BSCA1 1. ZH. 2011/12.

1C $\lim\limits_{n\to\infty} \frac{2n^2+n^2\sin^2 n}{n^3-8} = \lim\limits_{n\to\infty} \frac{2\frac{1}{n}+\frac{1}{n}\sin^2 n}{1-8\frac{1}{n^3}} = 0$ (1)

$\left|\frac{2n^2+n^2\sin^2 n}{n^3-8}\right| \overset{n\geq 3}{=} \frac{2n^2+n^2\sin^2 n}{n^3-8}$ (1) $\leq \frac{2n^2+n^2}{n^3-8}$ (1) $< \frac{3n^2}{\frac{1}{2}n^3}$ (1) $= \frac{6}{n} < \varepsilon \Rightarrow \frac{6}{\varepsilon} < n$

$N(\varepsilon) = \max\{3, [\frac{6}{\varepsilon}]+1\}$ (1)

1D $\lim\limits_{n\to\infty} \frac{n^2+n^2\cos^2 n}{2n^3-15} = \lim\limits_{n\to\infty} \frac{\frac{1}{n}+\frac{1}{n}\cos^2 n}{2-15\frac{1}{n^3}} = 0$ (1)

$\left|\frac{n^2+n^2\cos^2 n}{2n^3-15}\right| \overset{n\geq 2}{=} \frac{n^2+n^2\cos^2 n}{2n^3-15}$ (1) $\leq \frac{n^2+n^2}{2n^3-15}$ (1) $< \frac{2n^2}{n^3}$ (1) $= \frac{2}{n} < \varepsilon \Rightarrow \frac{2}{\varepsilon} < n$

$N(\varepsilon) = \max\{2, [\frac{2}{\varepsilon}]+1\}$ (1)

2C $(1+\frac{1}{2n})^n = \Big(\underbrace{(1+\frac{1}{2n})^{2n}}_{\to e}\Big)^{\frac{1}{2}} \mapsto \sqrt{e}$ (1) (1) $\quad \underbrace{\sqrt[n]{3}}_{\downarrow 1}\underbrace{(\sqrt[n]{n})^2}_{\downarrow 1} = \sqrt[n]{3n^2} \leq \underbrace{\sqrt[n]{3n^2+1}}_{\downarrow 1} \leq \sqrt[n]{4n^2} = \underbrace{\sqrt[n]{4}}_{\downarrow 1}\cdot\underbrace{\sqrt[n]{n}}_{\downarrow 1}\underbrace{\sqrt[n]{n}}_{\downarrow 1}$ (2)

$\lim\limits_{n\to\infty} a_n = 3\sqrt{e}+2$ (1)

2D $(1+\frac{1}{3n})^n = \Big(\underbrace{(1+\frac{1}{3n})^{3n}}_{\to e}\Big)^{\frac{1}{3}} \mapsto \sqrt[3]{e}$ (1) (1) $\quad \underbrace{\sqrt[n]{2}}_{\downarrow 1}\underbrace{(\sqrt[n]{n})^2}_{\downarrow 1} = \sqrt[n]{2n^2} \leq \underbrace{\sqrt[n]{2n^2+1}}_{\downarrow 1} \leq \sqrt[n]{3n^2} = \underbrace{\sqrt[n]{3}}_{\downarrow 1}\underbrace{\sqrt[n]{n}}_{\downarrow 1}\underbrace{\sqrt[n]{n}}_{\downarrow 1}$ (2)

$\lim\limits_{n\to\infty} a_n = 2\sqrt[3]{e}-3$ (1)

3C $\frac{3x^2-3x-6}{x^2+4x+3} = \frac{3(x-2)(x+1)}{(x+3)(x+1)}$ (1) $\quad \lim\limits_{x\to -1} \frac{3(x-2)(x+1)}{(x+3)(x+1)} = \lim\limits_{x\to -1} \frac{3(x-2)}{x+3}$ (1) $= \frac{-9}{2} = c$ (1)

$x=-3$-ban nem tehető ... (1) a nevezőnek ptlan multiplicitású gyöke (1)

3D $\frac{x^2-x-2}{3x^2+3x-18} = \frac{(x-2)(x+1)}{3(x+3)(x-2)}$ (1) $\quad \lim\limits_{x\to 2} \frac{(x-2)(x+1)}{3(x+3)(x-2)} = \lim\limits_{x\to 2} \frac{(x+1)}{3(x+3)}$ (1) $= \frac{3}{3\cdot 5} = \frac{1}{5} = c$ (1)

$x=-3$-ban nem ... (1) ... (1)

4C $f'(x) = \underbrace{2^{\frac{x}{x^2+1}}\cdot\ln 2\cdot\frac{x^2+1-x\cdot 2x}{(x^2+1)^2}}_{2} - \Big(\underbrace{3x\frac{1}{\sqrt{1-x^4}}\cdot 2x + 3\arcsin x^2}_{3}\Big)$

4D $f'(x) = \overbrace{3^{\frac{x^2+1}{x}}\cdot\ln 3\cdot\frac{2x\cdot x-(x^2+1)}{x^2}} + \overbrace{\Big(2\operatorname{arctg} x^3 + 2x\frac{1}{1+x^6}\cdot 3x^2\Big)}$