Problem Sheet 1-2 B

1. Suppose we toss two fair dice. Let E_1 denote the event that the sum of the dice is six and F denote the event that the first die equals four. Show that E_1 and F are not independent. Let E_2 be the event that the sum of the dice equals seven. Is E_2 independent of F?

2. Assume that an urn contains 5 red balls, 3 white balls and 7 green balls. Draw 3 balls from the urn without replacing the already drawn balls to the urn. Find the probability that "the first and the second draws are red and the third one is green".

3. In ping-pong the first player who achieves 11 points is the winner of the match. However, the winner should win with a difference of at least 2 points. Hence if the current state is 10-10 then the game continues until one of the players has an advantage of 2. In a championship the champion win 100000 HUF. The final match of this championship is played by two equally strong players. During this final match there is a powerbreak. Before the powerbreak the state of the game was 10-9. It is not possible to finish the game after the powerbreak. Hence, it is decided to divide the price money among the two finalists. In which proportion it would be fair to divide the price money?

4. In a certain country, 60 percent of the population is right-handed, 40 percent is left-handed. A right-handed person is able to hit a certain target with his left hand with a probability 0.2. For a left-handed person this probability is de?nitely larger, it is 0.7, A person is chosen at random.

   a) What is the probability that a person hits the target with his/her left hand?

   b) Assume that you get the information that a person hit the target with the left hand. What is the probability that that person is left-handed?

5. We roll two fair dice a blue and a red. We first roll the red die then we roll the blue die as many times as the outcome of the red die. Let Y denote the outcome of the red die and denote by X the sum of the outcomes on the blue die.

   a) Find $E(X)$ and $Var(X)$.

   b) What is the sign of $cov(X, Y)$?

6. Some years ago I met an old fisherman. He was fishing in a big lake, in which many small fish were swimming regardless of each other. He raised his net from time to time, and collected the fish if there were any. He told me that out of 100 cases the net is empty only 6 times or so, and then he added: "If you can guess the number of fish in the net when I raise it out of the water the next time, I will give you a big sack of money." I am sure he would not have said such a promise if he knew that his visitor was a well educated person in probability theory! I was thinking a little bit, then I made some calculation, and then a said a number. Which number did I say?

7. In the nearest forest a running contest were organized. We know that 300 contestants found 1 tick, 75 contestants found 2 ticks on their bodies. Can we guess the total number of contestants in the contest?

8. The distributions of random variables X and Y are exponential with parameters 2 and 3 respectively. What is the distribution of $\min\{X, Y\}$? Compute the probability that the minimum is attained by Y!

9. The number of failures $N(t)$, which occur in a computer network over the time interval $[0, t)$, can be described by a homogeneous Poisson process. On an average, there is a failure after every 4 hours. What is the probability of the following event: there is at most 1 failure in $[0, 8)$, at least 2 failures in $[8, 16)$, and at most 1 failure in $[16, 24)$ (time unit: hour)? What is the probability that the third failure occurs after 8 hours?

10. A hen wants to cross a one-way road, where cars drive according to a homogeneous Poisson process with intensity $\frac{1}{5}$ cars a time unit, all with the same speed. It takes 10 time units for the hen to cross the road. Assume that the hen starts to cross the road immediately when there is a chance to do it without being run over by a car. What is the probabily that the hen has to wait at most two cars before crossing the road? What is the expected number of cars driving through the cross before the hen is able to cross the road?