \lambda = \frac{\epsilon}{\gamma} \]

One solution is

\[ \hat{h}(y) = e^{-\epsilon y} \int \frac{\epsilon}{\gamma} \frac{d\Phi(x)}{dx} dx \]
The lumped differential equation applicable for any initial condition for the case of the point source of constant density is given by (16). For the zeroth order approximation, the solution is obtained by setting the coefficient equal to zero.

\[ \phi(q - v + \alpha) = (q-v) \phi(q - v + \alpha) \]

where \( \phi(q - v) \) is the initial value of \( \phi \) at \( q \), and \( v \) has been fixed by the second order approximation.

The lumped differential equation can be solved by integrating the equation over the domain of \( q \). The solution is given by (16).

\[ \frac{\partial}{\partial q} \left[ \frac{q}{q - v} \phi(q - v) \right] = \frac{\partial^2}{\partial q^2} \left[ \frac{q}{q - v} \phi(q - v) \right] \]

For the first order approximation, the solution is obtained by setting the coefficient equal to zero.

\[ \phi(q - v + \alpha) = (q-v) \phi(q - v + \alpha) \]

We will obtain a stable (as \( q \to 0 \)) equation for the particular case where \( \alpha = 0 \) and \( v = 0 \).
\[ \frac{d}{dp} \left( \frac{f}{g} \right) = \frac{g \frac{df}{dp} - f \frac{dg}{dp}}{g^2} \]

\[ \frac{d}{dp} \left( \frac{1}{g} \right) = -\frac{\frac{dg}{dp}}{g^2} \]

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(141) \[ \gamma \left( \frac{1}{1+\delta} \right)^2 + \left( \frac{1}{1+\delta} \right)^2 \left( \frac{1}{1+\delta} \right)^2 \left( \frac{1}{1+\delta} \right)^2 = (\theta + \theta) \frac{\partial}{\partial x} \]

(131) \[ \left( \frac{1}{1+\delta} \right)^2 \left( \frac{1}{1+\delta} \right)^2 \left( \frac{1}{1+\delta} \right)^2 = (\theta + \theta) \frac{\partial}{\partial x} \]

(13) \[ \left( \frac{1}{1+\delta} \right)^2 + \left( \frac{1}{1+\delta} \right)^2 \left( \frac{1}{1+\delta} \right)^2 \left( \frac{1}{1+\delta} \right)^2 = (\theta + \theta) \frac{\partial}{\partial x} \]

(12) \[ \left( \frac{1}{1+\delta} \right)^2 + \left( \frac{1}{1+\delta} \right)^2 \left( \frac{1}{1+\delta} \right)^2 \left( \frac{1}{1+\delta} \right)^2 = (\theta + \theta) \frac{\partial}{\partial x} \]

where

\[ \left[ \frac{1}{\gamma} + \frac{1}{\gamma} \right] \frac{\partial}{\partial x} (\theta + \theta) \frac{\partial}{\partial x} \]

(136) \[ \left[ \frac{1}{\gamma} + \frac{1}{\gamma} \right] \frac{\partial}{\partial x} (\theta + \theta) \frac{\partial}{\partial x} \]

Similarly, the results are the following

\[ \left[ \frac{1}{\gamma} + \frac{1}{\gamma} \right] \frac{\partial}{\partial x} (\theta + \theta) \frac{\partial}{\partial x} \]

The corresponding Jacobian matrices for \( \theta \) and \( \phi \)

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Relevant conditions to Eqs. (1) and (2), we need

\[ \left[ \frac{1}{\gamma} + \frac{1}{\gamma} \right] \frac{\partial}{\partial x} (\theta + \theta) \frac{\partial}{\partial x} \]

Similarly, we first define a purely real variable vector \( \phi \)

Relevant conditions to Eqs. (1) and (2), we need

\[ \left[ \frac{1}{\gamma} + \frac{1}{\gamma} \right] \frac{\partial}{\partial x} (\theta + \theta) \frac{\partial}{\partial x} \]
A singular perturbation method was used for the determination of a non-linear lumped model for a reaction system with kinetic equations in the form of Eqs. (1) and (2), containing a group of small positive parameters corresponding to different time scales. The key point of this approach is to separate a purely fast variable vector $\Phi(t)$ from the vector $\Phi(t)$ which contains all variables. The expansions of $\Phi(t)$ and $\Phi(t)$ can be determined by the singular perturbation method.

Two simple biochemical reactions show that this approach can construct a lumped differential equation system for any initial composition, as the small parameters can always be considered as such small parameters. The transformation from $\Phi(t)$ to $\Phi(t)$ has a clear geometrical explanation. Consider the equation

$$\Phi(t) = \Phi(t) + \Phi(t).$$

Notice that $\Phi(t)$ is an exponential function vector and approaches 0 at small $t$. Then

$$\Phi(t) = \Phi(t) + \Phi(t).$$

The lumped differential equation of $\Phi(t)$ is

$$\frac{dy}{dt} = a(y) + A(y)h(y) + A(y)\Phi(t) + \Phi(t) + \ldots$$

(147)

Let $\epsilon = 0.2$. This gives $\epsilon = 0.2$ and $\epsilon = 0.1$. In this case, we need consider only the first and zeroth order approximations. Actually, the first order approximation is almost exact. In Fig. 6, we show the profiles resulting from any other lumped approximation which is applicable for any initial composition, i.e., $\Phi(t) = 0$. The results are essentially the same.

The resulting approximation of $\Phi(t)$ is

$$\Phi(t) = \Phi(t) - b[y(t)].$$

(149)

The transformation from $\Phi(t)$ to $\Phi(t)$ has a clear geometrical explanation. Consider the equation

$$\Phi(t) = \Phi(t) + \Phi(t).$$

Notice that $\Phi(t)$ is an exponential function vector and approaches 0 at small $t$. Then

$$\Phi(t) = \Phi(t) + \Phi(t).$$

The lumped differential equation of $\Phi(t)$ is

$$\frac{dy}{dt} = a(y) - A(y)h(y) + A(y)\Phi(t) + \Phi(t) + \ldots$$

(150)

To obtain the solution for $\Phi(t)$, we substitute $\Phi(t)$ into the equations

$$\Phi(t) = \Phi(t) + \Phi(t).$$

(151)

The solution approach is

$$\Phi(t) = \Phi(t) - b[y(t)].$$

(152)

The resulting approximation of $\Phi(t)$ is

$$\Phi(t) = \Phi(t) - b[y(t)].$$

(153)

The solution approach is

$$\Phi(t) = \Phi(t) - b[y(t)].$$

(154)

The resulting approximation of $\Phi(t)$ is

$$\Phi(t) = \Phi(t) - b[y(t)].$$

(155)

The solution approach is

$$\Phi(t) = \Phi(t) - b[y(t)].$$

(156)

The resulting approximation of $\Phi(t)$ is

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(157)

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(158)

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(160)

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(161)

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$$\Phi(t) = \Phi(t) - b[y(t)].$$

(162)

The resulting approximation of $\Phi(t)$ is

$$\Phi(t) = \Phi(t) - b[y(t)].$$

(163)

The solution approach is

$$\Phi(t) = \Phi(t) - b[y(t)].$$

(164)

The resulting approximation of $\Phi(t)$ is

$$\Phi(t) = \Phi(t) - b[y(t)].$$

(165)

The solution approach is

$$\Phi(t) = \Phi(t) - b[y(t)].$$

(166)

The resulting approximation of $\Phi(t)$ is

$$\Phi(t) = \Phi(t) - b[y(t)].$$

(167)

The solution approach is

$$\Phi(t) = \Phi(t) - b[y(t)].$$

(168)

The resulting approximation of $\Phi(t)$ is

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The solution approach is

$$\Phi(t) = \Phi(t) - b[y(t)].$$

(170)

The resulting approximation of $\Phi(t)$ is

$$\Phi(t) = \Phi(t) - b[y(t)].$$

(171)
The difference between the real yield of \( z \) and the

\[ \frac{\partial G}{\partial z} = \frac{1}{2} \left( \frac{\partial f}{\partial z} \right)^2 \]

When \( z \) is taken from the application of the singular part.

\[ (z + \eta) = \frac{1}{2} \]

was exclusively devoted as

The application of the procedure saves the new variable

\[ \frac{\partial \mathcal{L}}{\partial \eta} = \frac{1}{2} \left( \frac{\partial f}{\partial \eta} \right)^2 \]

The partitioning method was used to test its

\[ \frac{\partial \mathcal{L}}{\partial \eta} = \frac{1}{2} \left( \frac{\partial f}{\partial \eta} \right)^2 \]

will apply.

In Fig. 1-4 we have presented a method of accounting

\[ \frac{\partial \mathcal{L}}{\partial \eta} = \frac{1}{2} \left( \frac{\partial f}{\partial \eta} \right)^2 \]

will appear on the right side, in which the application

\[ \frac{\partial \mathcal{L}}{\partial \eta} = \frac{1}{2} \left( \frac{\partial f}{\partial \eta} \right)^2 \]

is not clear, or interpreted the other terms

\[ \frac{\partial \mathcal{L}}{\partial \eta} = \frac{1}{2} \left( \frac{\partial f}{\partial \eta} \right)^2 \]

The partitioning method was used to test its

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