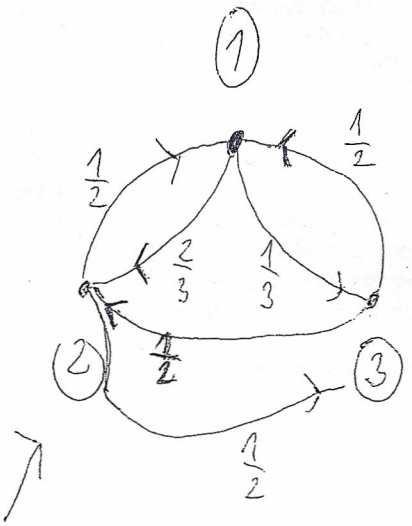
9) A professional appliance (e.g. a factory, a large scale computer network, a big block of flats) is controlled by a group of amateurs. From time to time several problems arise that affect the efficiency of the operation of the system. The real experts visit the system and solve the problems perfectly at the end of each week. When they arrive the actual state of the appliance falls into one of the following four categories: 1 - no problem; 2 - small correction is needed; 3 - big correction is needed; 4 - disastrous.
The costs of identifying the state and the correction in these states are: 10 (1), 30 (2), 100 (3), 1000 (4) units.
The states of successive weeks can be modeled by a four state Markov chain since we assume that the next week's state depends only on the previous week's state and the events occurred during the week. On the other hand, the amateur staff is able to handle the problems on a certain random level. The transition probabilities are summarized in the following matrix:

$$\begin{bmatrix} 0,4 & 0,4 & 0,2 & 0 \\ 0,2 & 0,6 & 0,1 & 0,1 \\ 0 & 0,3 & 0,4 & 0,3 \\ 0,5 & 0,4 & 0,1 & 0 \end{bmatrix}$$

(the order of the states are 1,2,3,4).

a) What is the probability that after two weeks the system is without any problem if the time period started with a small correction? (We have not registered the state of one intermediate week.)
b) What is the probability that a week without problems is followed by three weeks without problems?
c) In the long run find the percentage of time that the professional group finds the system in state i, $i = 1, 2, 3, 4$.
d) What is the long run average cost of the application of the professional group?



SOLVED EXERCISE

WE HAVE A MARKOV CHAIN GIVEN BY THIS GRAPH

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DISCRETE, HOMOGENOUS

1) IF THE INITIAL STATE OF THE MARKOV CHAIN IS 2 WHAT IS THE PROBABILITY THAT AFTER TWO STEPS WE FIND THE CHAIN AGAIN IN STATE 2 ?

2) IF THE INITIAL STATE OF THE MARKOV CHAIN IS 2 WHAT IS THE APPROXIMATE PROBABILITY THAT AFTER 100 STEPS WE FIND THE CHAIN IN STATE 3 ?