KONSTRUKTIVE
GEOMETRIE
Balatonföldvár, 05–09. 09. 2005

Vortragsauszüge
Abstracts
Előadáskivonatok

Budapest
September 2005

Dieser Band wurde in der Druckerei der
Szent István Universität Fakultät für Bauwesen hergestellt.

This volume has been printed by the printing-office of
Szent István University Ybl Miklós Faculty of Building Sciences

Ez a kötet a Szent István Egyetem
Ybl Miklós Műszaki Főiskolai Karának nyomdájában készült.

KONSTRUKTIVE GEOMETRIE

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In der Zeit vom 5. - 9. September 2005 veranstalten wir bereits unsere 5. Tagung über Konstruktive Geometrie im Hotel Jogar, Balatonföldvár. Diese Serie begann vor 12 Jahren, getragen von dem Ziel, die computergestützte Darstellende Geometrie, die Visualisierung und – ganz allgemein – die Geometrie und ihre Anwendungen bekannt zu machen und wissenschaftlich zu fördern.

Es ist gerade 10 Jahre her, dass uns einer der Initiatoren, Prof. Julius (Gyula) STROMMER, 1995 verlassen hat. Seit Mai dieses Jahr steht vor dem Gebäude H unserer Budapester Technischen und Ökonomischen Universität als Erinnerung an den unvergesslichen Lehrer und Freund eine Skulptur, gestaltet von Jozsef KAMPFL. Auch bei unserer diesjährigen Tagung werden wir die Erinnerung an ihn wachrufen und die Strommer Gedenkplakette samt Preis an eine/n junge/n Geometer übergeben. Alle diese Tätigkeiten sind nur möglich dank der schon von Prof Gyula STROMMER gegründeten Stiftung „Konstruktive Geometrie für eine visuelle Kultur“ und der von Dr. Mária KÕRÖSI gegründeten „Internationalen Geometrie-Stiftung Gyula Strommer“.

Es ist unser traurige Plicht, des verstorbenen ersten Direktors des Hotels Jogar, Herrn Ferenc KÓNYA, zu gedenken. Er war unseren Wünschen und Plänen gegenüber stets überaus entgegenkommend und hilfreich, so wie übrigens auch die jetzige Direktorin, Frau Éva KISSNÉ-PERJÉS, der wir in dieser Form recht herzlich danken.

Dieses Mal werden wir die Vorträge um die Plenarvorträge unserer prominenten ausländischen Professorkollegen gruppieren, nämlich um jene von Eike HERTEL (Jena), Konrad POLTHIER (Berlin), Helmut POTTMANN (Wien), Otto RÖSCHEL (Graz), Hellmuth STACHEL (Wien) und Gunter WEISS (Dresden). Fast alle haben schon einmal bei einer unserer Konferenzen teilgenommen.

Wir schließen mit der Hoffnung, dass auch unsere diesjährige Veranstaltung erfolgreich wird und wir die Begeisterung für die Geometrie an unsere jüngeren Kollegen weitergeben können, so dass diese vielleicht in Hinkunft unsere Traditionen übernehmen und fortsetzen.

Für die Tagungsleitung

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(Szent István Universität Fakultät für Bauwesen)

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KONSTRUKTIVE GEOMETRIE

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From September 5 to 9, 2005 we arrange the 5th International Conference on Constructive Geometry which is taking place in Hotel Jogar, Balatonföldvár. This series of conferences started 12 year before with the aim to promote the scientific fields computer-aided geometry, visualization, and – more general – geometry and its application.

10 years ago, one of the founders of this series, Prof. Julius (Gyula) STROMMER, passed away. In May 2005 a sculpture remembering on this outstanding teacher and friend has been placed and festively unvealed in front of the building H at the Budapest University of Technology and Economy. This scupture was designed and manufactured by József KAMPFL.

Also at our conference there will be an event in memory of our Prof. G. Strommer: The “Strommer Commemoration Plaque” will be presented together with the Prize to a young geometer. All these activites have only been made possible by both, by the “Funds for Constructive Geometry and Visual Culture” donated by G. STROMMER himself, and by the “International Geometry Endowment Gyula STROMMER” founded by Dr. Mária KÕRÖSI.

It is our duty to commemorate on the decease of Mr. Ferenc KÓNYA, the first director of Hotel Jogar. Whenever we confronted him with our particular requests, he was extremely obliging and helpful. And we are really grateful that we can meet the same kindness when contacting the recent director, Mrs. Éva KISSNÉ-PERJÉS.

This year, in the center of our conference there will be plenary lectures held by or prominent colleagues from abroad, as there are Eike HERTEL (Jena), Konrad POLTHIER Berlin), Helmut POTTMANN (Vienna), Otto RÖSCHEL (Graz), Hellmuth STACHEL (Vienna), and Gunter WEISS (Dresden). Almost all of them have already attended one of our previous conferences.

Let us conclude with the hope that also this conference will be succesful in promoting geometry. And in addition, we hope that our personal enthusiam on geometry will flash over to our young colleagues so that in the future they will take over and continue our traditions.

For the organizing committee

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KONSTRUKTIVE GEOMETRIE

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SOME REMARKABLE SPHERES IN CONNECTION WITH TETRAHEDRA

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Subject: solid geometry.

For triangles we have a well-known theorem, called Miquels theorem: Choose any point on each side of a triangle and draw through each vertex a circle passing through the two points on the sides which concur in that vertex. Then these three circles always have a point in common, called Miquels point.

For tetrahedra we have an exact analogon of this fact which seems – despite its beauty – to be almost forgotten. It is called ROBERTS' THEOREM: Choose any point on each edge of a tetrahedron ABCD and draw through each vertex a sphere passing through the three points on the edges which concur in that vertex. Then these four spheres always have a point in common.

The proof of this fact is quite esthetic and gives rise to the assumption that its method – using stereographic projection and several times the corresponding fact for the triangle – could possibly be applied for similar theorems for tetrahedra.

The main ideas of the proof are as follows: The spheres through A,B,D are intersected with face ABD, the spheres through B,C,D are intersected with face BCD and the spheres through A,C,D are intersected with face ACD, giving three times the plane situation of Miquels theorem. So we have three Miquel points MC, MA, MB on the faces ABD, BCD and ACD respectively. The three circles cB, cA, cC through MC, MA and point on edge DB, MC, MB and point on edge DA and MA, MB and point on edge DC lie on the sphere through vertex D. This sphere is intersected with faces ABD, BCD and ACD of the tetrahedron giving three circles through vertex D.

Via stereographic projection from D onto a plane normal to OD (O = center of the sphere through D), these three circles correspond to the sides of a plane triangle having the projected points on edge DB, DA, DC as vertices. Further, the circles cB, cA, cC correspond to circles through one vertex and two of the projections of points MC, MA, MB, which are points on the sides of the plane triangle. Thus by interpreting Miquels theorem on the sphere, we can say that circles cB, cA and cC have a point in common on the sphere through D. But by the previous constructions this point also lies on the spheres through A, B and C, thus being the common point of all four spheres.

ON A NEW RECTIFIABILITY CONDITION OF A SECOND ORDER ORDINARY DIFFERENTIAL EQUATION (GEOMETRICAL INTERPRETATION)

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Subject: second order differential equations, Douglas spaces.

In a famous book of Arnold [1] we can find the following theorem:
„An equation $y'' = \Phi(x,y,y')$ *can be reduced to the form* $\bar{y}'' = 0$ *if and only if the right-hand side is a polinomial in the derivative of order not greather 3 both for the equation and for its dual."*

The notion of Douglas spaces was defined by S. Bácsó and M. Matsumoto in the following way [2]: *A two-dimensional generalized metrical space is a Douglas space if the Douglas tensor vanishes identically.*

We use the following theorem [2]: *A two-dimensional generalised metrical space is a Douglas space if and only if (in a local coordinate system (x,y)) the right-hand side of the equation of geodesics* $y'' = \Phi(x,y,y')$ *is a polinomialin in* y' *of deegre at most three.*

From the previous theorem and definition we obtain a geometrical interpretation of the rectification: *An equation* $y'' = \Phi(x,y,y')$ *can reduced to the form* $\overline{y}'' = 0$ *if and only if the pathspace (determined by the equation* $y'' = \Phi(x,y,y')$*) is projective related to a two-dimensional Douglas space.*

In this lecture we give some example using the last theorem. For instance: the rectifiability condition of the equation $y'' = f(x,y)$ *is the following* $f(x,y) = A(x)y + B(x)$.

[1] V. I. Arnold: Geometrical methods in the theory of ordinary differential equations, Springer-Verlag (1983).

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ON FEUERBACH'S THEOREM
AND A PENCIL OF CIRCLES IN I_2

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Subject: analytic geometry with other transformation groups.

A real affine plane A_2 is called an isotropic plane I_2, if in A_2 a metric is induced by an absolute $\{f, F\}$, consisting of the line at infinity f of A_2 and a point $F \in f$.

A triangle in I_2 is called allowable if no one of its sides is isotropic. Each allowable triangle of an isotropic plane can be set in a standard position, in which it is possible to prove analytically in a simplified and easier way the geometric properties by means of the algebraic theory we have developed in [2].

Using that very method for adapting the well-known Euler and Feuerbach theorems for the isotropic plane, the connection among the circumcircle, Euler circle, tangential circumcircle, and the polar circle of a given allowable triangle will be shown. It will be proved that all four circles belong to the same pencil of circles. There are two more interesting circles in this pencil.

[1] I. M. Jaglom: *Relativity Principle of Galilean and Non-Euclidean Geometry* (in Russian), Nauka, Moskva 1969.

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ISOMETRIES OF AFFINE PLANE AG(2, q) AND CORRESPONDING MIQUELIAN MÖBIUS PLANE

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Subject: finite geometry.

We observe the Miquelian Möbius plane M of order q (where q is a power of a prime) by associating it with the projective line $PG(1, q^2)$ in the way that the points of M are all the points from that line (represented by the set $GF(q^2) \cup \{\infty\}$) and circles are all the Baer sublines of $PG(1, q^2)$.
With a use of irreducible polynomial of second degree over the field $GF(q)$ we construct the finite field $GF(q^2)$ and the affine plane $A = AG(2, q)$, which we build into the Möbius plane M.
Since circles of the plane M are exactly regular Hermitian varieties in $PG(1, q^2)$, we express them with the operator of conjugation of the finite field $GF(q^2)$ (unique involutory automorphism of $GF(q^2)$). With that operator, we also define the "square of the length" of any vector from M and the "square of the distance" between any of two points in M.
Circles from that Möbius plane which are affine ellipses in $A = AG(2, q)$ we name the "regular circles" of M $(AG(2, q))$. We observe the automorphisms of planes A and M which map "regular circles" to "regular circles", as well as automorphisms of those planes which preserve the "square of the distance" between any of two points (isometries).
Matrical forms for such automorphisms, witch depend on a choice of an irreducible polynomial used to construct the finite field of order $GF(q^2)$, are found, as well as vector forms, which do not depend on such a choice.

KIEPERT'S TRIANGLES IN HEXAGONAL QUASIGROUP

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An idempotent, semisymmetric and medial quasigroup is called hexagonal quasigroup. Hexagonal quasigroups were studied in [1], [2].

Let $q = \frac{1}{2} + \frac{\sqrt{3}}{2} i$, and let $a \circ b = (1 - q)a + qb$. Then $(\mathbb{C}, \circ)$ is an example of hexagonal quasigroup, which inspires defining various geometric terms in hexagonal quasigroup.

In 1868 Lemoine proposed the following problem: *If three distinct points P, Q, R are given, construct points A, B, C so that P, Q and R be vertices of equilateral triangles BCP, CAQ and ABR constructed over the sides of a triangle ABC.* The problem was solved by Kiepert [3], and later Steiner [4] showed that there are in general eight such triangles.

We call those triangles "Kiepert's triangles" and using tools from hexagonal quasigroups we prove some results on the position of the vertices of the eight Kiepert's triangles.

[1] V. Volenec: Hexagonal quasigroups, *Archivum Mathematicum* (Brno), Tomus 27a (1991), 113–122.

[2] V. Volenec: Regular triangles in hexagonal quasigroups, *Rad HAZU* [467] 11 (1994), 85–93.

[3] M. Kiepert: Question 864, *Nouv. Ann. Math.*, 8(1869), 40–42.

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GEODESICS AND THE FRENET FORMULAS IN SOL GEOMETRY

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Subject: differential geometry, Sol geometry

The homogeneous geometries have main roles in the modern theory of three-manifolds, regardless being isotropic or not. It is well-known, that there exist only three homogeneous two-dimensional geometries, namely the elliptic, the Euclidean and the hyperbolic geometry. They are isotropic geometries, as well. Homogeneous, but non isotropic geometries can be found in three or higher dimensional manifolds. (Note, that the word geometry means the family of geometries.)

Homogeneous Riemannian manifolds play important role in topology, e.g. in the famous Thurston conjecture. The conjecture states that a three-manifold with a given topology has a canonical decomposition into a connected sum of 'simple three-manifolds', each of which admits one, and only one, of eight homogeneous geometries: H^3, S^3, E^3, $S^2\times S^1$, $H^2\times S^1$, Sol, Nil and SL(2, R). [1]

The purpose of the presentation is to study – probably the oddest one among them – the Sol geometry.

The illustration of Sol geometry is really a hard task, but E. MOLNÁR elaborated the projective interpretations of the eight homogeneous geometries, e.g. of the Sol geometry. [2]

As for our results in one hand we carry out the computation of the Frenet frame (tangent, normal and binormal unit vectors) furthermore with the help of the Frenet formulas we present the curvature and the torsion of a curve, on the other hand we describe the geodesics of Sol.

[1] W. P. Thurston: Three dimensional manifolds, Kleinian groups and hyperbolic geometry. *Bull. Amer. Math. Soc.* **6** (1982), 357-381.

[2] E. Molnár: The projective interpretation of eight 3-dimensional homogeneous geometries. *Beiträge zur Algebra und Geometrie* **38** (1997), 261–288.

ON ION LIKE PACKINGS

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Let be given two positive numbers $r < R$. A packing of circular discs of radius r on a surface of constant curvature is called ion like with parameter R, if the family of discs can be decomposed into two groups, into the so called positive and negative ions in such a way that the centres of any two discs of the same charge are of distance at least $2R$. We determine the densest ion like packings in the Euclidean plane if $r = 1$ and either $1 < R \leq \sqrt{2}$ or $\sqrt{3} \leq R$.

Theorem 1 Let us consider ion packings with parameters $r = 1$ and $1 < R \leq \sqrt{2}$. Then the following lattice-like packing provides a densest ion packing: A fundamental parallelogram of the lattice is a rhomb whose sides are of length 2 and whose not longer diagonal is of length $2R$.

Theorem 2 Let us consider ion packings with parameters $r = 1$ and $\sqrt{3} \leq R$. Then the following packings obviously provide densest ion packings: The centres of the positive ions are the vertices of regular triangle tile whose sides are of length $2R$, and the centres of negative ions are translates of the centres of the positive ions, and the smallest centre distance is at least 2.

Regarding a surface of constant curvature, we write s to denote the circumradius of the square with side lengths $2r$. For $r < R < 2r$, let $T(r, R)$ be a triangle with side lengths $2r$, $2r$, $2R$. We consider the circles of radius r centred at the vertices of $T(r, R)$, and write $d(r, R)$ to denote the density of the parts of the circles in $T(r, R)$ with respect to $T(r, R)$.

Theorem 3 Let us consider ion packings with parameters r and let $R \leq s$.
(i) The density of any such ion packins is at most $d(r, R)$.
(ii) If congruent copies of $T(r, R)$ tile the corresponding surface of constant curvature, then the density bound $d(r, R)$ is optimal.

We note that Károly Bezdek has obtained results about ion like packings in that case when the circular discs are not congruent.

WEINGARTEN SURFACES IN SOME PROJECTIVE-METRIC SPACES

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Subject: non-Euclidean geometry, differential geometry.

In differential geometry of surfaces in Cayley-Klein spaces, i.e. projective-metric spaces, it is of interest to investigate surfaces having some relation between Gaussian and mean curvature. These surfaces are called Weingarten surfaces. We study them in simply isotropic space I_3^1, in double isotropic space I_3^2, in Galilean G_3 and pseudo-Galilean space G_3^1. There are several important classes among these surfaces, such as surfaces with constant Gaussian curvature, minimal surfaces, surfaces with linear connection between Gaussian and mean curvature etc. Some of the results of these types are available in [1], [3], [7] and [8].

Further, we investigate the geometrical interpretation of Gaussian curvature in Galilean and pseudo-Galilean space in connection with the spherical image of a surface.

There are special types of mappings other than (local) isometry that are of interest in Cayley-Klein geometries. Since we described isometry and specially Minding isometry in Galilean, pseudo-Galilean and isotropic space in [4] and [6] we will define conformal and equiareal mappings and find a relationship between them and Gaussian and mean curvature of a surface. The concepts of conformal and equiareal mappings depend only on the first fundamental form of the surface. Conformal and equiareal mappings are further types of equivalences in the study of surfaces.

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[4] B. Divjak, Ž. Milin Šipuš, Minding Isometries of Ruled Surfaces in Pseudo-Galilean Space, *J. geom.* (2003) **77**, 35-47.

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[7] O. Röschel, *Die Geometrie des Galileischen Raumes*, Habilitationsschrift, Institut für Math. und Angew. Geometrie, Leoben, 1984.

[8] H. Sachs, *Isotrope Geometrie des Raumes,* Vieweg, Brauenschweig/Wiesbaden, 1990.

VISUAL ILLUSIONS AND EXTRA DIMENSIONS

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Visual illusions and extra dimensions have been investigated for three decades, namely how to make the - not perceivable by the everyday senses - spatial world visible. Its roots are fed on partly from the arts (e.g., Escher), partly from the sciences (e.g., multidimensional geometry, the world of abstract symmetries). The created *graphical world* is not a simple play with forms and colours, rather it is built by grave scientific regularities.

This *form-world* is composed of one or more (2, 3, 4, 6) closed quadratic prism lines woven into themselves, each returning into itself. These lines, woven also into each other, build a system characterised by an internal regularity. Applying the developed regularities, more and more complicated structures have been created, within the limits of the representation in 2 dimensions. At the same time, the forms seem to emerge from the plane into the space before the spectator's eyes. These complicated structures keep their perspicacity and can easily be surveyed by the general observer, because order and symmetry prevail in them. Most often the 2-, 3-, 4- and 6-fold rotational symmetries are applied, and in certain cases mirror-symmetry from among the geometrical symmetry transformations. The dimensional abundance of planes are increased - to perpendicular in pairs or triplets to each other in multiple dimensions - by the applied colours and their shades. The investigations include special organisations of plane forms, that can be repeated, not only in the plane, but even into higher dimensions, in principle up to infinite.

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CONNECTING SIMPLICES WITH MAXIMAL CONVEX HULL

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How do we have to place two simplices with common vertex or common origin so as to maximize the volume of their convex hull? In this lecture we solve some cases of this problem in E^3, when the dimension of the simplices are not greater than three.

We remark that the best solutions for this problem sometimes distinct the corresponding best solutions of the general problem “How do we have place $v = 6, 7, 8$ points on the unit sphere so as to maximize the volume of their convex hull”

THE SPIRALLOHEDRON
(A SPACE FILLING BODY)

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We demonstrate, through an example of investigating a geometrical construction, how the possibilities of a CAS (Computer Algebra System), in particular, the Maple software can be used to analyse, visualize and clarify a not very simple geometric problem.

The geometric figure that we investigate is as follows.

Take a single-turn helical arc (for example that is obtained from a diameter of a rectangle by preparing the right circular cylinder the mantle of which is the given rectangle). Rotate this helical arc about the straight line passing through its endpoints by the $1/\boldsymbol{n}$, $2/\boldsymbol{n}$, …, $(\boldsymbol{n}-1)/n$ part of the full angle, where $\boldsymbol{n}=3, 4, \ldots$.

Any plane perpendicular to the axis of rotation intersects all the $\boldsymbol{n}$ helical arcs. The points of intersections form the set of vertices of a regular $\boldsymbol{n}$-gon.

Let the point of intersection of the axis of rotation and the perpendicular plane run along the line segment from one endpoint to the other. Meanwhile, the sides of the regular $\boldsymbol{n}$-gons sweep out a closed surface. This surface, and the body bounded by this surface as well, is called an ***n**-armed spirallohedron*.

We show that the Euclidean 3-space can be tiled with the copies of either the 3-armed or the 4-armed spirallohedron, as well as with the copies of 3-armed and 6-armed spirallohedra together.

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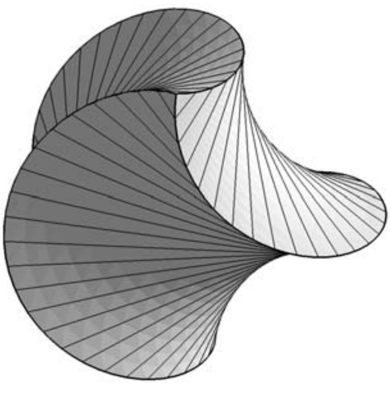
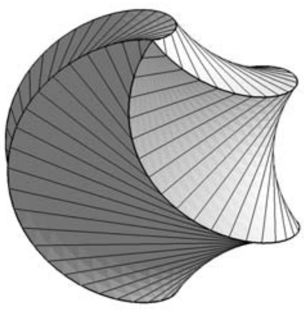
Webpages: http://home.inreach.com/rtowle/Polytopes/spirallo/spiral.html
http://home.inreach.com/rtowle/Polytopes/spirallo/close_pack.html





ON FISHES AND FISHEYE PERSPECTIVES

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Subject: computational geometry.

Humans and animals of all kind – including fishes and insects – have a complicated method to develop images in their brains. They measure angles, not lengths. Together with nonlinear projections onto curved surfaces, impressions are transformed into spatial imagination.

Nevertheless, photos, movies and computer generated animations almost exclusively use classic perspectives, i.e., central projections of space onto an image plane. Such perspectives are linear since straight lines in space appear as straight lines in the image. Classic perspective images will only lead to comparable results in our brains when they are viewed from a distinct viewpoint.

In the general case, however, we need nonlinear perspectives in 2-space, when it comes to 2D-reproduction of visualizing processes. They usually look like fisheye-photos, i.e., projections of space onto a plane via a not symmetric, extremely refracting spherical lens. Similar distortions occur when we look out of still water or into reflecting spheres. In fine Arts, the angle measuring was intuitively applied by artists. In geometry, the inversion at a circle (sphere), several models of non-Euclidean geometries and the stereographic projection onto the plane or mappings of the sphere respectively lead to comparable results.

Finally, realtime algorithms are presented that transform primary fisheye perspectives and other special refractions into classic perspectives. Therefore, we work with Taylor series (or, if possible, with accurate formulas) and – for speed reasons – with precalculated tables.

nth DEGREE INVERSIONS IN PROJECTIVE SPACE

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Subject: projective geometry, analitic geometry, programming.

In this paper we define the transformations of 3-dimensional projective space into itself (nth degree inversions) where corresponding points lie on the rays of 1st order and nth class congruences C_n^1 and are conjugate with respect to quadric Ψ. It is shown that these inversions transform a staight line into the nth degree space curve and a plane into the nth degree surface with one $(n-2)$-ple straight line. Some properties of these surfaces are proved.

In 3-dimensional Euclidean space we shown that nth degree inversions with respect to any sphere with center P transform the plane at infinity into the pedal surfaces of congruences C_n^1 and pole P. We made the program for drawing these surfaces in *Mathematica*.

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[2] A. Gray: *Modern Differential Geometry of Curves and Surfaces with Mathematica.* CRC Press, Boca Raton, 1998.

[3] G. Salmon: *A Treatise on the Analytic Geometry of Three Dimensions, Vol.II.* Chelsea Publishing Company, New York, 1965.

[4] R. Sturm: *Die Lehre von den geometrischen Verwandtschaften, Band IV.* B. G. Taubner, Leipzig-Berlin, 1909.

TATLIN'S NEVER BUILT IMPOSSIBLE TOWER

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Subject: computational geometry.

In 1919, the Russian constructivist artist Vladimir Tatlin submitted a proposal for Monument to the Third International, known as "Tatlins Tower". The tower should be built in glass and steel. Its main form was kind of a double helix which spiraled up to 400 meters high. Tatlin made drawings, and he even built a 10 meter high model of the tower. The model slightly differed from the drawings – obviously due to problems in statics. A few photographs were made. It was strongly doubted whether the tower could ever be built in reality or not. To give a solution to the answer, a group of architects at the University of Applied Arts in Vienna tried to reconstruct the tower and then do calculations with modern software.

A major problem was the geometric reconstruction of the model. Out of three photos with poor quality, this seemed to be an impossible task. Several specialists on 3D-reconstruction were asked for their help and declared it impossible to get the 3D-shape out of the images. With the help of classic geometric ideas, however, the task could be achieved. By means of variation of several uncertainties like vanishing points and image distortions, the total errors could be minimized. E.g., it was possible to locate the cameras positions precisely.

The tower then was digitalized and tested by professional construction software. The result was astonishing: Despite all estimations, the huge tower could have been built with some minor corrections. Its weight, however, would have been 10 times the weight of the Eiffel Tower. Also, the building carried by the tower would for sure have remained an illusion.

HELLY TYPE THEOREMS FOR LINE TANSVERSALS IN THE PLANE

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Consider a family $\mathcal{F}$ of domains of certain property P in the plane. We say that $\mathcal{F}$ is a $T(k)$ family if to any k-member subset there exists a line meeting all of them. The families of property P have Helly number h if h is the smallest number for which $T(h)$ implies the existence of a line meeting all members of the family. In the talk a survey of related problems and results will be given with special emphasis on families of congruent discs.

ZERLEGUNGSPROBLEME IN DER DISKRETEN GEOMETRIE

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Subject: discrete geometry.

Wir betrachten zwei Arten von Zerlegungen von Punktmengen A eines (topologischen) Raumes R mit Transformationsgruppe G:

A *disjunkt zerlegt* in Teilmengen A_i

$$A = A_1 \sqcup ... \sqcup A_n \quad :\Longleftrightarrow \quad A = \bigcup_{i=1}^{n} A_i \quad \wedge \quad A_i \cap A_k = \emptyset \quad (1 \leq i < k \leq n),$$

A *elementar zerlegt* in Teilmengen A_i

$$A = A_1 \uplus ... \uplus A_n \quad :\Longleftrightarrow \quad A = \bigcup_{i=1}^{n} A_i \quad \wedge \quad \mathrm{int}(A_i \cap A_k) = \emptyset \quad (1 \leq i < k \leq n).$$

Es werden neuerer Ergebnisse und offene Probleme vorgestellt zu Fragen der folgenden Art:

1. *(Quadratur des Kreises)* Unter welchen Bedingungen für die Transformationsgruppe G und die Art der Zerlegungsmengen sind ein Kreis K und ein (flächengleiches) Quadrat Q *disjunkt zerlegungsgleich*

$$K \overset{G}{\underset{d}{}} Q \quad :\Longleftrightarrow \quad K = \bigsqcup_{i=1}^{n} K_i \quad \wedge \quad Q = \bigsqcup_{i=1}^{n} \gamma_i(K_i) \quad (\gamma_i \in G;\ i = 1, ..., n)?$$

2. *(Verdoppelung des Würfels)* Unter welchen Bedingungen sind zwei Polyeder eines d-dimensionalen Raumes konstanter Krümmung (elementar) zerlegungsgleich?

3. *(Dreiteilung des Winkels)* Unter welchen Bedingungen kann eine Punktmenge K *disjunkt gepflastert* werden

$$K = \bigsqcup_{i=1}^{n} K_i \quad \wedge \quad K_i = \gamma_i(K_1) \quad (\gamma_i \in G;\ i = 1, ..., n)?$$

[1] E. Hertel; H. E. Debrunner: Zur Rolle von Subtraktionssatz und Divisionssatz in Zerlegungsstrukturen. *Beiträge Algebra Geom.* **10** (1980), 145–148.

[2] E. Hertel: Disjunkte Pflasterungen konvexer Körper. *Stud. Sci. Math. Hung.* **21** (1986), 379–386.

[3] E. Hertel.; Ch. Richter: Squaring the Circle by Dissection. *Beiträge Algebra Geom.* **44** (2003), 47–55.

SHAPE MODIFICATION OF CUBIC B-SPLINE CURVES WITH CURVATURE CONTROL

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Subject: geometric modelling, B-spline curve, curvature, shape control.

B-spline curves are standard description methods in computer aided geometric design today. The data structure of the cubic B-spline curve is fairly simple containing control points and knot values, which are section points of the domain of definition. Any change of these data obviously affects the shape of the curve. Repositioning of a control point is a well-known tool and widely used for shape control by CAD users. The effects of knot modification are less obvious but have been extensively studied in the last couple of years [1-4]. Both methods, the control point repositioning and the knot modification is capable to solve shape control problems given by geometric constraints, like changing the curve to pass a given point or move a point of a curve to a specified location etc.

In this presentation we describe the effects of these shape control methods to the curvature and monotonicity of the curve. Advantages and disadvantages of both of the methods are discussed. Finally a mixed technique is presented with the help of which users can choose among curves with different curvature properties meanwhile the predefined geometrical constraints are always satisfied.

[1] I. Juhász, M. Hoffmann: The effect of knot modifications on the shape of B-spline curves. *Journal for Geometry and Graphics* **5** (2001), 111–119.

[2] I. Juhász, M. Hoffmann: Modifying a knot of B-spline curves, *Computer Aided Geometric Design*, **20** (2003), 243–245.

[3] M. Hoffmann, I. Juhász: On the knot modification of a B-spline curve, *Publicationes Mathematicae*, **65** (2004), 193–203.

[4] I. Juhász, M. Hoffmann: Constrained shape modification of cubic B-spline curves by means of knots, *Computer-Aided Design*, **36** (2004), 437–445.

* Supported by the Hungarian National Research Fund (OTKA) grant No. T048523

CUTTING THREE-DIMENSIONAL BODIES INTO SMALLER PIECES

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Subject: combinatorial geometry, packing and covering, numerical methods

From the practial and theoretical points of view it is important to know how to cut a three-dimensional compact convex body into smaller pieces. There are different ways to measure the size of a piece. The number of pieces and their sizes must be as low as possible. We list some classical and new results related to the famous Borsuk problem.

[1]Partially supported by Hungarian OTKA grants T-038397 and T-047340

ON SINGULARITY OF PARAMETRIC CURVES

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Subject: computer aided geometric design.

For many applications in geometric modeling it is often necessary to detect cusps, loops and zero curvature points of curves. There are quite a few publications on this topic for cubic parametric curves, e.g. [6-7]. In [3] there is a study for polynomial and rational parametric curves of arbitrary degree, in [2] for rational curves, while in [5] for rational Bézier curves.

We examine a more general case, curves that can be described by combination of control points $\mathbf{d}_j$ and basis functions $F_j(u)$

$$\mathbf{g}(u) = \sum_{j=0}^{n} F_j(u)\,\mathbf{d}_j, u \in [a,b].$$

We fix all control points but one, and we show that the locus of the varying control point that yields a zero curvature point on the curve is a developable surface. The regression curve of this developable surface is the locus of those positions of the moving control point that yields a cusp on the curve. This curve is called discriminant, and for the moving control point $\mathbf{d}_i$ has the form

$$\mathbf{c}_i(u) = -\frac{\dot{\mathbf{r}}_i(u)}{\dot{F}_i(u)},\ \mathbf{r}_i(u) = \sum_{j=0, j\neq i}^{n} F_j(u)\,\mathbf{d}_j.$$

Those positions of the moving control point that yield a loop on the curve is the surface

$$\mathbf{l}_i(u,\delta) = -\frac{\mathbf{r}_i(u+\delta) - \mathbf{r}_i(u)}{F_i(u+\delta) - F_i(u)}, u \in [a,b], \delta \in (0, b-u].$$

[1] Y. M. Li, R. J. Cripps: Identification of inflection points and cusps on rational curves. *Computer Aided Geometric Design* **14** (1997), 491–497.

[2] D. Manocha, J. F. Canny: Detecting cusps and inflection points in curves. *Computer Aided Geometric Design* **9** (1992), 1–24.

[3] J. Monterde: Singularities of rational Bzier curves *Computer Aided Geometric Design* **18** (2001), 805–816.

[4] M. Sakai: Inflection points and singularities on planar rational cubic curve segments *Computer Aided Geometric Design* **16** (1999), 149–156.

[5] M. C. Stone, T. D. DeRose: A Geometric Characterization of Parametric Cubic Curves *ACM Transactions on Graphics* **8** (1989), 147–163.

[1]Supported by the Hungarian Scientific Research Fund (OTKA T048523)

THE BUTTERFLY THEOREMS IN THE HYPERBOLIC PLANE

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Subject: synthetic geometry.

There are many varieties of Butterfly Theorem in the Euclidean plane, [1], [3], [4]. Here some analogous theorems in the hyperbolic plane, [2], are proved. It is shown that proofs do not depend on the class of circle into which complete quadrangle is inscribed. The only difference is between the case when quadrangle is inscribed into absolute conic and the case when it is inscribed into one of three classes of circles. The construction of the points with butterfly property is given. It is proved that with any quadrangle an infinite number of butterfly points is associated which are located on a second order curve.

[1] H. S. M. Coxeter, S. L. Greitzer: *Geometry Revisited.* MAA, 1967.

[2] L. Rajčić: Obrada osnovnih planimetrijskih konstrukcija geometrije Lobačevskog sintetičkim sredstvima. *Glasnik mat. Fiz. Astr.* **V** (1950), 57–120.

[3] A. Sliepčević: A New Generalization of the Butterfly Theorem. *Journal for Geometry and Graphics* **VI** (2002), 61–68.

[4] V. Volenec: A generalization of the butterfly theorem. *Mathematical Communications* **V** (2000), 157–160.

THE TRISECTION OF THE ANGLE

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Subject: combinatorial geometry.

In this paper there was presented method of the trisection of the angle. The idea of this method is presented in figure 1.

In this configuration the following relations appear:

- The section |OD| = |DE|= r $= \frac{1}{2} * \left| \frac{R}{\cos(\alpha/3)} \right|$,
- The length of the orthogonal projection |DE| section on the **x** axis is $\frac{R}{2}$ = constant,
- The point D is moving on the curve with equation $y_4 = \pm(x+1) * \sqrt{\frac{1-2x}{3+2x}}$.

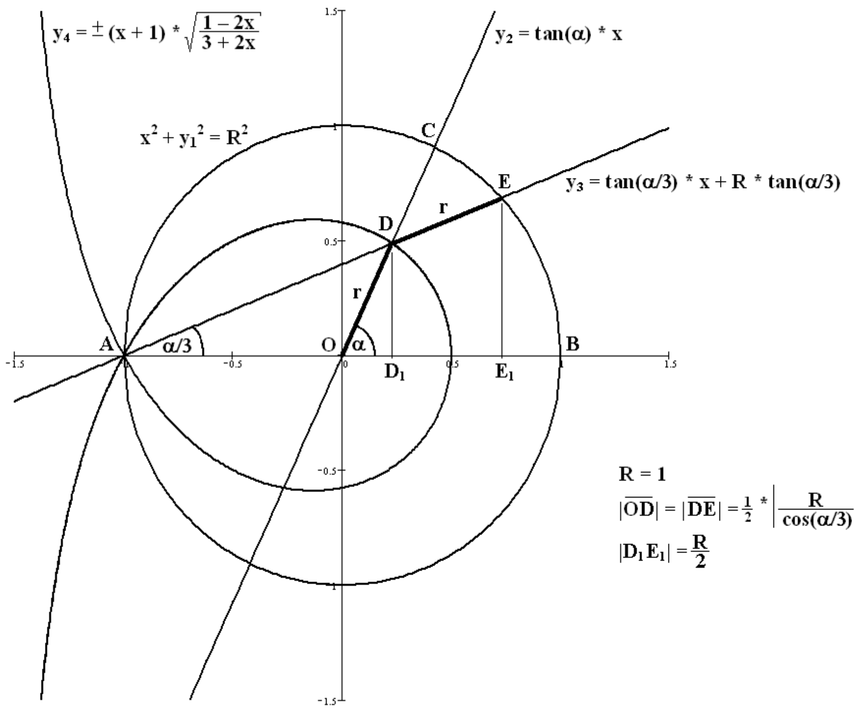

Figure 1. The trisection of α angle.

The correctness of the presented relations was checked in the following way:

- The Cartesian co-ordinate system was applied in the middle of circle $x^2 + y_1^2 = R^2$ with radius R=1,
- The intersection of the straight lines y_2 and y_3 is point D:

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$$D\left(\frac{R*\cos(\alpha)}{2*\cos(\alpha/3)};\frac{R*\sin(\alpha)}{2*\cos(\alpha/3)}\right),$$

- The point D is moving on the curve with equation:

$$x_4(\alpha)=\frac{R*\cos(\alpha)}{2*\cos(\alpha/3)}\ ;\ y_4(\alpha)=\frac{R*\sin(\alpha)}{2*\cos(\alpha/3)}$$

$$y_4(x)=\pm(x+1)*\sqrt{\frac{1-2x}{3+2x}}$$

- The intersection of the straight lines y_1 and y_3 is point E:

$$E\left(\frac{-R*\left(\tan^2(\alpha/3)-1\right)}{\tan^2(\alpha/3)+1};\frac{2R*\tan(\alpha/3)}{\tan^2(\alpha/3)+1}\right),$$

- The length of the section |OD| = $\frac{1}{2}*\left|\frac{R}{\cos(\alpha/3)}\right|$,
- The length of the section |DE| = $\frac{1}{2}*\left|\frac{R}{\cos(\alpha/3)}\right|$,
- The length of the section $|D_1E_1|=\frac{1}{2}R$.

KREISE BEINHALTENDE FRAKTALE

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Subjekt: Fraktal, Kreis, Logo

Die Frage der Fraktale beschäftigt die Mathematiker (Georg Cantor, Helge von Koch, Lewis F. Richardson, Benoit B. Mandelbrot…) schon Jahrzehnte, wenn nicht seit Jahrhunderten, weil sie in unserer Umgebung und in der Natur oft vorkommen.

Die Fraktale zu verstehen, ist nicht gerade einfach, aber einige Aufgaben in Zusammenhang mit bestimmten Fraktalen können sogar Schüler der allgemeinbildenden Schulen mit Hilfe von Logo-Programmen einfach lösen. Die Schüler lernen dabei nicht nur das Programmieren, sondern auch verschiedene Fraktale kennen. Dabei nutzen sie geometrische Kenntnisse und lernen den Begriff der Rekursion.

Außer den Grundaufgaben der Fraktale (Cantor-Menge, Koch-Kurve, Koch-Schneeflocke, Sierpinski-Dreieck und -Teppich) stellt das Zeichnen von Baum-Verzweigungen, -Zweige und -Wurzeln einen großen Bereich dar. Allerdings können solche Rekursiv-Abbildungen, ebenfalls nach Zahl und Position verschiedene Vielecke und Kreise gezeichnet werden.

In meinem Vortrag sind Beispiele von Fraktale so zu sehen, wo nach bestimmten Regeln berührende Kreise gezeichnet werden, deren Radien immer eine kleinere Gestalt annehmen.
Die Kreise mit ungleichem Radius haben verschiedene Farben, wobei hingegen Kreise mit gleichem Radius dieselben Farben haben. Die Abbildungen wurden mit Comenius Logo 3.0 hergestellt.

NEW WAY IN TEACHING CONSTRUCTIVE/DESCRIPTIVE GEOMETRY

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Subject: constructive geometry, descriptive geometry, teaching geometry

The presentation introduces a digital constructive/descriptive geometry teaching programme recently developed by the staff of the Department of Descriptive Geometry and Informatics at Szent István University, Ybl Miklós School, Hungary.

As opposed to the traditional printed media teaching methods that dominantly concentrate only to the end solutions, this computer aided curriculum applies digital presentational techniques (2D drawings, animations and interactive 3D models) to lead the user through every step of construction.. The user works not only with projections, but in addition a 3D model assists him/her in gaining a full understanding of the operations. Various manipulations (zoom, move, rotate) and display options (wireframe, flat or Gouraud shade) are available to further ease comprehension. Thus this method is suitable for studying geometry alone, students need only Internet connection and a plug-in to display 3D models in a browser window.

The new programme is part of an E-learning frame, called Content Management and Collaboration System for eLearning of Natural Sciences, a project granted by EU Socrates-Minerva. In order to test this method, it has been introduced to the curriculum of architecture, civil and urban engineering students at our School this semester.

ISOGONALITY IN AN ISOTROPIC PLANE

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Subject: Analytic geometry with other transformation groups

For each triangle in an isotropic plane whose vertices are not parallel, with the suitable choice of the coordinate system it can be achieved that its vertices are of the form $A = (a, a^2)$, $B = (b, b^2)$, $C = (c, c^2)$ where $a + b + c = 0$.
The isogonality with respect to the triangle ABC is the mapping $T \to T'$, where with $T = (x, y)$ and $T' = (x', y')$ the equalities

$$x' = \frac{xy + qx - p}{y - x^2}, \quad y' = \frac{px - qy - y^2}{y - x^2}$$

are valid.
The images of some points and lines with respect to the isogonality will be studied.

[1] R. Kolar–Šuper, Z. Kolar–Begović, J. Beban–Brkić, V. Volenec: Metrical relationships in standard triangle in an isotropic plane. (to appear)

DIPENDENCY STRUCTURE BASED DYNAMIC DESCRIPTIVE GEOMETRY

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Subject: dynamic geometry, descriptive geometry, teaching geometry.

We show in this lecture, how we can use dynamic geometry softwares as digital aid in descriptive geometrical constructions. We point out the limits of this application and show a way of improves.

The base of these researches is a digital insert of a descriptive geometry course book. The insert contains descriptive geometrical constructions created with Cabri dynamic geometry software. Using this program we can create elemental geometrical constructions. It is possible to "drag" the base/independent elements of it. The geometrical position of all the other elements will change without hurting the geometrical relations. In descriptive geometrical constructions there are geometrical relations belonging to the construction and to the properties of the projection. If we use Cabri, we are able the build both kind of relationships. In this way we can create the whole dependency structure of a descriptive geometrical construction and as it's accepted the "dragging mode" still be enabled.

With CabriWeb we can export our work to Html-format. If users want to check the figures it is necessary to have an installed web-browser and Java VM, Cabri is not needed. They can use "dragging mode" and "step by step mode". During the construction with Cabri we can also fix independent elements. Fixed base elements and dependent elements are not "draggable".

We would like to show our work and results till now, and mention some unanswered questions.

ON THE FIXED POINT OF A COLLINEATION OF THE REAL PROJECTIVE PLANE

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Subject: projective plane geometry, fixed point of a collineation.

Using the "extended Euclidean plane" model we prove the existence of the fixed point (invariant line) of a collineation of the real projective plane. Our construction isn't a proper projective one because we use metrical concepts.

At first, we obtain the given (non-affine, non central-axial) collineation as a product of an opposite isometry and either a homology with positive characteristic cross ratio or an elation. We have to examine only the case when the first factor is a glide reflection, and we express this mapping as a product of a reflection in line and a half-turn. The basis of the final steps of our proof is the fact that the center of the latter one may be arbitrarily chosen on the invariant line of the glide reflection. We show the existence of the fixed point of the product of the half-turn and the homology (elation). Finally, we prove that it is possible to choose the center of the half-turn so that the previously mentioned fixed point lies on the axis of the reflection in line. Thus this point is fixed under the given collineation. We visualize the last steps of the proof by using the "Euklides" dynamic geometry software [1].

[1] www.euklides.hu

RELIEFS AND WHAT THEY SHOW ABOUT VISION

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Subject: projection, constructive geometry.

Some artists show a special interest in problems of projection. With a few examples we will detect the connection between different approaches. They as well have their parallel in spatial transformations. The transformations shall be classified by means of comprehensibility. We ask: "Can the relief or model be seen as a – more or less – strange representation of a familiar object or is it a strange object in itself?"

A CLASS OF AFFINE SURFACES

WITH RANK ONE SHAPE OPERATOR

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Subject: affine differential geometry.

Let *M* be a connected 2-dimensional manifold and $f : M \to R^3$ an immersion so that *f(M)* is an affinely admissable surface. If the the affine normal $\xi : M \to R^3$ has no critical points then the affine normal image $\xi(M)$ is a curve iff the affine shape operator has rank 1 ([2]). The special case when the values of ξ are in a 2-dimensional subspace of R^3, that means the affine normals are parallel to a plane, can be characterized by the following two conditions: rank $S = 1$ and im S is parallel with respect to the affine connection induced by ξ ([1]).
In the lecture we will classify surfaces of the above type within several classes.

[1] B. OPOZDA: A class of projectively flat surfaces. *Math. Z. 219,* (1995), 77–92.

[2] B. OPOZDA, T. SASAKI: Surfaces whose affine normal images are curves. *Kyushu J. of Math. Vol. XLXV No. 1,* (1995), 1-10.

LOCAL PSEUDODISTANCE IN THE CONFIGURATION SPACE OF THE KINEMATIC CHAINS AND ITS OPTIMIZATION FOR SOLVING THE INVERSE KINEMATIC PROBLEM

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Subject: CAD/CAM Systems.

One of the important tasks for applications is the inverse kinematic problem. The problem is defined as calculation of the set of joints' displacements of the given kinematic chain, which corresponds to a position and orientation (pose) of the end-effector. The pose of the end-effector is described as the position and orientation of a rigid body. In the 6-dimensional configuration space of a rigid body we introduce a local pseudodistance to measure the displacement between 2 rigid body configurations. In this paper we propose several ways of defining such a local pseudodistance and we choose the best one for our needs.

Having the distance defined, we can measure the displacement between a destination configuration (the requested pose of the end-effector) and the current one. This displacement (error) is a function f of the number n of joints in the kinematic chain. We then minimize this error. If we find such a $\mathbf{x} \in \mathbf{R}^\mathbf{n}$ that $f(\mathbf{x}) = 0$ (with the given accuracy), then the inverse problem will be solved.

In the paper we discuss the behaviour of the pseudodistance (goal) function. We also assess effectiveness of several optimization methods. Convergence of the method and the cost of calculations, as well as the admissible accuracy are also considered.

The presented solution turned out to be useful as a tool for rapid kinematic chain testing and evaluation.

1. I. A. Sultan, J. G. Wager: A ϕ-model solution for the inverse position problem of calibrated robots using virtual elementary motions. *Inverse problems in Engineering* **Vol. 9** (2001), 261–285.

2. A. A. Goldenberg, B. Benhabib, R. G. Fenton: A Complete Generalized Solution to the Inverse Problem of Robots. *IEEE Journal of Robotics and Automation* **Vol. RA-1**, (March 1985), 14–20.

3. P. E. Gill, W. Murray, M. H. Wright: *Practical Optimization.* Stanford, 1981.

4. D. Manocha, J. F. Canny: Real Time Inverse Kinematics for General 6R Manipulators. *Proceedings of the 1992 IEEE International Conference on Robotics and Automation, Nice, France - May 1992* , 383–389

GENERALIZED POLYGONAL WANKEL ENGINES

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Subject: Kinematical geometry.

The trigonal Wankel engine is kinematically based on the motion where a circle P_m of radius $3d$, as the moving pool curve, rolles on the circle P_s of radius $2d$, as the standing pole curve in the interior. Then the regular trigonal rotor with outcircle of radius $\rho > 3d$, fixed concentrically t_0 the moving pole circle, describes its orbit curve c_ρ . This orbit curve c_ρ is crutial in forming the engine space.

Answering a question of István Revuczky we prove and animate by computer that c_ρ is a convex curve iff $\rho \geq 9d$. The parallel curve $c_{\rho+r}$ with distance r will be the solution to the engine space if the triangle rotor touches $c_{\rho+r}$ with small roller circles of radius r centred in the vertices of the triangle.

All these concepts will be generalized – with animation – to a k-gonal rotor ($3 \leq k \in \mathbf{N}$ natural numbers) in a natural way.

[1] Gy. Strommer: *Geometria.* Műszaki Könyvkiadó, Budapest, 1985.

[2] W. Wunderlich: *Ebene Kinematik.* Bibliographisches Institut, Mannheim–Wien–Zürich, 1970.

[1]Motivated and inspired by ideas of István Revuczky (Dunaújváros).

ON THE HYPERBOLIC CUBE MOSAICS

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Subject: hyperbolic geometry, hyperbolic mosaics.

There is one cube mosaic in the 3-dimensional and one hypercube mosaic in the 4-dimensional hyperbolic space [1]. For the examination of these mosaics first we create belts.

Let the point *P* be one of the finite vertices of a mosaic. Let this point be called the belt 0. Let the belt 1 be formed by all the elements of the mosaic whose common vertex is the point *P*. The belt 2 is constructed by all the elements which have common finite vertices with one of the elements of the belt 1 and the point *P* is not their vertex. If the belt *i* is given, let the belt *i*+1 be formed by the elements which have common finite vertices with some elements of the belt *i*, but have not any of the belt *i*-1.

Let R_i denote the volume of the belt *i* and $S_i := \sum_{j=0}^{i} R_j$, it is the volume of the union of the belts. KÁRTESZI [3], HORVÁTH [2] and VERMES [4] dealt with the ratios of the volumes of the belts in the hyperbolic plane and ZEITLER [5] did it in the hyperbolic space for the mosaic with the regular hexahedron (cube). They showed that $\lim_{i\to\infty} \frac{R_i}{S_i} \neq 0$, which limit is obviously zero in the Euclidean space.

We determine the limits $\lim_{i\to\infty} \frac{R_i}{S_i}$ and $\lim_{i\to\infty} \frac{R_{i+1}}{R_i}$ for the 4-dimensional case.

[1] H. S. M. Coxeter: Regular honeycombs in hyperbolic space, *Proc. Int. Congress of Math. Amsterdam* Vol. III. (1954), 155-169.

[2] J. Horváth: Über die regulären Mosaiken der hyperbolishen Ebene, *Annales Univ. Sci.*, Budapest, Sectio Math., 7 (1964), 49-53.

[3] F. Kárteszi: Eine Bemerkung über das Dreiecksnetz der hyperbolischen Ebene, *Publ. Math.*, Debrecen, 5 (1957), 142-146.

[4] I. Vermes: Über die Parkettierungsmöglichkeit der hyperbolischen Ebene durch nicht-total asymptotische Vielecke. *Wissenschaftliche Zeitschrift der Martin-Luther-Universitat Halle*, 1 (1971), 9-13.

[5] H. Zeitler: Über eine Parkettierung des dreidimensionalen hyperbolischen Raumes, *Annales Univ. Sci.*, Budapest, Sectio Math., XII (1969), 3-10.

NEARLY CONFORMALLY SYMMETRIC RIEMANNIAN SPACES

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Subject: differential geometry, Riemann space, geodesic mapping.

A Riemannian manifold is a manifold possessing a metric tensor. The metric tensor g_{ij} is symmetric and positive definite and tells how to compute the distance between any two points in a given space.

A geodesic is a locally length-minimizing curve. In the plane, the geodesics are straight lines. On the sphere, the geodesics are great circles. The geodesics in a space depend on the Riemannian metric, which affects the notions of distance and acceleration.

The notions of distance, length and angle can be explained by using the metric tensor. We can define coefficients of connection of Riemannian space from the partial derivatives of metric tensor, $\Gamma^i_{jk} = \frac{1}{2} g^{i\alpha}(\partial_j g_{k\alpha} + \partial_k g_{j\alpha} - \partial_\alpha g_{jk})$ is called Christoffel-symbols of V^n.

A geodesic mapping between Riemannian spaces V^n and $\bar{V}^n$ is a one to one mapping between points of spaces, where all geodesic curves of V^n correspond some geodesic curves of $\bar{V}^n$.[2]

All geodesic mappings of a given V^n make a group, where the operation is "carrying out consecutive". The geodesic mappings induce a classification in the set of Riemannian spaces. A class is determined by the spaces having a geodesic mapping onto the same Riemannian space V^n. This class is called to be the geodesic class of V^n. [2]

A Riemannian space V^n has a non-trivial geodesic mapping to another Riemannian space iff the differential equations

$$a_{ij,k} = \lambda_i g_{jk} + \lambda_j g_{ik} \tag{1}$$

$$n\lambda_{i,l} = \mu g_{il} + a_{\alpha i} R^\alpha{}_l - a_{\alpha\beta} R^\alpha{}_{il}{}^\beta \tag{2}$$

$$(n\text{-}1)\mu_{,k} = 2(n+1)\lambda_\alpha R^\alpha_k + a_{\alpha\beta}(R^\alpha_{k,}{}^\beta - R^{\alpha\beta}_{,k}) \tag{3}$$

have non-trivial solutions in a_{ij} (a symmetric tensorfield), λ_i (non-zero vectorfield) and the scalarfield μ. [2]

An *n*-dimensional (*n*>3) Riemannian space V^n is called nearly conformally symmetric if its Ricci-tensor satisfies the condition

$$R_{ij,k} - R_{ik,j} = \frac{1}{2(n-1)}\left(R_{,k} \cdot g_{ij} - R_{,j} \cdot g_{ik}\right),$$

where the symbol "," means the covariant derivative in V^n. [1]

I investigated the nearly conformally symmetric $(NCS)^n$ spaces. I studied the following question: Which property has the nearly conformally symmetric space if it admits a geodesic mapping into another Riemannian space. I proved the following

Theorem. Let us assume that a nearly conformally symmetric Riemannian space $(NCS)^n$ admits a geodesic mapping to another Riemannian space. Then in this $(NCS)^n$ space it necessarily holds the following:

$$-a_i^{\alpha} R_{,\alpha} + 2(n-1)^2 \lambda_{\alpha} R^{\alpha\ \beta}_{\ i\ \beta} = 0 .$$

(a_{ij} and λ_i are non-trivial solutions of (1), (2), (3) differential equations.)

[1] A. Gebarowski: *Nearly conformally symmetric warped product manifolds*. Bulletin of the Institute of Mathematica Academia Sinica **20** (1992) 289-303.

[2] N. Sz. Szinjukov: *Geodezicseszkije otobrazsenyija rimanovih prosztranszv* (Riemann terek geodetikus leképezései), Moszkva, Nauka, 1979.

QUARTIC CURVES OF GENUS 3

constructive classification

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Subject: projective geometry, curves theory.

Quartic projective plane curves (4th degree curve in the projective P^2 space) are divided according to the type of singular points they have, which can bee real or complex. Quartics of genus 3 are a group of curves sufficiently restricted to enable the classification but also sufficiently rich for this classification to be interesting. The theory of this group of quartics has been little studied except properties of bitangents [1]. They are divided in 4 classes with 9 types of curves.

Quadratic transformations are available and useful for construction of quartics with singularities but it seems to be difficult to deduce quartics of genus 3 in a constructive way.

In this talk the real smooth non-singular quartics are classified by points at infinity, type of their branches and we look for constructive deduction of this group of quartics. There are M-curves (with a maximal number of branches) and maximally inflected curves.

[1] G. Salmon: Higher Plane Curves. *Chelsea Publishing Company*, *New York*

[1] Supported by Faculty of Civil Engineering, Rijeka

QUARTIC CURVES
IN MULTIMEDIAL GEOMETRY TEACHING

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Civil Engineering Faculty, University of Rijeka

K. Jurasic [1]
Technical Faculty, University of Rijeka

Subject: quartic curves in multimedial education

As an additional part for the talk about quartics, we present the role and position of quartics in mathematical education of our students. The presentation contains some examples about curvature of fourth degree curves, animated using software *Mathematica.* Future engineers use also the constructive way to get space quartic curves in E^3 and their plane projections, using CAD software. Learning materials available on Internet, lectures in Power point combining CAD, *Mathematica* and photos, examination using computer, that are new media in teaching of Constructive and Descriptive geometry.

Examples are prepared as a lesson for students, within the project “Actual parts of mathematic and geometry in engineering graphics and practice”, which is supported by Foundation “Zaklada” of University of Rijeka.

[1] Supported (partially) by “Zaklada”, University of Rijeka

DISCRETE MINIMAL SURFACES

AND CONVERGENCE

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Subject: discrete differential geometry, surface theory, constant mean curvature.

Discrete minimal surfaces are a generalization of minimal surfaces to polyhedral surfaces which are critical points of the discrete area functional [1][2][5]. In this talk we present a new set of triply periodic discrete minimal surfaces [4] which are embedded and metrically complete. We also discuss new results on discrete curvature operators for polyhedral meshes [3] which include the convergence of discrete minimalsurfaces.

[1] K. Polthier, W. Rossman: *Index of Discrete Constant Mean Curvature Surfaces*. J. Reine und Angew. Math., 549 (2002), 47-77.

[2] K. Polthier: *Unstable Periodic Discrete Surfaces*. in: Nonlinear Partial Differential Equations, S. Hildebrandt and H. Karcher (Eds.) Springer Verlag (2002), 127-143.

[3] K. Hildebrandt, K. Polthier, M. Wardetzky: *On the Convergence of Metric and Geometric Properties of Polyhedral Surfaces*. ZIB Preprint 05-24, 2005.

[4] M. Nieser, K. Polthier: *Construction of Explicit Discrete Minimal Surfaces*. In preparation.

[5] K. Polthier: *Computational Aspects of Minimal Surfaces*. In: Global Theory of Minimal Surfaces, D. Hoffman (Ed.) Clay Mathematics Proceedings, AMS (to appear: Sept. 2005).

[1] Supported by DAAD and the DFG Research Center ”Mathematics for Key Technologies” (MATHEON) in Berlin.

MODELING OF SHAPES IN THE GEOMETRIC SIMULATION

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Subject: geometric modeling, designing CAD/CAM systems.

The module of geometric simulation of machining enables verification of NC programs at the design stage. The main role here is played by the calculation of the tool path and the updating of the stock shape as the cutter moves along the designed path. This is necessary to detect collisions between the tool holder and the machined part. The only data is the initial shape of the stock and the definition of the tool trajectory. The final shape of the part may differ considerably from the designed object; therefore, no assumption about the current shape can be made. Hence, corresponding data structures are either based on explicit solid representation or on an approximation of the machined part, such as surface discretization or volume decomposition.

In the explicit solid representation, the changing shape is defined as a Boolean difference between the stock and the tool movement envelope. It is a natural way to provide a verification module in systems, which support direct CSG modeling. However, this technique is computationally expensive and not suitable for complicated tool shapes or trajectories despite its high accuracy.

Surface discretization methods define a set of vectors at selected points on the designed model surface and then cut the vectors at intersections with the current envelope of the moving cutter [1]. The precision of the verification is determined here by the input surface mesh, which is generated before the simulation starts and fixed afterwards.

Solid volume partition methods extensively exploit the solid model decomposition mostly in the XY-plane in cells containing: the corresponding heights over this plane (Z-values) or a set of line segments (ray-rep). Another solution recommended here is dividing the changing shape into spatial cubic cells stored in a three-colored oct-tree. Grey leaves contain the current boundary of the changing stock and can be seen as its approximation, mostly as a polyhedral boundary representation (B-rep) [2]. Alternatively, a new method based on the lazy-programming and adaptive definition has also been proposed [3]. The necessary accuracy improvement is made here locally (e.g. just in the vicinity of a suspected collision region) without changes to the rest of the stock.

[1] B. K. Choi, R. B. Jerard: *Sculptured surface machining – Theory and applications*. Kluwer Academic Publishers, 1998.

[2] U. Roy, Y. Xu: Computation of a geometric model of a machined part from its NC machining programs. *Computer-Aided Design* (1999), 31, 401–411.

[3] J. Porter-Sobieraj, K. Marciniak: Design and exploitation of a simulation module for injection moulds machining. *Computer Integrated Manufacturing Advanced Design and Management* (2003), 449–454.

FEATURES, SIGNATURES AND FLOWS OF SHAPES

Helmut Pottmann

Methods for global shape understanding are required in a variety of applications including shape segmentation, retrieval of geometric objects from databases, matching of shapes or morphing between different objects. In the past, a lot of work in this area has been based on curvature analysis on multiple resolutions of the object and on feature reducing flows. In the present talk, the speaker will outline some new directions and especially focus on work currently done at TU Vienna: We will study integral invariants for multiscale curvature analysis; those can be used for the generation of so-called shape signatures and for the detection of persistent features, which provide an approach to robust global shape matching. Moreover we will describe initial results on morphing via geodesic flows in Riemannian shape manifolds and address directions for future research.

CONSTUCTION OF A FAMILY OF FOUR-DIMENSIONAL UNIFORM POLYTOPES

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Subject: computational geometry, 4-dimensional geometry.

Starting with a 3-dimensional regular polyhedron, certain truncation methods derive Archimedian polyhedra. For example truncating a cube at the midpoints of edges we get a cuboctahedron (3, 4, 3, 4), or cutting the vertices of an icosahedron at appropriate division points we obtain a (5, 6, 6) classical football polyhedron.

In the talk we look for the analogous procedures to create 4-dimensional polytopes. We can find 14 possibilities which will be demonstrated on the cube tiling (as regular infinite polytope). In a similar way we get a lot of polytopes (84 ones)from the six regular ones by a computer program. Usually these polytopes have too many cells (facets or 3-faces), faces (2-faces), edges and vertices, so it is very difficult to recognize their similarities and differences. In order to examine their structure we shell display a characteristic part of them. This part is built from the cells near the vertices of a characteristic simplex of the original regular polytope. In this way we can classify the constructed polytopes and find 42 different ones among them forming convex uniform polytopes of Archimedian cells.

NEW MEDIA IN GEOMETRY TEACHING

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Subject: new media, information technology, didactics, geometry

A course "New media in geometry teaching" was recently established in the curriculum for geometry teachers at Vienna University of Technology. In winter term 2004 this course was held by the author for the first time. Here an overview of topics and experiences in this course and further teacher training courses shall be given.

The main topics covered in the courses - beside geometric modelling, construction and problem solving - are creation of worksheets, presentations, realistic images and animations with appropriate software as presentation software (e.g. PowerPoint), vector graphics software (e.g. CorelDraw), CAD software (e.g. MicroStation) and software to generate realistic images and animations (e.g. PovRay, MicroStation). These techniques are important either for preparation of teaching materials or as subjects of geometry curricula. It is also important to focus on data management and transfer, software techniques and didactic methods for any of the above mentioned topics.

It is obvious that the meaning of "new" is dependent on the current period. So in the future also the following topics will become more and more important: e-learning, learning management systems, video sequences generated with screen capture software, webquests,... and more.

Also some examples of student works and teaching materials will be presented.

RECONSTRUCTION OF SURFACES FROM POINT CLOUDS

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Subject: reconstruction, surface fitting

Reconstruction of complex surfaces is an important technique mainly in computer aided design and advanced medical technologies, and also occurs in creating computer representations of artistic objects and in animation.

The process of reconstruction converts a large set of discrete data points generated by an appropriate measurement (usually laser scanning) into continuous surfaces. The quality of the conversion depends on two basic factors: from functional point of view, the deviation of the surface from the data points should be as small as possible (accuracy); from the point of view of aesthetic appearance, the smoothness and fairness of the surfaces are most important. Unfortunately, these two factors are in fundamental contradiction to each other. If a surface is `too smooth', it cannot meet tight tolerances at highly curved areas. On the other hand, if the surface is `too accurate', it will remain close to the data points, but unwanted oscillations may occur. To resolve this contradiction is one of the key problems in reconstruction.

The reconstruction process consists of several steps. It starts with filtering the (usually large) set of data points and eliminating outlying points. The next step is registration, i.e. matching and fusing of point sets arising from separate measuring directions. Then a topology (usually a triangulation) is built over unstructured set of points, which reflects neighborhood relations. The central problem of reconstruction is segmentation and surface fitting. Segmentation means to group and separate data points that belong to different surfaces as constituting geometrical elements. Finally surface fitting provides the parameters of the mathematical representation of surfaces that approximate each separated point set. This later is solved by minimizing a functional which depends on the surface parameters, and consists of two terms, reflecting the above two requirements (accuracy and smoothness).

Several methods have been developed and implemented to perform the above process of reconstruction. In many cases they work well, however there are cases (especially object surfaces with complex shape and topology), when they fail, or the quality of the reconstructed surface is not satisfactory.

In the presentation we briefly summarize mathematical and computational techniques implemented in most powerful reconstruction software systems. We analyze conditions when they work sufficiently, and describe geometrical situations where they fail. After that we discuss some of our improvements, such as maintaining continuity over the whole surface and adjusting smoothness automatically.

POLYHEDRAL ORNAMENTS
WITH MOVEABLE MOTIVES

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Institute of Geometry, University of Technology, Graz, Austria

Subject: Kinematic Geometry (AMS Classification: 53 A 17).

Our basic tool is a given (finite) polyhedral group *g* of the Euclidean 3-space, which consists of the set of automorphic displacements of a polyhedron *P**. Application of all transformations from *g* to an arbitrary motif *M* generates an **ornament of the group** g **with the motif** *M*. We start with a one-parametric motion of a moving frame Σ with respect to a fixed frame Σ^* and call it **moveable motif** $M = \Sigma / \Sigma^*$. Application of the transformations from the group *g* yields an ornament which is called **polyhedral ornament with a moveable motif**. This concept generalizes ideas of R. CONNELLY et al. [1] and their generalizations by H. STACHEL, K. WOHLHART and O. RÖSCHEL published in several papers not cited in this abstract. The corresponding ornament determines a kinematic chain of congruent one – parametric motions (even congruent with respect to their parametrisation!), which are linked via the transformations of the group *g*. It is quite natural to start the considerations with "simple" motions *M* as motives: We will give examples for planar motions (extended into the 3-space), later we will take Darboux – motions as motives *M*. There the general points paths are ellipses.

If a **Darboux – motion as motif** *M* is transformed by displacements from *g*, we gain a kinematic chain. Any two copies $M_1 = \Sigma_1 / \Sigma^*$ and $M_2 = \Sigma_2 / \Sigma^*$ of the motif *M* (gained by two different transformations from *g*) can be seen as **linked Darboux – motions** [2]. In general the relative motion Σ_2 / Σ_1 is a rational motion of degree 4 – the point paths being rational curves of degree 4. In [2] I showed that in general there will be an (at least) two - parametric family of points in Σ_2 at a constant distance from certain corresponding points in Σ_1. The points are situated on a one – sheet – hyperboloid, which may split into a pair of planes. This way we are able to determine a framework with rigid rods, which does not disturb the one – parametric motion of the ornament with the moveable motif.

Some examples of these new overconstrained mechansims with high order of symmetry will be presented in this paper (we will focus on motions as motives and on ornaments with respect to the octahedral and the icosahedral group, resp.).

[1] R. Connelly, B. Hendrickson, M. Terrell: Kaleidoscopes and Mechanisms. *In K. Böröczky et al. (eds.): Intuitive Geometry, Colloquia Mathematica Soc. J. Bolyai,* **63** (1991), 67 – 75.

[2] O. Röschel: Linked Darboux – motions. *Mathematica Pannonica* **7**/2 (1996), 291 – 301.

MATRIX REPRESENTATION OF CENTRAL-AXONOMETRIC MAPPING

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Subject: computer graphics, descriptive geometry

The degenerated projective, or so called *central-axonometric* mapping of the space to the plane can be given by the images of the origin, the unit-points and the infinite points of the axes of the Cartesian coordinate system, using homogeneous coordinates:

$$\kappa:\phi:\left(O;E_1,E_2,E_3,U_{1\infty},U_{2\infty},U_{3\infty}\right)\to\phi^c:\left(O^c;E_1^c,E_2^c,E_3^c,U_1^c,U_2^c,U_3^c\right)$$

This mapping between the protectively embedded Euclidian spaces $\mathrm{P}^3\to\mathrm{P}^2$ can be written by a 3×4 type real matrix as a linear mapping $\mathrm{E}^4\to\mathrm{E}^3$. In general this mapping is not a central projection, however, the kernel of the mapping holds as a center. In this paper we will give a method to find the matrix of the mapping if ϕ^c is given. We will show that the kernel of κ is a point in P^3, and we calculate this point. Using this method we can solve such problems as back face culling, visibility problems, shading without using central projection. Second, there are more criteria when ϕ^c is a basic figure of a central projection. Using the so-called Szabó condition, choosing a comfortable coordinate describing ϕ^c, we calculate the matrix of the central projection by finding a "good" U_3^c if the $O^c;E_1^c,E_2^c,E_3^c,U_1^c,U_2^c$ points are given.

THE NEGATIVE PEDAL CURVES OF THE CONICS

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Subject: synthetic geometry.

Although the curves of order two are also the curves of the class two, it is shown that their negative pedal curves can have class two, three and four and orders two, three, four, five and six, in different combinations. The properties of that line curves can be derived from the properties of their corresponding curves in the polarity. Line curves are constructed as the curves corresponding to the conics in the composition of inversion, with different position of the pole, and polar reciprocation.

[1] A. A. Savelov: *Ravninske krivulje.* translation from Russian, Školska knjiga, Zagreb, 1979.

[2] H. Wieleitner: *Spezielle Ebene Kurven.* G.J. Goeschensche Verlagshandlung, Leipzig, 1908.

ON STUDY'S PRINCIPLE OF TRANSFERENCE

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Subject: Euclidean analytic geometry, kinematics.

Normalized Plücker coordinates give rise to a one-to-one correspondence between directed lines (spears) in the Euclidean 3-space and dual unit vectors, i.e., points of the "dual unit sphere". Thus theorems from spherical geometry can be transferred into spatial geometry just by replacing the field $\mathbb{R}$ by the ring $\mathbb{D}$ of dual numbers.

But this principle of transference is not only restricted to unit vectors. Each infinitesimal spatial motion (screw) can be represented by a dual vector, which arises when the dual unit vector of the screw axis is multiplied with the dual angular velocity. Hence any result from spherical kinematics has a counterpart in spatial kinematics. This is demonstrated by transfering the Euler-Savary-formula as well as the Camus principle of gearing into line geometry.

[1] W. Blaschke: *Kinematik und Quaternionen.* VEB Deutscher Verlag der Wissenschaften, Berlin 1960.

[2] H. Stachel: Instantaneous spatial kinematics and the invariants of the axodes. Proceedings Ball 2000 Symposium, Cambridge 2000, no. 23, 14 p.

SYMMETRY GROUPS FOR THE COMPACT SURFACE OF GENUS 3^{-}

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University of Kaposvár

Subject: Symmetry groups, topology of the surfaces.

In this talk the complete diagram of metric normalizers of the fundamental group $\mathbf{G} = \otimes^3$ in Isom H^2 will be determined. We completely classify the symmetry groups **N/G** of the 3^{-} surface, i.e. the connected sum of 3 projective planes, into 12 normalizer classes, up to topological equivariance, by the algorithm for fundamental domains, developed in [1], [2], [3] and [4], aided by computer. Our algorithm is applicable for any compact surface with exponential complexity by the genus g.

The most important new results are formulated in the following two theorems:

Theorem 1.3 : *The* 3^{-} *surface, as a connected sum of* 3 *projective planes, allows hyperbolic* (H^2) *metric structures such that* 12 *isometry groups* **N**/**G** *can act on the* 3^{-} *surface, induced by normalizers* **N** *of the fundamental group* $\mathbf{G} = \otimes^3$ *in the isometry group of* H^2*, up to homeomorphism equivariance. These* 12 *normalizers* **N** *provide* 65 + 58 *fundamental tilings for our* 3^{-} *surface* $H^2/\mathbf{G}$**.**

Theorem 1.2 : *The surface* 3^{-} *has 2 maximal, i.e. not extendable, symmetry groups:* ****2223/G*** *of order 12 and* ****2224/G*** *of order 8. The other groups* ***N/G*** *(*$\mathbf{G} = \otimes^3$*) are their subgroups, having a lattice structure.*

Finally we can formulate two conjectures on the basis of given results for any surface g^{-}, $g \geq 3$.

[1] Z. Lučić – E. Molnár, Combinatorial classification of fundamental domains of finite area for planar discontinuous isometry groups, *Arch. Math.* **54** (1990), 511–520.

[2] Z. Lučić – E. Molnár, Fundamental domains for planar discontinuous groups and uniform tilings, *Geometriae Dedicata*, **40** (1991), 125–143.

[3] Z. Lučić – E. Molnár – N.Vasiljević, Combinatorial structure of fundamental polygons of finite area for plane discontinuous groups, *Manuscript,* 1998.

[4] E. Stettner, Die Computergestützte Klassifizierung der Flächeneinwickelungen in einem Vieleck vorgegebener Seitenanzahl, *Annales Univ. Sci. Budapest,* **41** (1998), 103–115.

[5] E. Stettner, Symmetriegruppen und fundamentale Pflasterungen der Fläche vom Geschlecht –3, I. Maximale Gruppen mit Sechseckbereichen, *Studia Sci. Math. Hung.***40** (2003), 41–57.

[6] E. Molnár – E Stettner, Symmetry Groups and Fundamental Tilings for the Compact Surface of Genus 3^{-}, 2. The normalizer diagram with classification, *Beiträge zur Algebra und Geometrie*, **46** (2005), 19–42.

SUBGROUPS WITH A TETRAHEDRAL FUNDAMENTAL DOMAIN OF THE COXETER SPACE GROUPS

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In their papers E. Molnár, I. Prok and I. K. Zhuk classified all tetrahedral fundamental domains. Looking to symmetries of these domains, it is possible to find all their supergroups, especially Coxeter groups if they exist. In that way it is possible to find all subgroups with a simplicial fundamental domain of Coxeter space groups.

ZENTRALAXONOMETRIE
UND DIE GENAUIGKEIT DER ABBILDUNGEN
DER DIGITALEN FOTOMASCIENE

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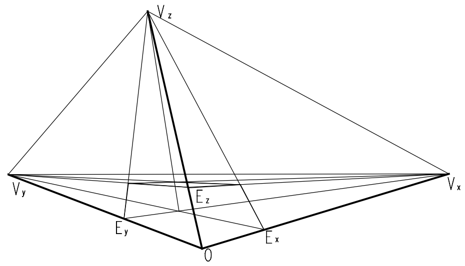
Lehrstuhl für Computergraphik, Universität Debrecen

Subject: descriptive geometry, projektive geometry, computer graphics.

In Arbeit [1] ist eine Kondition dafür, wann eine Zentralaxonometrie Zentral-projeztion ist.



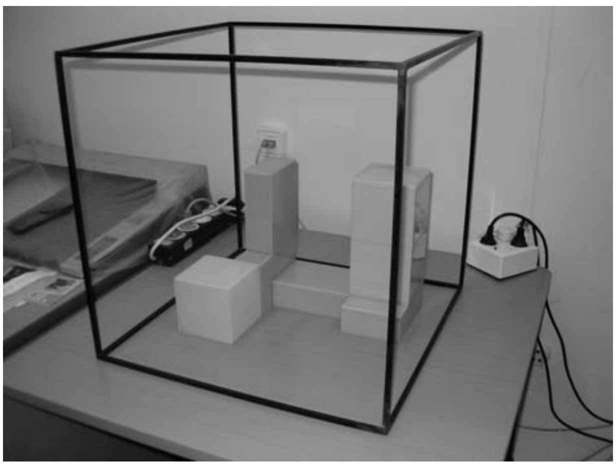
Die degenerierte projektive Abbildung des Raumes auf die Bildebene ist dann und nur dann eine Zentralprojection des Raumes, wenn für die Hautbildfigur $O(E_x, E_y, E_z, V_x, V_y, V_z)$

$$\left(\frac{OE_x}{E_xV_x}\right)^2 : \left(\frac{OE_y}{E_xV_y}\right) : \left(\frac{OE_z}{E_xV_x}\right)^2 = \tan(V_yV_xV_z) : \tan(V_xV_yV_z) : \tan(V_yV_zV_x) \text{ gültig ist.}$$

Wir machten ein Programm dafür, dass aus einem Würfelbild die obige Formula anwenden zu können. Dazu ist es nötig nur 6 Punkte digitalizieren. Daraus durch homogene Koordinaten mit einem Befehl bekommen wir die Bilder der Fernelemente. Daraus die Winkel des Dreieks $V_xV_yV_z$. Dann bekommen wir 3 Werte. Zum Bp.

$$\left(\frac{OE_x}{E_xV_x}\right)^2 : \left(\frac{OE_y}{E_xV_y}\right)^2 - \tan(V_yV_xV_z) : \tan(V_xV_yV_z) = \Delta_x \quad \text{Ebenso } \Delta_y \text{ und } \Delta_z .$$

Endlich bilden wir einen Wert: $\frac{\Delta_x + \Delta_y + \Delta_z}{3}$. Wenn für eine Maschiene dieser Wert kleiner ist als für andere, diese Maschiene gibt geometrisch besseres Bild.



[1]. Ein Satz über die Zentralaxonometrie (J. Szabó, H. Stachel, H. Vogel) Sb. Akad. Wiss. Wien (math.-nat. Kl.) 203 (1994), 3–11.

IMPROVING DISCRETE REPRESENTATIONS OF SURFACES BY SLICING

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Department of Geometry, Budapest University of Technology and Economics

Subject: CAD, triangulated surfaces.

Many surface-oriented applications, e.g. finite element methods and layered manufacturing use triangle meshes, which are often the only "reliable" approximations of continuous surfaces. In CAD systems a number of techniques have been developed to create discrete representations of surfaces using different triangulation algorithms. Unfortunately, the datastructures of such meshes may contain errors in numerical data and topological relations [3]. Discrete geometry algorithms, and discrete differential-geometry operators for estimating simple geometric attributes such as curvatures [5], for generating characteristic surface curves [2] and for smoothing of meshes [1] require error-free representations.

With an appropriate polyhedral datastructure built on a triangular mesh topological errors such as holes and gaps can be detected efficiently [4]. For removing defect triangles and filling simple holes a slicing algorithm will be presented. It works semi-interactively, and computes in each step only with the triangles that are effected by the slicing plane.

[1] M. Desbrun, M. Meyer, P. Schröder and A. H. Barr: Implicit Fairing of Irregular Meshes using Diffusion and Curvature Flow. *Computer Graphics Proceedings* (Annual Conference Series, 1999), 317–324.

[2] K. Polthier and M. Schmies: Straightest Geodesics on Polyhedral Surfaces. In H. C. Hege and K. Polthier, editors, *Mathematical Visualization* Springer Verlag, 1998.

[3] M. Szilvási-Nagy and Gy. Mátyási: Analysis of STL Files. *Mathematical and Computer Modelling* **38** (2003), 945–960.

[4] M. Szilvási-Nagy, I. Szabó and Gy. Mátyási: A polyhedral data structure for shape characterization of meshed surfaces. *II.Magyar Számítógépes Grafika és Geometria Konferencia* (Budapest, 30.06.–1.07. 2003), 71–77.

[5] G. Taubin: Estimating the Tensor of Curvature of a Surface from Polyhedral Approximation. In *Proc. 5th Intl. Conf. on Computer Vision (ICCV'95)* (June 1995), 902–907.

[1] Supported by a joint project between the TU Berlin and the BUTE and the Hungarian National Foundation OTKA No. T047276

THE OPTIMAL HOROBALL PACKINGS OF THE COXETER TILINGS IN THE HYPERBOLIC n-SPACE

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Subject: Hyperbolic geometry, horoball packings.

In this talk I describe a method – based on the projective interpretation of the hyperbolic geometry – that determines the data and the density of the optimal horoball packings of each well-known Coxeter tiling (Coxeter honeycomb) in the hyperbolic n-space $\mathbf{H}^n$.

[1] Szirmai, J. Determining the optimal horoball packings to some famous tilings in the hyperbolic 3-space, *Studies of the University of Zilina, Mathematical Series* (2003) **16**, 89–98.

[2] Szirmai, J. Horoball packings for the Lambert-cube tilings in the hyperbolic 3-space, *Beiträge zur Algebra und Geometrie*, (2005) **46/1**, 43–60.

THE TOUCHING NUMBER OF A TYPICAL CONVEX BODY

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Subject: convex and discrete geometry

The touching number $t(K)$ of a d-dimensional convex body K is the largest number of pairwise touching translates of K that can be arranged in $\mathbb{R}^d$. By Danzer and Grünbaum [1], it is known that $t(K) \leq 2^d$. By Groemer [2], the equality $t(K) = 2^d$ occurs only for parallelotopes. On the other hand, it is easy to see that for an Euclidean ball B in $\mathbb{R}^d$, we have $t(B) = d+1$, while there exist convex bodies in $\mathbb{R}^d$ with exponentially large touching numbers which are simultanoeusly smooth and strictly convex, so smoothness or strict convexity in themselves do not imply a relatively small touching number. The question arises that what is the order of magnitude of the touching number for the majority of d-dimensional convex bodies?

We show that a typical convex body K in $\mathbb{R}^d$ has a touching number $t(K)$ which is at most $2d$, that is, most d-dimensional convex bodies have touching numbers less than or equal to $2d$. Note, that we say that a typical convex body has a certain property if every d-dimensional convex body has that property except a family of convex bodies in $\mathbb{R}^d$ which is the countable union of nowhere dense subsets of the collection of all convex bodies in $\mathbb{R}^d$, in the topology induced by the Hausdorff metric (cf. Schneider [3]).

More precisely, we prove that there exists a dense open subset of convex bodies in $\mathbb{R}^d$, in the sense of the topology induced by the Hausdorff metric on the collection of all convex bodies in $\mathbb{R}^d$, such that for every convex body K in that subset, it holds that $t(K) \leq 2d$.

[1] L. Danzer and B. Grünbaum: Über zwei Probleme bezüglich konvexer Körper von P. Erdös und von V. L. Klee. *Mathematische Zeitschrift* **79** (1962), 95–99.

[2] H. Groemer: Abschätzungen für die Anzahl der konvexen Körper, die einen konvexen Körper berühren. *Monatsh. Math.* **65** (1961), 74–81.

[3] R. Schneider: *Convex bodies: The Brunn-Minkowski theory*, Cambridge Univ. Press, Cambridge, 1993.

[1]Supported by grant no. T038397 of the Hungarian National Science Foundation (OTKA).

ÜBER EIN PROBLEM VON MEHRFACHEN GITTERFÖRMIGEN KREISANORDNUNGEN

Ágota H. Temesvári
WU Sopron

Eine Menge von kongruenten offenen Kreisen bildet eine k-fache Packung, wenn jeder Punkt der Ebene zu höchstens k der Kreisen gehört. In einer k-fachen Überdeckung mit kongruenten abgeschlossenen Kreisen überdecken die Kreise jeden Punkt der Ebene mindestens k-fach.

L. FEJES TÓTH hat die Definition der mehrfachen Packung und Überdeckung angegeben. Bilden die Kreismittelpunkte in einer Kreisanordnung von kongruenten Kreisen ein ebenes Punktgitter, dann ist die Kreianordnung gitterförmig.

Wir behandeln solche, in erster Reihe gitterförmige mehrfache Kreisanordnungen, wobei die Kreise eine k-fache Überdeckung (mit abgeschlossenen Kreisen) und eine $(k+1)$-fache Packung (von offenen Kreisen) für irgendeinen Wert von k bilden.

Wir geben die möglichen gitterförmigen Kreisanordnungen mit der obigen Eigenschaft für $0 \le k \le 8$ and $k \ge 2 \cdot 10^5 - 1$ an. Daneben werden auch einige nicht gitterförmige Kreisanordnungen vom obigen Typ angegeben.

Die Mittelpunkte von diesen speziellen Kreisanordnungen bilden ein solches ebenes Punktsystem, für das die Grezvarianz im Fall eines speziellen ebenen stochastischen Prozesses minimal ist.

REGULÄRE KÖRPER UND MEHRDIMENSIONALE WÜRFEL

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Subject: konstruktive Geometrie

Wenn wir die regulären (platonischen) Körper in- oder aufeinander konstruieren, bestimmen sie in gewisser Maßen reguläre Körper. Zwei von ihnen, sind von Rauten gegrenzt und wenn wir deren Kanten, die verschiedene räumliche Stände haben in die Körperspitzen schieben, erhalten wir die dreidimensionalen Raumgitter von mehrdimensionalen Würfeln.

Die Kanten dieser Gitter haben die gleiche Länge und die benachbarten Kanten treffen sich in gleichen Winkeln in ähnlicher Weise zu der isometrischen Axonometrie.

Die Titelblattfigur des Journal of Geometry kann offenbar als ein zweidimensionales Bild des vierdimensionalen Würfels verstanden werden, besser gesagt als dessen dreidimensionales Gitter, wenn wir die dargestellten Deckungen der Kanten in Betracht ziehen.

In der von den Bildern der vier verschiedenen Kanten gegebenen Richtungen sind zwei-zwei kongruenten Monge-Bilder des 3D Raumgitters aufzunehmen so, daß wir voraussetzen, daß die Kanten mit gleichen Längen zur ersten Bildebene in gleichem, freigewähltem Winkel stehen. Auf Grund dieser Voraussetzungen, kann solch ein *k*-dimensionaler Würfel in einem 2D Bild dargestellt werden, das ähnlich zu der isometrischen Axonometrie, die folgenden Eigenschaften hat: die Bilder der benachbarten Kanten schließen gleiche Winkel ein, jede Bildkante hat die gleiche Länge, das Bildkontur zeigt ein reguläres Polygon, dessen Seitenzahl *k* oder um die zusammenfallenen Bildpunkten möglichst zu reduzieren, 2***k*** ist. Um den 3D Gitter zu konstruieren müssen die Kanten in gleichem Winkel von der Bildebene aufgestellt werden.

Die Kanten können natürlich mit beliebigen Längen und Winkeln gewählt werden und mit entsprechenden Verschiebungen sind die dreidimensionalen „axonometrischen Gitter” der mehrdimensionalen Würfel herzustellen, wie auch die früher erörterten, von den regulären Körper stammenden Formen. Im Interesse der schnellen Konstruktion und um die diesbezüglichen darstellerischen Möglichkeiten des AutoCAD auszunutzen, habe ich ein Autolisp-Program geschrieben.

Die Frage, in wie weit – nicht nur auf grund der Analogien – diese zwei- und dreidimensionalen Bilder als durch mehrdimensionale parallele Projektionen entstandene (isometrische) Axonometrien benannt werden können, ist noch offen, da die Gültigkeit des Satzes von K. Pohlke in mehrdimensionalen euklidischen Räumen beschränkt ist. [1], [2]

[1] Heinrich Brauner: *Zum Satz von K. Pohlke in n-dimensionalen euklidischen Räumen* ÖAdW Math-naturw. Klasse *Sonderdruck aus Sitzungsberichte, Abt. II*, Band 195, Heft 8-10 *Wien 1986 in Komm. bei Springer-Verlag Wien New York*

[2] Hellmuth Stachel: *Mehrdimensionale Axonometrie,* Proceedings of the Congress of Geometry, *Thessaloniki*, 159–168 (1987)

DISCOVER AND FORGET INSTEAD OF APPLY – THE BASIC PROBLEM OF TODAY'S GEOMETRIC RESEARCH

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Subject: classical geometry in general, projective geometry, geometry of linear and nonlinear mappings, algebraic line geometry, politics

Geometric research was and is mostly concerned with tiny scientific details, even they are diamonds to us. Of course, some few discoveries and ideas have turned out to be trailblazing, like e.g. the concept of Bézier curves, and they continuously stimulate further research on "details" by generalizing prerequisites or improving applicability or other modifications of the original results. But even we praise such results by emphasizing its obvious (but unfortunately still theoretical) multiple applications here and there, we find them hardly integrated into manufacturing procedures and industrial software. Why it is so difficult to convince industry people of the advantages of our discoveries? The reasons could be:

- Lack of communication abilities (on both sides!)
- Insufficient presentation / immature status of the result
- Lack of advertisement
- Our results are freeware versus applications are patented / industrial secrets

This will be illustrated by some examples of recent "practical" geometric research at diploma thesis level.

Lack of communication is a key problem not only between us and industry, but also between us and other parts of Mathematics, and even between ourselves. Nowadays scientific community lives in a climate coined by "publish or perish", "hire and fire", "fund rising or death", "outsource and privatise". The logical answers are

- Elite Universities; Centres of Competence and/or Innovation; M.Planck-, Fraunhofer-, Boltzmann-Institutes; private MBA-Institutions;…
- Isolation by creating new words for old concepts
- Neglect of basic education at 'classical' European Universities.

Also here some seemingly absurd examples will illustrate the already critical state of the art (of misunderstanding each other).

KONSTRUKTIVE GEOMETRIE

Balatonföldvár, 05–09. 09. 2005

Die Tagungsleitung möchte hiemit ihren herzlichen Dank aussprechennan die Kollegen der Druckerei an der Szent István Universität Ybl Miklós Fakultät für Bauwesen. Wir danken Ihnen für Ihre stets überaus sorgfältige und präzise Arbeit, mit der Sie zum äußeren Erscheinungsbild und damit zum Gelingen unserer traditionellen Tagungen so wesentlich beitragen. Nicht ganz uneigennützig hoffen wir, dass Sie diese Arbeit bei guter Gesundheit auch noch viele Jahre lang für unsere Konferenzen verrichten können.

Für die Tagungsleitung

We, members of the Organizing Committee of the 5th Conference of Constructive Geometry, want to express our sincere gratitude to the printing office at the Szent István University Ybl Miklós Faculty of Building Sciences. We feel deeply thankful for your careful and precise elaboration of the conference materials. This contributes essentially to the success of our conferences. We wish you good health and hope that also in the future you will able to support our conference by your proficiency.

For the organizing committee

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