1. Let G be a group and suppose that $x^2 = 1$ for every element $x \in G$. Prove that G is Abelian.

2. Determine all possible orders of elements in the groups
a) $(\mathbb{R} \setminus \{0\}, \cdot)$;
b) $(\mathbb{R}, +)$;
c) $(\mathbb{C} \setminus \{0\}, \cdot)$;
d) $GL(2, \mathbb{R})$;
e*) $GL(2, \mathbb{Q})$?

3. Prove that every group of even order has an element of order 2.

4. Let g be an element of order $o(g) = n$ in a group, and let $k \in \mathbb{Z}$. Prove that
a) $o(g^k) = \dfrac{n}{(n,k)}$;
b) $\langle g^k \rangle = \langle g \rangle \Leftrightarrow (n,k) = 1$.
Formulate and prove the corresponding statements for elements of infinite order.

5. Let $\emptyset \neq H \subseteq G$. Prove that $H \leq G$ if and only if $HH = H$ and $H^{-1} = H$.

6. Let A and B be subgroups of G. Prove that $AB = \{ab \mid a \in A,\ b \in B\}$ is a subgroup of G if and only if $AB = BA$.

7. Let G be a finite group and A and B two subgroups of G. Prove that $|AB| = \dfrac{|A| \cdot |B|}{|A \cap B|}$.

8. Determine the order of the permutation $(1345)(236)(41)$.

9. Determine the number of cyclic subgroups in the symmetric group S_4.

10. Prove that any infinite group has infinitely many subgroups.

11. Let K be a field with q elements. What is the order of the general linear group $GL(n, K)$ of all invertible $n \times n$ matrices over K and of the special linear group $SL(n, K)$ of all $n \times n$ matrices over K with determinant 1?

12*. Find elements of order 2, 3 and 7 in the group $GL(3, 2)$. Prove that $GL(3, 2)$ has no element of order 6.

HW1. Do the element of the open interval $(-1, 1)$ form a group with respect to the multiplication $a * b = \dfrac{a+b}{1+ab}$?

HW2. Determine the number of elements of order 6 in S_7.

HW3. Prove that $o(ab) = o(ba)$ for any elements a, b of a group G.