BSCA1 1. ZH pótló 2011. december

1. $\lim_{n\to\infty} a_n = \frac{1}{2}$ ①

$$\left|\frac{n^3-2n^2+4n-1}{2n^3+3n^2-8n+1} - \frac{1}{2}\right| = \left|\frac{2n^3-4n^2+8n-2-2n^3-3n^2+8n-1}{2(2n^3+3n^2-8n+1)}\right| ① = \left|\frac{-7n^2+16n-3}{2(\underbrace{2n^3+3n^2-8n+1}_{>0\ n\geq 3\ ①})}\right| \overset{*}{=}$$

$7n^2-16n+3=0 \qquad n_{12} = \frac{16\pm\sqrt{256-84}}{14} < \frac{16+14}{14} \leq 3$

$$\overset{*}{\underset{n\geq 3}{=}} \frac{7n^2+16n-3}{2(2n^3+3n^2-8n+1)} \cancel{①} < \frac{8n^2}{2\cdot 2n^3} ① = \frac{2}{n} < \varepsilon \quad \Rightarrow \quad n > \frac{2}{\varepsilon}$$

$n^2+16n-3=0 \qquad n_{12} = \frac{-16\pm\sqrt{256+12}}{2} < \frac{1}{2}$

$N(\varepsilon) = \max\left\{ \left[\frac{2}{\varepsilon}\right]+1,\ 3 \right\}$ ①

2. $$\frac{\sqrt{n+1}-\sqrt{n-1}}{\sqrt{2n}-\sqrt{2n-1}} \cdot \frac{\sqrt{n+1}+\sqrt{n-1}}{\sqrt{2n}+\sqrt{2n-1}} \cdot \frac{\sqrt{2n}+\sqrt{2n-1}}{\sqrt{n+1}+\sqrt{n-1}} ② = \frac{\cancel{n}+1-\cancel{n}+1}{\cancel{2n}-\cancel{2n}+1}\ \frac{\sqrt{2n}+\sqrt{2n-1}}{\sqrt{n+1}+\sqrt{n-1}} ① =$$

$$= 2\cdot\frac{\sqrt{2}+\sqrt{2-\frac{1}{n}}}{\sqrt{1+\frac{1}{n}}+\sqrt{1-\underbrace{\frac{1}{n}}_{\to 0}}} ① \xrightarrow[\infty]{n} 2\cdot\frac{2\sqrt{2}}{2} \cancel{①} = 2\sqrt{2} ①$$

3. $$\lim_{x\to 2}\frac{x-2}{\sin(x-2)} = \lim_{y\to 0}\frac{1}{\underbrace{\frac{\sin y}{y}}_{\to 1\ ①}} = 1 ①$$

$$\lim_{x\to 2}\frac{x^2}{(2-x)^2} = \infty ① \qquad \lim_{x\to\infty} \operatorname{arctg} x = \frac{\pi}{2} ①$$

$$\lim_{x\to 2}\frac{x-2}{\sin(x-2)}\cdot \operatorname{arctg}\frac{x^2}{(2-x)^2} = \frac{\pi}{2} ①$$

4. $y^2 + x\cdot 2y y' + 3y^2 y' = 36x^2$ ②

$$y' = \frac{36x^2-y^2}{2xy+3y^2} ① \qquad y'(x=1, y=2) = \frac{36-4}{4+12} = \frac{32}{16} = 2 ①$$

$y - y_0 = f'(x_0)(x-x_0) \quad \Rightarrow \quad y-2 = 2(x-1)$ ①