1. For a group G let $G^\circ = (G, *)$ be the group on the same underlying set with the multiplication performed in the reverse order: $a * b := b \cdot a$. Prove that $G^\circ \cong G$.

2. Let K be a subset of the group G. Prove that at most one subgroup of G can have K as a left coset, furthermore, if K is a left coset then it is also a right coset of an appropriate subgroup of G.

3. Let $\mathbb{Z}$ be the additive group of integers and $m\mathbb{Z}$ its subgroup generated by a number $m > 1$. Describe the cosets of $m\mathbb{Z}$ in $\mathbb{Z}$.

4. Prove that every nontrivial subgroup of C_∞ has a finite index, i. e. it has finitely many cosets.

5. Which of the following statements are true?

a) If $|G| = 81$ and there exists an element $g \in G$ with $g^{29} \neq g^2$ then G is cyclic.
b) If $|G| = 54$ and there exists an element $g \in G$ with $g^{29} \neq g^2$ then G is cyclic.
c) If $|G| = 81$ and there exists an element $g \in G$ with $g^{29} = g^2$ then G is not cyclic.

6. Prove the Dedekind law: for any subgroups A, B and C of a group G, the condition $A \leq C$ implies that $A(B \cap C) = AB \cap C$.

7. Let $\langle X \rangle = G$ and let $\varphi, \psi : G \to H$ be group homomorphisms with $\varphi(x) = \psi(x)$ for all $x \in X$. Prove that $\varphi = \psi$.

8. Let $\varphi : G \to H$ be a group homomorphism and $g \in G$. Prove that $o(\varphi(g)) \mid o(g)$.

9. What can be the order of the image of an element of order 6 when we apply homomorphisms between the following pairs of groups?

a) $C_6 \to C_{15}$ b) $C_6 \to C_{12}$ c) $C_{12} \to C_6$

10. What is the number of different homomorphisms between the following groups?

a) $C_{10} \to C_{33}$ b) $C_n \to C_n$ c) $C_n \to C_m$ d) $C_\infty \to C_n$ e) $C_n \to C_\infty$

11. Prove that conjugate elements of a group always have the same order, i. e. $o(x) = o(y^{-1}xy)$ for every x, y.

12. a) Determine the conjugate $b^{-1}ab$ of the permutation $a = (132)(45)$ by the permutation $b = (254)$.
b) Which of the elements (123), $(4523)(16)$, (4321), and $(24)(314)$ are conjugate to (1234) in S_6?
c) Determine the number of elements g of S_7 such that conjugation by g maps the element $(123)(45)(67)$ to itself. And how many elements map it to $(125)(63)(47)$?

13. Prove the following statements about the conjugacy classes of a group G.

a) The product of two conjugacy classes is the union of some conjugacy classes.
b) If $\mathcal{K}$ is a conjugacy class then so is $\mathcal{K}^{-1}$.
c) A subgroup H is normal if and only if it is the union of some conjugacy classes.
d) Any subgroup generated by conjugacy classes is normal.

14. Let f be a rotation of order 4 in the dihedral group D_4 and t one of the reflections. Prove that $t^{-1}ft = tft = f^{-1}$. Determine the conjugacy classes of D_4. What can we say about the conjugacy classes of D_n in general?