Courses Offered in English for Incoming Students
by the Mathematical Institute of BME

Spring 2014

Parameters: CODE, lectures / practical lectures / laboratory / f= term mark, v = exam / ECTS credit points

1. Basis Courses for Students of Engineering

Mathematics A2a – Vector Functions
BMETE90AX02, 4/2/0/v/6

Solving systems of linear equations: elementary row operations, Gauss-Jordan- and Gaussian elimination. Homogeneous systems of linear equations. Arithmetic and rank of matrices. Determinant: geometric interpretation, expansion of determinants. Cramer's rule, interpolation, Vandermonde determinant. Linear space, subspace, generating system, basis, orthogonal and orthonormal basis. Linear maps, linear transformations and their matrices. Kernel, image, dimension theorem. Linear transformations and systems of linear equations. Eigenvalues, eigenvectors, similarity, diagonalizability. Infinite series: convergence, divergence, absolute convergence. Sequences and series of functions, convergence criteria, power series, Taylor series. Fourier series: expansion, odd and even functions. Functions in several variables: continuity, differential and integral calculus, partial derivatives, Young's theorem. Local and global maxima / minima. Vector-vector functions, their derivatives, Jacobi matrix. Integrals: area and volume integrals.

Calculus 2 for Informaticians
BMETE90AX05, 4/2/0/v/7

Differential equations: Separable d.e., first order linear d.e., higher order linear d.e. of constant coefficients. *Series:* Tests for convergence of numerical series, power series, Taylor series. *Functions of several variables:* Limits, continuity. Differentiability, directional derivatives, chain rule. Higher partial derivatives and higher differentials. Extreme value problems. Calculation of double and triple integrals. Transformations of integrals, Jacobi matrix. *Analysis of complex functions:* Continuity, regularity, Cauchy - Riemann partial differential equations. Elementary functions of complex variable, computation of their values. Complex contour integral. Cauchy - Goursat basic theorem of integrals and its consequences. Integral representation of regular functions and their higher derivatives (Cauchy integral formulae).

Mathematics EP2 for Architects
BMETE90AX4, 0/2/0/f/2

Limit, continuity, partial derivatives and differentiability of functions of multiple variables. Equation of the tangent plane. Local extrema of functions of two variables. Gradient and directional derivative. Divergence, rotation. Double and triple integrals and their applications. Polar coordinates. Substitution theorem for double integrals. Curves in the 3D space, tangent line, arc length. Line integral. 3D surfaces. Separable differential equations, first order linear differential equations. Algebraic form of complex numbers. Second order linear differential equations with constant coefficients. Taylor polynomial of exp(x), sin(x), cos(x). Eigenvalues and eigenvectors of matrices.

Mathematics A4 – Probability Theory
BMETE90AX08, 2/2/0/f/4

Notion of probability. Conditional probability. Independence of events. Discrete random variables and their distributions (discrete uniform distribution, classical problems, combinatorial methods, indicator distribution, binomial distribution, sampling with/without replacement, hypergeometrical distribution, Poisson distribution as limit of binomial distributions, geometric distribution as model of a discrete memoryless waiting time). Continuous random variables and their distributions (uniform distribution on an interval, exponential distribution as model of a continuous memoryless waiting time, standard normal distribution). Parameters of distributions (expected value, median, mode, moments, variance, standard deviation). Two-dimensional distributions. Conditional distributions, independent random variables. Covariance, correlation coefficient. Regression. Transformations of distributions. One- and two-dimensional normal distributions. Laws of large numbers, DeMoivreLaplace limit theorem, central limit theorem. Some statistical notions. Computer simulation, applications.

Numerical Methods for Engineers
BMETE91AX30, 1/1/0/f/2

Basic notions of numerical computations (types of errors, error propagation). Fundamentals of metric spaces, Banach's fixed point theorem. Iterative methods for solving nonlinear equations and their convergence properties (regula falsi, Newton's method, successive approximation). Extreme value problems (e.g., gradient method for nonlinear systems of equations). Systems of linear equations (some iterative methods, least square solution for over- and underdetermined systems). Orthogonal systems of functions (dot product for functions, orthogonal polynomial systems for different dot products, Chebyshev- and Legendre polynomials). Interpolation and approximation of functions (by polynomials, by orthogonal system of functions). Numerical differentiation and integration (Gauss quadratures).

2. Advanced Courses for Students of Engineering

Mathematics M1c - Differential Equations
BMETE90MX44, 2/1/0/v/3, Thursdays, 12-14 Building H, Room 61

Explicit first order ordinary differential equations and their solution. Simple types. Linear systems. Higher order equations. Laplace transform, properties and applications. Elements of the qualitative theory. On partial differential equations. Elements of variational calculus.

Mathematics M1 – Differential Equations and their Numerical Methods
BMETE90MX46, 4/2/0/v/8

Ordinary differential Equations. Well-posedness of initial value problems. Various types of stability. Stability of equilibria and Liapunov functions. Phase space analysis near equilibria and periodic orbits. The loss of stability in parametrized families of equations. Explicit/implicit Euler and Runge-Kutta methods. Comparing explicit and approximate dynamics, error estimate between exact and approximate solutions. Retarded equations. Partial differential equations. The standard initial and boundary value problems of mathematical physics. Separation of variables. Fourier series as coordinate representation in Hilbert space. The method of finite differences for the heat equation: error estimate and the maximum principle.

3. Advanced Courses for Students in Mathematics

Limit- and Large Deviation Theorems of Probability Theory
BMETE95MM10, 3/1/0/v/5

Limit theorems: Weak convergence of probability measures and distributions. Tightness: HellyPtohorov theorem. Limit theorems proved with bare hands: Applications of the reflection principle to random walks: Paul Lévy's arcsine laws, limit theorems for the maximum, local time and hitting times of random walks. Limit theorems for maxima of i.i.d. random variables, extremal distributions. Limit theorems for the coupon collector problem. Proof of limit theorem with method of momenta. Limit theorem proved by the method of characteristic function. Lindeberg's theorem and its applications: Erdős-Kac theorem: CLT for the number of prime factors. Stable distributions. Stable limit law of normed sums of i.i.d. random variables. Characterization of the characteristic function of symmetric stable laws. Weak convergence to symmetric stable laws. Applications. Characterization of characteristic function of general (non-symmetric) stable distributions, skewness. Weak convergence in non-symmetric case. Infinitely divisible distributions: Lévy-Hinchin formula and Lévy measure. Lévy measure of stable distributions, self-similarity. Poisson point processes and infinitely divisible laws. Infinitely divisible distributions as weak limits for triangular arrays. Applications. Introduction to Lévy processes: Lévy-Hinchin formula and decomposition of Lévy processes. Construction with Poisson point processes (a la Ito). Subordinators and Lévy processes with finite total variation, examples. Stable processes. Examples and applications. Large deviation theorems: Introduction: Rare events and large deviations. Large deviation principle. Computation of large deviation probabilities with bare hands: application of Stirling's formula. Combinatorial methods: The method of types. Sanov's theorem for finite alphabet. Large deviations in finite dimension: Bernstein's inequality, Chernoff's bound, Cramer's theorem. Elements of convex analysis, convex conjugation in finite dimension, Cramer's theorem in R^d. Gartner-Ellis theorem. Applications: large deviation theorems for random walks, empirical distribution of the trajectories of finite state Markov chains, statistical applications. The general theory: general large deviation principles. The contraction principle and Varadhan's lemma. large deviations in topological vector spaces and function spaces. Elements of abstract convex analysis. Applications: Schilder's theorem, Gibbs conditional measures, elements of statistical physics.

Dynamical Systems
BMETE93MM02, 3/1/0/v/5

Continuous-time and discrete-time dynamical systems, continuous versus discrete: first return map, discretization. Local theory of equilibria: Grobman–Hartman lemma, stableunstable-center manifold, Poincaré's normal form. Attractors, Liapunov functions, LaSalle principle, phase portrait. Structural stability, elementary bifurcations of equilibria, of fixed points, and of periodic orbits, bifurcation curves in biological models. Tent and logistic curves, Smale horseshoe, solenoid: properties from topological, combinatorial, and measure theoretic viewpoints. Chaos in the Lorenz model.

Mathematical Statistics and Information Theory
BMETE95MM05, 3/1/0/v/5

Multivariate statistical inference in multidimensional parameter spaces: Fisher's information matrix, likelihood ratio test. Testing hypotheses in multivariate Gauss model: Mahalanobis' distance, Wishart's, Hotelling's, Wilks' distributions. Linear statistical inference, Gauss–Markov theorem. Regression analysis, one- and two-way analysis of variance as a special case of the linear model. ANOVA tables, Fisher–Cochran theorem. Principal component and factor analysis. Estimation and rotation of factors, testing hypotheses for the effective number of factors. Hypothesis testing and Idivergence (the discrete case). I-projections, maximum likelihood estimate as I-projection in exponential families. The limit distribution of the I-divergence statistic. Analysis of contingency tables by information theoretical methods, loglinear models. Statistical algorithms based on information geometry: iterative scaling, EM algorithm. Method of maximum entropy.

Extreme Value Theory
BMETE95MM16, 2/0/0/v/3

Review of the limit theorems, normal domain of attraction, stable low of distributions, alpha-stable domain of attractions. Max-stable distributions, Fisher-Tippet theorem, standard extreme value distributions, regularly varying functions and their properties, Frechet and Weibull distributions and characterization of their domain of attraction. Gumbel distribution. Generalized Pareto distribution. Peak over threshold. Methods of parameter estimations. Applications in economy and finance.

4. A Course of the Program Mathematica for Everyone

Applied Computational Analysis
BMETE927205, 0/0/2/f/3, Tuesdays, 14-16 Building H, Room 27

It is about the application of the program Mathematica (see e.g. http://demonstrations.wolfram.com) in the fields of engineering, physics, chemistry and mathematics - depending on the audience. Those enrolled will get a copy of the program for personal use. The main goal is to produce and present a program at the end of the course about your own work (which you may--are even encouraged--to use in other subjects).