BSCA1 12. 2011/12

1A $\lim\limits_{n\to\infty} \frac{2n^2-18}{n^3-n^2\cos^2 n} = \lim\limits_{n\to\infty} \frac{2\cdot\frac{1}{n}-18\cdot\frac{1}{n^3}}{1-\frac{1}{n}\cos^2 n} = 0$ ①

$\left|\frac{2n^2-18}{n^3-n^2\cos^2 n}\right| \overset{n\geq 3}{=} \frac{2n^2-18}{n^3-n^2\cos^2 n}$ ① $< \frac{2n^2}{n^3-n^2\cos^2 n}$ ① $\leq \frac{2n^2}{n^3-n^2}$ ① $= \frac{2}{n-1} < \varepsilon \Rightarrow \frac{2}{\varepsilon}+1 < n$

$N(\varepsilon) = \max\left\{3, \left[\frac{2}{\varepsilon}+1\right]+1\right\}$ ①

1B $\lim\limits_{n\to\infty} \frac{n^2-9}{2n^3-n^2\sin^2 n} = \lim\limits_{n\to\infty} \frac{\frac{1}{n}-9\frac{1}{n^3}}{2-\frac{1}{n}\sin^2 n} = 0$ ①

$\left|\frac{n^2-9}{2n^3-n^2\sin^2 n}\right| \overset{n\geq 3}{=} \frac{n^2-9}{2n^3-n^2\sin^2 n}$ ① $< \frac{n^2}{2n^3-n^2\sin^2 n}$ ① $\leq \frac{n^2}{2n^3-n^2}$ ① $= \frac{1}{2n-1} < \varepsilon \Rightarrow \left(\frac{1}{\varepsilon}+1\right)\cdot\frac{1}{2} < n$

$N(\varepsilon) = \max\left\{3, \left[\frac{1}{2}\left(\frac{1}{\varepsilon}+1\right)\right]+1\right\}$ ①

2A $\left(1+\frac{2}{n}\right)^n = \left(\left(1+\frac{1}{\frac{n}{2}}\right)^{\frac{n}{2}}\right)^2 \to e^2$ ①, where $\left(1+\frac{1}{\frac{n}{2}}\right)^{\frac{n}{2}} \to e$ ①

$\sqrt[n]{3}\left(\sqrt[n]{n}\right)^2 = \sqrt[n]{3n^2} \leq \sqrt[n]{3n^2+n} \leq \sqrt[n]{4n^2} = \sqrt[n]{4}\cdot\sqrt[n]{n}\cdot\sqrt[n]{n}$ ②, where $\sqrt[n]{3}\to 1$, $\sqrt[n]{n}\to 1$, $\sqrt[n]{3n^2+n}\to 1$, $\sqrt[n]{4}\to 1$

$\lim\limits_{n\to\infty} a_n = 3e^2+2$ ①

2B $\left(1+\frac{3}{n}\right)^n = \left(\left(1+\frac{1}{\frac{n}{3}}\right)^{\frac{n}{3}}\right)^3 \to e^3$ ①, where $\left(1+\frac{1}{\frac{n}{3}}\right)^{\frac{n}{3}} \to e$ ①

$\sqrt[n]{2}\left(\sqrt[n]{n}\right)^2 = \sqrt[n]{2n^2} \leq \sqrt[n]{2n^2+n} \leq \sqrt[n]{3n^2} = \sqrt[n]{3}\,\sqrt[n]{n}\,\sqrt[n]{n}$ ②, where $\sqrt[n]{2}\to 1$, $\sqrt[n]{n}\to 1$, $\sqrt[n]{2n^2+n}\to 1$, $\sqrt[n]{3}\to 1$

$\lim\limits_{n\to\infty} a_n = 2e^3-3$ ①

3A $\frac{2(x^2+x-2)}{x^2+5x+6} = \frac{2(x+2)(x-1)}{(x+3)(x+2)}$ ① $\lim\limits_{x\to -2} \frac{2(x+2)(x-1)}{(x+3)(x+2)} = \lim\limits_{x\to -2} \frac{2(x-1)}{(x+3)}$ ① $= \frac{-6}{1} = -6$ ① $= c$

$x=-3$ -ban nem tehető folytonossá ①, ptlan multiplicitású gyöke a nevezőnek ①

3B $\frac{x^2+x-2}{2(x^2+2x-3)} = \frac{(x-1)(x+2)}{2(x+3)(x-1)}$ ① $\lim\limits_{x\to 1} \frac{(x-1)(x+2)}{2(x+3)(x-1)} = \lim\limits_{x\to 1} \frac{x+2}{2(x+3)}$ ① $= \frac{3}{8} = c$ ①

$x=-3$ -ban nem tehető ... ① ... ①

4A $f'(x) = \underbrace{\frac{1}{\frac{x}{x^2+1}\ln 2}\cdot\frac{x^2+1-x\cdot 2x}{(x^2+1)^2}}_{②} + \underbrace{3\,\mathrm{arctg}\,x^2 + 3x\cdot\frac{1}{1+x^4}\cdot 2x}_{③}$

4B $f'(x) = \frac{1}{\frac{x^2+1}{x}\cdot\ln 3}\cdot\frac{2x\cdot x-(x^2+1)}{x^2} - \left(6x\arcsin x^3 + 3x^2\frac{1}{\sqrt{1-x^6}}\cdot 3x^2\right)$