BSCAI. 3. vizsga 2011-12. (jan. 18.)

1. $\lim_{x\to-\infty} \frac{e^x}{1+e^x} = \frac{1}{2}$ (1) (indoklás (1)) $\lim_{x\to\infty} \frac{e^x}{1+e^x} = \frac{1}{2}$ (1), $\lim_{x\to0-0} \frac{e^{\frac{1}{x}}}{1+e^{\frac{1}{x}}} = 0$ (1)

$\lim_{x\to0+0} \frac{e^{\frac{1}{x}}}{1+e^{\frac{1}{x}}} \overset{\text{B-L}}{=} \lim_{x\to0+0} \frac{e^{\frac{1}{x}}\left(-\frac{1}{x^2}\right)}{e^{\frac{1}{x}}\left(-\frac{1}{x^2}\right)} = 1$ (1)

2. $x(t) = 2\sin t\cos t = \sin 2t$ (1) $\quad \dot{x}(t) = 2\cos 2t$ (1)

$y(t) = 2\sin t\sin t = 2\sin^2 t$ (1) $\quad \dot{y}(t) = 4\sin t\cos t = 2\sin 2t$ (1)

$\frac{\dot{y}(t)}{\dot{x}(t)} = \frac{\sin 2t}{\cos 2t}$ (1)

$x\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$, $y\left(\frac{\pi}{6}\right) = \frac{1}{2}$ (1) $\quad \frac{\dot{y}(t)}{\dot{x}(t)}\Big|_{\frac{\pi}{6}} = \frac{\sqrt{3}}{2}\cdot\frac{2}{1} = \sqrt{3}$ (1)

$y - y_0 = f'(x_0)(x - x_0) \Rightarrow y - \frac{1}{2} = \sqrt{3}\left(x - \frac{\sqrt{3}}{2}\right)$ (1)

3. $f'(x) = \frac{3x^2-3}{x^2(x^2-3)^2}$ (2) $= 3\frac{x^2-1}{x^2(x^2-3)^2} = 0 \Rightarrow x_1 = 1 \quad x_2 = -1$

	$(-\infty,-\sqrt{3})$	$(-\sqrt{3},-1)$	-1	$(-1,0)$	$(0,1)$	1	$(1,\sqrt{3})$	$(\sqrt{3},\infty)$
f'	+	+	0	−	−	0	+	+
f	↗	↗	lokális max	↘	↘	lokális min	↗	↗

(1) (2) (2) (1)

4. $m = \varrho V \qquad m = \pi\int_a^b \varrho f^2(x)\,dx \sim V$ (2)

$\varrho \sim 1$

$V = \pi\int_0^1 \frac{x^2}{1+x^2}\,dx = \pi\int_0^1 1 - \frac{1}{1+x^2}\,dx = \pi\left[x - \operatorname{arctg} x\right]_0^1 = \pi\left(1-\frac{\pi}{4}\right)$

(2) (1) (2) (1)

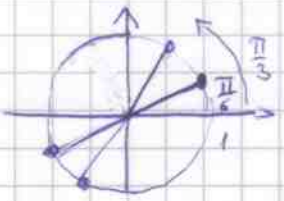
5. $z_{12}^2 = \frac{i\sqrt{3} \pm \sqrt{i^2 3 + 4}}{2} = \frac{i\sqrt{3} \pm \sqrt{-3+4}}{2} = \frac{i\sqrt{3} \pm 1}{2}$ (1)

$z_1^2 = \frac{1}{2} + i\frac{\sqrt{3}}{2} = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$ $\quad z_1 = \cos\frac{\frac{\pi}{3}+2k\pi}{2} + i\sin\frac{\frac{\pi}{3}+2k\pi}{2}$ $\quad k=0,1$ (1)

$z_{11} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} + i\frac{1}{2}$ (1) $\quad z_{12} = \cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6} = -\frac{\sqrt{3}}{2} - i\frac{1}{2}$ (1)

$z_2^2 = -\frac{1}{2} + i\frac{\sqrt{3}}{2} = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$ $\quad z_2 = \cos\frac{\frac{2\pi}{3}+2k\pi}{2} + i\sin\frac{\frac{2\pi}{3}+2k\pi}{2}$ $\quad k=0,1$ (1)

$z_{21} = \cos\frac{2\pi}{6} + i\sin\frac{2\pi}{6} = +\frac{1}{2} + i\frac{\sqrt{3}}{2}$ (1) $\quad z_{22} = \cos\frac{8\pi}{6} + i\sin\frac{8\pi}{6} = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$ (1)



(1)