

Stochastics
Midterm test 2
Fall 2021

1. A face paint artist makes two types of face paint: tiger or butterfly. A tiger takes 10 minutes to paint and a butterfly takes 20 minutes to paint. Each child will ask for tiger with probability $2/3$ and for butterfly with probability $1/3$, independently from the others. There is a long line of children waiting for face paint, so the face paint artist works all the time.
 - (a) Model the face paint artist's activity with a discrete time Markov chain. What are the states? Compute the transition probability matrix.
 - (b) What is the ratio of time she spends with painting tigers?
 - (c) The cost of a tiger is 1200 HUF and the cost of a butterfly is 1800 HUF. Calculate the long term average income of the face paint artist per hour.
2. The parking lot of a small store has room for 2 cars. A customer arrives by car every 10 minutes on average. If the parking lot has room for a car, they park and enter the store. If the parking lot is full, they leave immediately, without entering the parking lot or the store. Each customer who enters the store spends on average 5 minutes inside, then leaves.
 - (a) Let X_t denote the number of cars in the parking lot at time t . What assumptions do we need to make so that the Markov property holds for X_t ? Calculate the generator.
 - (b) What is the probability that at a random time, we find the parking lot empty?
 - (c) What is the long term average ratio of customers lost due to a full parking lot?
3. A translator can translate 10 pages of text per day. She receives a job offer every 4 days on average. If she is idle, she takes the first incoming offer, but she declines all incoming offers while she is working on a job. Each job is translating a text; the average length of the text is 80 pages.
 - (a) Model the translator's activity with a continuous time Markov chain. State the assumptions we need to make to ensure that the Markov property holds. Calculate the generator.
 - (b) Right now, she is idle. Estimate the probability that 12 hours from now, she will be working on a job.
 - (c) She charges 30 euros per page for translation. Calculate her long-term average daily income.
4. Each student passes a certain exam with probability p , but p is unknown. In each of the last 5 semesters, 10 students have taken the exam, and the number of students passing the exam was 5, 7, 5, 9, 6 for each semester respectively. Based on this sample, calculate either the moment estimate for p or the maximum likelihood estimate for p (one of them is enough, you may choose).
5. The box of a board game states that the average game length is 60 minutes. We play 4 games and the length of the games turns out to be 42, 54, 60, 48 minutes each. Based on this sample, test the hypothesis that the average game length is 60 minutes versus the hypothesis that the average game length is not equal to 60 minutes on a 95% confidence level. Also state the type of test used explicitly.