BSCAI 22H. 2011. uos ~~A~~ csoport és D csoport

1C $x(t_0) = 8\,\mathrm{ch}\ln 4 = 8\,\dfrac{e^{\ln 4}+e^{-\ln 4}}{2} = 8\cdot\dfrac{4+\frac{1}{4}}{2} = 8\cdot\dfrac{16+1}{8} = 8\,\dfrac{17}{8} = 17$ ①

$y(t_0) = 16\,\mathrm{sh}\ln 4 = 16\,\dfrac{e^{\ln 4}-e^{-\ln 4}}{2} = 16\,\dfrac{4-\frac{1}{4}}{2} = 16\cdot\dfrac{16-1}{8} = 16\cdot\dfrac{15}{8} = 30$ ①

$\dot{x}(t) = 8\,\mathrm{sh}\,t \qquad \dot{x}(t_0) = 8\,\mathrm{sh}\ln 4 = 8\cdot\dfrac{15}{8} = 15$ ①

$\dot{y}(t) = 16\,\mathrm{ch}\,t \qquad \dot{y}(t_0) = 16\,\mathrm{ch}\ln 4 = 16\cdot\dfrac{17}{8} = 34$ ① $\qquad \dfrac{\dot{y}(t_0)}{\dot{x}(t_0)} = \dfrac{15}{34}$

$y - y(t_0) = \dfrac{\dot{y}(t_0)}{\dot{x}(t_0)}\,(x - x(t_0)) \implies y - 30 = \dfrac{15}{34}\,(x-17)$ ①

2C $f'(x) = 2\,e^{\frac{1}{x}} + (2x+12)\,e^{\frac{1}{x}}\cdot\left(-\dfrac{1}{x^2}\right) = \dfrac{2e^{\frac{1}{x}}}{x^2}\,(x^2-x-6) = 0$ ① $\iff x_{1,2} = \dfrac{1\pm\sqrt{1+24}}{2} = \begin{cases}3\\-2\end{cases}$

	$(-\infty,-2)$	-2	$(-2,0)$	$(0,3)$	3	$(3,\infty)$
f'	+	0	−	−	0	+
f	↗	lmax	↘	↘	lmin	↗

② ②

3C $\displaystyle\int -4x\,e^{6x}\,dx = -4\int x\,e^{6x}\,dx = -4\left\{\frac{x}{6}\,e^{6x} - \int\frac{e^{6x}}{6}\,dx\right\} = -4\left\{\frac{x}{6}\,e^{6x} - \frac{e^{6x}}{36}\right\} + C$ ② ②

$x = u \qquad e^{6x} = v' \qquad v = \dfrac{e^{6x}}{6}$ ① és (*) alkalmaz!

4C $f\quad \dfrac{1}{x(x+7)} = \dfrac{A}{x} + \dfrac{B}{x+7} = \dfrac{Ax+7A+Bx}{x(x+7)}$ ① $\qquad \begin{array}{l} A+B=0\\ 7A=1\end{array} \implies A = \dfrac{1}{7} \quad B = -\dfrac{1}{7}$

$\displaystyle\int -\frac{1}{x(x+7)}\,dx = \frac{1}{7}\int -\frac{1}{x} + \frac{1}{x+7}\,dx = \frac{1}{7}\left(-\ln x + \ln(x+7)\right) + C$ ② ②

1D $x(t_0) = 8\,\mathrm{sh}\ln 2 = 8\,\dfrac{e^{\ln 2}-e^{-\ln 2}}{2} = 8\,\dfrac{2-\frac{1}{2}}{2} = 8\,\dfrac{4-1}{4} = 8\cdot\dfrac{3}{4} = 6$ ①

$y(t_0) = 4\,\mathrm{ch}\ln 2 = 4\,\dfrac{e^{\ln 2}+e^{-\ln 2}}{2} = 4\,\dfrac{2+\frac{1}{2}}{2} = 4\,\dfrac{4+1}{4} = 4\,\dfrac{5}{4} = 5$ ①

$\dot{x}(t) = 8\,\mathrm{ch}\,t \qquad \dot{x}(t_0) = 8\,\mathrm{ch}\ln 2 = 8\cdot\dfrac{5}{4} = 10$ ① $\qquad \dfrac{\dot{y}(t_0)}{\dot{x}(t_0)} = \dfrac{3}{10}$

$\dot{y}(t) = 4\,\mathrm{sh}\,t \qquad \dot{y}(t_0) = 4\,\mathrm{sh}\ln 2 = 4\cdot\dfrac{3}{4} = 3$ ①

$y - y(t_0) = \dfrac{\dot{y}(t_0)}{\dot{x}(t_0)}\,(x - x(t_0)) \implies y - 5 = \dfrac{3}{10}\,(x-6)$ ①

2D $f'(x) = -4\,e^{-\frac{1}{x}} + (-4x+8)\,e^{-\frac{1}{x}}\left(\dfrac{1}{x^2}\right) = -4\,\dfrac{e^{-\frac{1}{x}}}{x^2}\,(x^2+x-2) = 0$ ① $\iff x_{1,2} = \dfrac{-1\pm\sqrt{1+8}}{2} = \begin{cases}1\\-2\end{cases}$

	$(-\infty,-2)$	-2	$(-2,0)$	$(0,1)$	1	$(1,\infty)$
f'	−	0	+	+	0	−
f	↘	lmin	↗	↗	lmax	↘

② ②

3D $\displaystyle\int 8x\,e^{-2x}\,dx = 8\int x\,e^{-2x}\,dx = 8\left\{\frac{x}{-2}\,e^{-2x} - \int\frac{e^{-2x}}{-2}\,dx\right\} = 8\left\{\frac{x}{-2}\,e^{-2x} - \frac{e^{-2x}}{4}\right\} + C$ ② ②

$u = x \qquad v' = e^{-2x} \qquad v = \dfrac{e^{-2x}}{-2}$ ① és (*) alkalmaz!

4D $\dfrac{1}{x(x-3)} = \dfrac{A}{x} + \dfrac{B}{x-3} = \dfrac{Ax-3A+Bx}{x(x-3)}$ ① $\qquad \begin{array}{l} A+B=0\\ -3A=1\end{array} \implies A = -\dfrac{1}{3} \quad B = \dfrac{1}{3}$

$\displaystyle\int -\frac{1}{x(x-3)}\,dx = \frac{1}{3}\int \frac{1}{x} - \frac{1}{x-3}\,dx = \frac{1}{3}\left\{\ln x - \ln(x-3)\right\} + C$ ② ②